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FUNDAMENTALS OF MACHINE DESIGN

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BY

C. A. NORMAN

PROFESSOR OF MACHINE DESIGN, THE OHIO STATE UNIVERSITY

E. S. AULT

PROFESSOR OF MACHINE DESIGN, PURDUE UNIVERSITY

I. F. ZAROBSKY

PROFESSOR OF MECHANICAL ENGINEERING, UNIVERSITY OF TOLEDO

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PREFACE

In the preparation of this book, emphasis has been placed on the *fundamentals of machine design*, as well as on the application of those fundamentals to the design of machine elements. The various elements are discussed as fully as space permits, and in addition, many illustrative examples have been included to show the application of the elements in composite machinery. Design examples have been given in which not only the element under consideration has been computed and dimensioned, but also related members which may control or influence the design. This procedure is illustrated in the design of the various parts of a screw press as given on page 133.

Throughout the book it has been the aim of the authors to show by means of typical problems the basic principles underlying machine design, and the necessity, in many instances, of modifying a strictly theoretical design when practical limitations are taken into consideration. In conjunction with the analytical treatment, therefore, certain practical data have been included which are not only useful but quite necessary in the solution of many problems in machine design.

Chapter II is devoted to a very brief treatment of the more important principles of the strength of materials. It is hoped that the material in this chapter may be used as a reference for such basic equations as are frequently encountered in design. A chapter on manufacturing processes has been included, since the actual details of design are affected, to some extent at least, by the probable method of manufacture, cost, degree of accuracy obtainable, etc.

The training derived from a study of machine design is exceedingly valuable in developing a good sense of judgment and proportion; the ability to evaluate facts critically and effect compromises; the capacity to analyze a situation logically; in short, to render intelligent decisions and opinions.

C. A. Norman
E. S. Ault
I. F. Zarobsky

November, 1937

ACKNOWLEDGMENT

The illustrations in this book have all been made from tracings furnished by the authors and made from their own drawings. In numerous cases, however, these drawings were detail reproductions of manufacturers' designs, and were mostly intended to show such designs. In these cases credit to the manufacturers has been given immediately under the illustrations, or in the accompanying text. The authors wish to express their great obligation for this material, and for the unfailing courtesy with which it has been furnished.

There are a number of instances, however, in which direct credit has not been given, since the illustrations involved modification of existing designs—modifications for which the makers should not be made responsible. Reference to such designs was, however, often very helpful in such cases and the authors wish to express their obligation to the following firms and individuals:

Professor Oscar D. Rickly of the Ohio State University and Mr. W. E. Miller of the Jeffrey Manufacturing Company for work samples showing designs to meet manufacturing requirements.

Baker-Perkins Manufacturing Corporation, Saginaw, Michigan, for design of screw press.

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Socony-Vacuum Oil Corporation, New York, for bearing details.

Clipper Belt Lacer Company, Grand Rapids, Michigan, and Crescent Belt Fastener Company, New York, for belt fasteners.

Stewart-Bolling & Company, Cleveland, Ohio, for brake design.

Whitney Manufacturing Company, Hartford, Conn., for chain details.

Gleason Works, Rochester, New York, for gear details.

Niles-Bement-Pond Company, Philadelphia, Pa., for crane details.

The authors have drawn copiously on available text books on strength of materials—those of Timoshenko, Morley, Bach, Boyd, and others—and on articles in periodicals. They have also, of course, consulted and found much inspiration in the books of other teachers of machine design, without endeavoring in any case to copy them except for original contributions for which credit is given.

The authors are indebted to Professor C. D. Albert of Cornell University for suggestions; to Dr. A. Nadai of the Westinghouse Electric and Manufacturing Company; and to Mr. H. A. S. Howarth of the Kingsbury Machine Works for their generous help in the elucidation of certain specific points.

THE AUTHORS

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FUNDAMENTALS OF MACHINE DESIGN

CHAPTER 1

DESIGN PROCEDURE AND STANDARDIZATION

1. The Subject of Elementary Machine Design. Machine design has for its object the planning and developing of machines. This planning must be done with full consideration of the natural laws of forces and materials, human needs and desires, and the processes and costs of manufacture. The resultant product must be made available at a reasonable cost and operate properly over a useful life without undue attention. It must be safe to use and be attractive in appearance. Above all, it must perform its functions in such a way as to justify the expenditure of labor and material necessary to plan and to build it.

A thoroughly competent machine designer should have, therefore, a broad scientific training, a keen and practical understanding of human wants, a versatile knowledge of manufacturing methods, and sufficient ingenuity and resourcefulness to combine this knowledge into the production of useful and practical things. To possess all these qualifications is highly desirable; however, the fundamental training in machine design involves a consideration merely of the more specifically technical phases of the subject.

It is these phases that elementary machine design develops as a course of study. Such a study is confined to simple elements that are regularly used in machines of different kinds, parts that recur so frequently that the design of these elements has become somewhat standardized. After the proper mechanism or type of construction has been selected, an analysis must be made to determine the forces that are acting upon the different members. When these forces are known, suitable materials may be selected and the members proportioned to withstand the loads.

Although the design should be such that a minimum of effort and cost will be expended in its manufacture, a study of elementary machine design cannot consider at length the exact methods of production that may be used. Production methods are influenced by the manu-

facturing equipment available, the quantity to be made, the precision required, and other factors. All these factors may not be known at the time of the original design; therefore it may be necessary subsequently to change the design to meet existing conditions. Certain manufacturing processes require special design; if such processes are to be used, the designer should be so informed.

Even though the *method* of manufacture may not be prescribed, the designer must definitely control the character of the product to the minutest detail. He will specify, for instance, the material and its treatment, give complete dimensions and the permissible deviations therefrom, degree of finish, radii of fillets, drill sizes for tapped holes, and other similar information. Many such details were formerly left to the foundry bosses, shop foremen, or even to the workers themselves. Now such items are settled in great detail in the design or production departments, in order that the product may be uniform and according to predetermined specifications.

2. Classification of Machine Drawings. Machine drawings are generally classified as *assembly*, *sub-assembly*, and *detail* drawings. Drawings made for the purpose of determining space requirements or for installation purposes are called *layout* drawings.

An *assembly drawing* is made to show clearly how the various parts of a machine or device are fitted together, and to identify all the component parts in such a way that they may be recognized quickly on detail drawings. Usually, no dimensions are given on an assembly drawing except perhaps overall and locating dimensions.

A *sub-assembly drawing* is an assembly of a small unit of the machine, as for instance the carburetor of an engine.

A *detail drawing* should give all the information necessary to control the fabrication of the part. It should include the specifications of the material, treatment, finish, dimensions, tolerances, quantity required per unit, part number, and any other information that may be necessary. Occasionally, special operations may be listed with a tabulation of the tools required. Generally, a single detail drawing is used for both the forging or casting and the finished part. In some cases, the amount of material to be removed in finishing is indicated on the drawing. Many engineering departments require each part to be detailed on a separate sheet, although the more economical practice of including a number of related parts on one sheet is quite common. Every individual part of the machine must be detailed, except parts which are commercially standard and readily procurable in the open markets, like bolts, nuts, washers, etc.

3. Design Procedure—Theoretical. When the need for a new machine arises, we may modify the design of an existing machine, or design a new machine entirely from a fundamental analysis. Usually we are able to combine in a new design many proven devices, and thus accomplish our purpose without the necessity of developing entirely new ones.

As in all good engineering procedure, we start with the known facts, decide upon what we wish to accomplish, and devise the best practical means of satisfying the requirements of the problem. If moving parts are involved, we must first consider the form and source of our operating energy, and the form in which we wish to deliver energy. We must then decide on a suitable mechanism or combination of mechanisms to effect this change. Having selected a suitable mechanism, we may proceed with the second step, an analysis of the forces acting upon the members of the machine.

The loads acting upon machine members may be a combination of the forces resulting from the energy transmitted, the weight of parts, the forces of assembly, inertia forces, friction, forces due to temperature changes, impact, etc. Obviously, all these different forces may not be present in a given machine. Yet sometimes the incidental forces may be of greater magnitude than those essentially required for the transmission of power. In an automobile engine, the forces on the bearings due to the inertia of the moving parts may exceed the forces resulting from the working fluid. Valve-seat inserts in an aluminum cylinder head are subject to shrinkage stresses from assembly, temperature stresses due to different rates of expansion of the head and seat, and impact stresses due to valve closure.

Knowing the resultant maximum forces acting upon each member, we may select suitable materials after carefully considering such requirements as hardness, wear resistance, corrosion resistance, weight, electrical properties, cost, etc. A suitable factor of safety is then selected, either on the basis of other proven designs or by an analytical investigation. This factor depends upon the reliability and characteristics of the material and workmanship, the degree of accuracy in determining stresses, hazards that would result if failure occurred, and the presence of impact, and repeated variations of load. By dividing the ultimate strength of the material by the factor of safety, the working stress is established.

After determining the loads and allowable working stresses, we may compute the necessary proportions of the component parts by applying the principles of the Strength of Materials. Many of these calculations are relatively simple, but some become very involved. Nevertheless, proportions should be computed when possible.

4. Design Procedure—Practical. Even when the parts are carefully designed upon a theoretical basis, it frequently becomes necessary to alter the computed proportions to conform with past experience or with factors not allowed for in the calculations, such as the corroding of thin parts, refinishing, or difficulties of casting. As an example, in determining the wall thickness of a cast-iron pump cylinder subjected to a pressure of several hundred pounds per square inch, it would be better to make the thickness about 3/8 of an inch even though the theoretical thickness might be 1/8 inch. In casting a cylinder block of this type, a slight displacement of the cores would produce a defective casting if the walls were too thin, whereas with 3/8 inch walls this condition would not be objectionable. Furthermore, thin walls do not possess sufficient rigidity for machining true surfaces and would not permit reboring when worn. In addition to these manufacturing problems, there is also the tendency for thin walls to "bleed" (allow water to seep through under pressure).

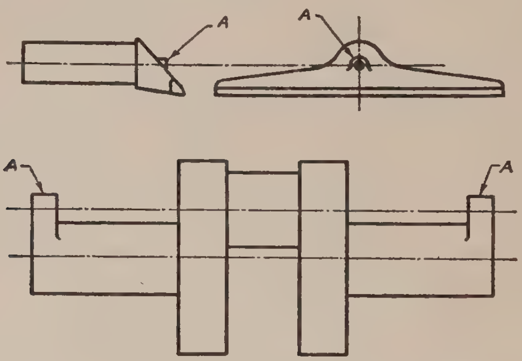


FIG. 1. BOSSES (A) PROVIDED FOR CENTERING.

Dimensions should be rounded out when possible to permit the use of stock sizes or standard parts. In some cases, the design of a machine part is influenced by the requirement that it must be used interchangeably in several different types or sizes of machines. In other cases, the design is purposely altered to facilitate production processes. Bosses or holding flanges are frequently provided on a casting or forging for the purpose of holding it while machining operations are performed, such as the bosses A in Fig. 1, the hook holes in Fig. 2, and the raised lugs for chucking in Fig. 3. In many cases these temporary devices are subsequently removed from the finished part.

5. Standardization. Standardization in engineering may be defined as the adoption of prescribed regulations and specifications as

they pertain to materials, methods, and equipment. This important phase of engineering is evident in such matters as the rules and regulations covering methods of design, symbols to represent forms and materials, operation and maintenance of machinery, material specifications, and standardized shapes and sizes of various kinds of materials and parts as commonly used in practice. Standardization is accompanied by simplification, or the elimination of unnecessary variations and sizes. In order to promote economy, we should require only as many different sizes as are necessary and have these retained sizes as universal as possible. Systems of "preferred numbers," or sizes, have been established for this purpose. The relatively few types of gear-tooth forms, standard pressure angles, standard pitches, and the accompanying standard cutters are examples of simplification to a few variants and the standardization of those retained.

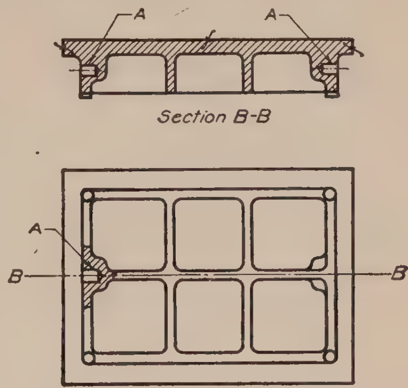


FIG. 2. HOLES (*A*) PROVIDED FOR HOOK BOLTS.

Standards may be established internationally, as for ball bearings; nationally, as for screw threads; within an industry, as for hoisting rope; or within a company, as illustrated by the various drafting-room standards of machine parts (hand wheels, for instance). The advantages of standardization in reducing cost, simplifying replacement, and reducing the quantities of materials carried in stock are so evident that no lengthy discussion is necessary. Standardizing upon particular shapes may enable the use of extruded or rolled sections with a consequent elimination of machining operations. Machinery may be so designed that parts are interchangeable in different sizes and models. The designer should always be on the alert to use parts for which patterns and dies are available, instead of making new designs which serve the purpose no better.

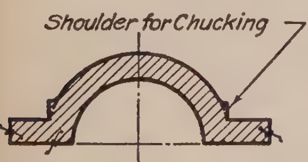


FIG. 3. RAISED LUGS PROVIDED FOR CHUCKING.

Naturally, the more diversified the group cooperating in establishing standards, the fewer are the items that can be considered. The number of topics covered in the standards issued by the American Standards Association in conjunction with the American Society of

Mechanical Engineers is rather small and covers, in general, fired and unfired pressure vessels, drawing methods, conventional symbols, metal fits, pipe and pipe fittings, milling cutters, taps, T-slots, and transmission shafting. Many of these standards are only tentative; some are not observed by manufacturers through ignorance, indifference, or disagreement. One of the most useful and important standards should be the one on metal fits, yet it is not universally used because of lack of agreement. It is difficult, of course, to persuade manufacturers to discontinue the use of their own standards and to adopt those of a competitor. One should realize that a standard is never more than an expression of the best judgment available at the time of its adoption. It is not only possible, but highly probable, that further developments will lead to a deeper insight and improved practice. Standards should never be allowed to impede such progress.

6. Standard Metal Fits. Many standards which pertain to particular machine elements will be referred to in the chapters devoted to these elements. We shall confine our discussion at present to standard metal fits, selected because of their wide application. Although the standard for "Tolerances, Allowances, and Gages for Metal Fits" has been much criticized, it is well to explain its principles here.

The tentative American Standard, as established, is the so-called *basic hole* standard. This designation means that the nominal diameter, which is usually a round number, is the minimum diameter of the hole and the base from which the other dimensions are derived. From this minimum diameter there is subtracted, for running fits, an allowance to give the maximum shaft diameter. The *allowance*, therefore, is the minimum diametral clearance provided to give the proper functioning of the shaft in the hole. For *running fits*, the dimension given is always the maximum shaft size supplemented by the *tolerance*, or permissible inaccuracy, in such a direction as to decrease the shaft size and increase the clearance. Thus the hole has a *plus* tolerance, that is, the hole may be slightly larger than the nominal diameter, and the shaft has a minus tolerance, that is, the shaft may be slightly smaller than the nominal diameter.

For *tight fits*, the nominal diameter is the minimum hole size and a *plus* tolerance is applied to this hole, as for running fits. The minimum shaft diameter is obtained by adding an "average interference of metal" to the minimum hole size, and the tolerance *added* to this dimension gives the maximum shaft diameter. Observe in Table 1, which gives the values for the various fits, that the hole tolerance is the same in the different classes of tight fits and only the shaft tolerance

TABLE 1

FORMULAS FOR RECOMMENDED ALLOWANCES AND TOLERANCES
(From Bulletin B, 4a-1925, American Engineering Standard Committee)

Class of Fit	Method of Assembly	Allowance	Selected Average Interference of Metal	Hole Tolerance	Shaft Tolerance
(1) Loose	Strictly interchangeable	$0.0025D^{2/3}$		$+0.0025D^{1/3}$	$-0.0025D^{1/3}$
(2) Free	Strictly interchangeable	$0.0014D^{2/3}$		$+0.0013D^{1/3}$	$-0.0013D^{1/3}$
(3) Medium	Strictly interchangeable	$0.0009D^{2/3}$		$+0.0008D^{1/3}$	$-0.0008D^{1/3}$
(4) Snug	Strictly interchangeable	0.0000		$+0.0006D^{1/3}$	$-0.0004D^{1/3}$
(5) Wringing	Selective assembly		0.0000	$+0.0006D^{1/3}$	$+0.0004D^{1/3}$
(6) Tight	Selective assembly		$0.00025D$	$+0.0006D^{1/3}$	$+0.0006D^{1/3}$
(7) Medium force	Selective assembly		$0.0005D$	$+0.0006D^{1/3}$	$+0.0006D^{1/3}$
(8) Heavy force or shrink	Selective assembly		$0.0010D$	$+0.0006D^{1/3}$	$+0.0006D^{1/3}$

D = diameter of fit in in.

and interference vary. It will be noted that all tight fits with the exception of the snug fit are for selective assembly.

Criticism has been made of this system that a variety of reamers is required to provide the tolerances necessary for the same basic hole diameters in the different classes of running fits. Many engineers feel that this requirement is an unnecessary complication, and that the accuracy of the hole should always be taken as the one provided by a standard reamer. A more general criticism of the basic-hole system is that it is impractical to follow at times, and in such cases the shaft should furnish the basic dimension. Consider, for instance, a piston pin that is to be a tight fit in the piston bosses and is to carry a free-running connecting rod between the bosses. The basic-hole system would require the holes in the piston and in the connecting rod to be of the same basic diameter, while the diameter at the ends of the pin would have to be slightly larger than the diameter at its midlength. This condition would be an impractical arrangement for both manufacture and assembly. The simplest solution is to make the pin of uniform diameter and to provide the necessary variation in the holes to give the proper fit.

EXAMPLES. (1) An 8 in. shaft is to have a medium running fit in its bearing. From Table 1,

$$\text{Hole tolerance} = 0.0008D^{1/3} = 0.0008 \times 2 = 0.0016 \text{ in.}$$
$$\text{Shaft tolerance} = 0.0008D^{1/3} = 0.0008 \times 2 = 0.0016 \text{ in.}$$
$$\text{Allowance} = 0.0009D^{2/3} = 0.0009 \times 4 = 0.0036 \text{ in.}$$

The dimensioning of these two parts is shown in Fig. 4.

(2) A 1 in. shaft is to have a snug fit in its bearing. The hole tolerance is + 0.0006, giving hole dimensions of 1.0000 to 1.0006 in. The maximum shaft size is 1.0000 in. and the shaft tolerance − 0.0004, which gives a minimum shaft dimension of 0.9996 in. The tightest fit occurs with the maximum shaft combined with the minimum hole, resulting in no clearance whatever. The freest fit occurs with the minimum shaft combined with the maximum hole, giving a clearance of 1.0006 − 0.9996 = 0.0010 in. This fit is *interchangeable*, meaning that any combinations of hole and shaft within the indicated limits will function satisfactorily. The snug fit is intended to hold parts together without perceptible looseness and without permitting free motion under load, yet to assemble and adjust readily.

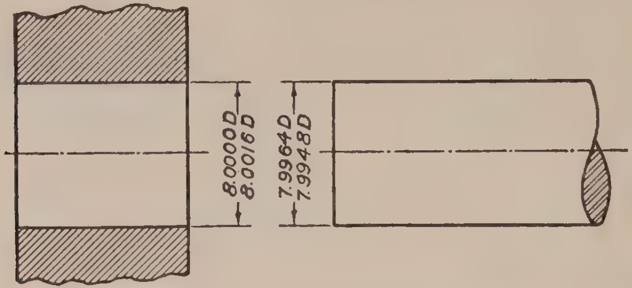


FIG. 4. DIMENSIONS FOR A MEDIUM RUNNING FIT.

(3) With the force fit, the shaft is larger than the hole, and the parts are held together in a fixed connection, not readily taken apart or adjusted after assembly. With reasonable accuracy of manufacture, the tolerances will be of the same order of magnitude as those specified, and unless the two engaging parts are selected to give the desired amount of force fit, the connection may be too loose; or, if we take the other extreme, when the parts are forced together, the stresses set up may be too great.

Consider the medium force fit as an example. For a 1 in. shaft, the minimum hole diameter is 1.0000 in. and the maximum hole is 1.0006 in. The desired metal interference for a 1 in. shaft is 0.0005, which gives a minimum shaft diameter of 1.0005 in. It will be seen that the minimum shaft size is actually 0.0001 in. loose in the maximum hole, whereas the maximum shaft is 0.0011 in. tight in the minimum hole. The latter is more than twice the desired amount of tightness. Both conditions are equally undesirable and should be avoided by selective fitting of the mating parts.

PROBLEMS

1. Define: Tolerance, allowance, force fit.
2. Give the dimensions for hole and shaft for the following conditions:
   (a) A 1/2 in. journal and bearing in a domestic machine.
   (b) A 2 in. journal and bearing in an agricultural machine.
   (c) A 1½ in. journal and bearing in an electric generator.
   (d) A wringing fit for a 7/8 in. shaft and hub.
   (e) A railroad car wheel with a force fit on an 8 in. shaft.
   (f) A snug fit for a 5/8 in. pin in a hole.
   (g) A medium running fit for a 4½ in. crankpin and bearing.

3. Give an important reason for preferring the hole as a basis for a system of fits to that system using the pin as a basis. What difficulty sometimes arises with the hole basis system?

4. What kind of a fit would you select for the following:

(a) Motor armature on shaft.
(b) Gear keyed to shaft.
(c) Machine-tool journal.
(d) Close hand fit.
(e) Engine parts.

5. What are component drawings? What are the two major functions of component drawings?

6. What determines the degree of interchangeability desired in a manufactured product? How is interchangeability secured?

CHAPTER 2

STRENGTH OF MATERIALS

7. Failure. Machine parts may cease to function either by rupture or by permanent deformation. It is necessary to know at what stresses these conditions occur.

If a test specimen is stretched in a tension-testing machine by the application of an increasing load, intermittently relieved, a stress is finally reached at which the specimen does not return to its original dimensions when the load is released. This stress is called the *elastic limit.* This stress is not far from the rupturing stress for cast iron, and it may be fairly close to the rupturing stress for other hard and brittle materials like high-carbon steel, especially when hardened. It is considerably below the rupturing point for ductile materials, like low-carbon steel and, particularly, annealed low-carbon steel.

If a steadily increasing load is applied to a test specimen and the *tensile stress* s_t is plotted against the *strain* e (the elongation per unit length), there may be a linear relation between stress and strain at the beginning as shown in Fig. 5. In this region the relation is expressed by *Hooke's Law:*

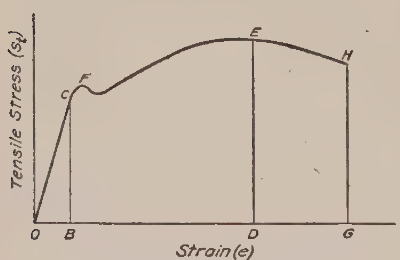


FIG. 5. STRESS-STRAIN CURVE FOR DUCTILE STEEL.

$$s = Ee. \tag{1}$$

The quantity E is known as the *modulus of elasticity in tension.* The stress at which linear proportionality ceases is called the *limit of proportionality* (C in Fig. 5). For steel and some other materials it happens to be very nearly equal to the limit of elasticity and it has become customary to refer to it as the *proportional elastic limit,* or simply as the *elastic limit.* This practice is unfortunate. In the case of cast iron the limit of proportionality is practically zero (see Fig. 6), whereas the elastic limit is close to the rupturing stress. In many other materials the two limits differ. It is better to distinguish between them by using different terms.

If permanent deformation is to be avoided, no stress in a machine member must exceed the elastic limit.

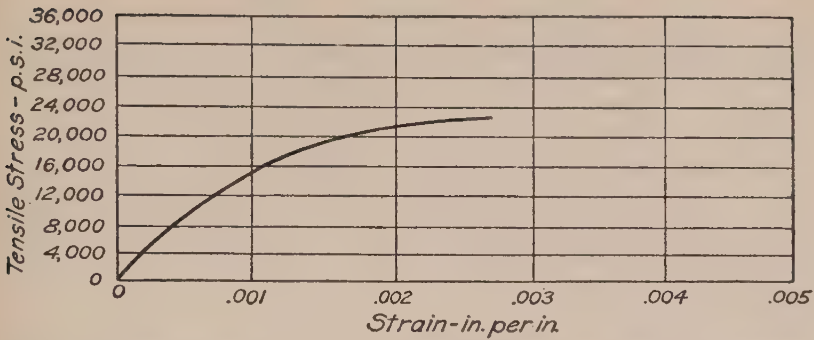


Fig. 6. Stress-Strain Curve for Average Gray Cast Iron in Tension.

In ductile materials, the material may *yield* or *flow* without increase of stress at a point somewhat above the limit of proportionality (F in Fig. 5). The stress at which this condition occurs is called the *yield point*. The material may flow even if the stress, after reaching the yield point, is somewhat reduced.

Hard or inelastic materials exhibit no true yield point, but it has become quite customary to call the limit of proportionality, or a somewhat arbitrary point slightly above the limit of proportionality, the yield point for such materials. This practice obtains, for instance, on data given in the *Handbook of the Society of Automotive Engineers*, on the properties of materials.

After passing through the yield point, the test specimen exhibits increasing resistance until the *ultimate strength* or *maximum stress* is reached (E in Fig. 5). Beyond the maximum stress, the curve *may* drop as at H. This is due to the fact that the specimen *necks* and the cross-sectional area is reduced. The stress, however, is defined as the pulling force divided by the original area. Consequently, even though the actual stress in the material is increased, the plotted stress decreases. Hard and brittle materials exhibit no necking and the plotted stress increases to the rupturing point (Fig. 7).

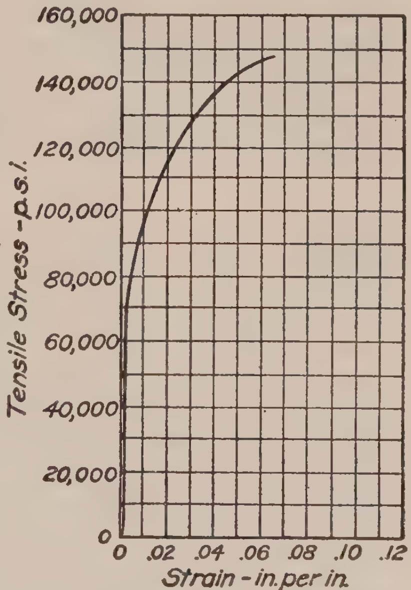


Fig. 7. Stress-Strain Curve for Hard Steel.

8. Tensile and Shear Strength. The tensile stress s_t on the bar shown in Fig. 8 is defined as the force F divided by the area A at right angles to the force. Along any area as A_s inclined to A, the force F has a shear component F_s. When a force causes a sliding action, parallel to the section considered, it is termed a shear force. The *shear stress* s_s on this area is F_s/A_s. The shear stress is a maximum at 45 deg. and is equal to $(F/\sqrt{2})/(A\sqrt{2}) = F/(2A)$. Hence

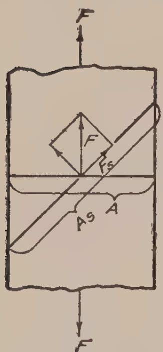


FIG. 8. TENSILE AND SHEAR STRESSES.

$$(2) \qquad s_s = \frac{s_t}{2}.$$

The cupped or conical fracture of ductile materials seems to indicate that such materials fail along shear areas rather than along areas in straight tension. Equation (2) indicates that the material in such cases must *be assumed to be half as strong in shear as in tension.*

Direct experiments by J. J. Guest * indicate that when a ductile material is subjected to various stresses simultaneously the resultant maximum shear stress determines failure, whereas with a brittle material the maximum direct stress (tension or compression) determines failure.

In case there is a tensile or compressive stress s_t and a pure shear stress s_s, then the resultant *maximum shear stress* is

$$(3) \qquad s_{s\ max} = \sqrt{\frac{s_t^2}{4} + s_s^2},$$

and the resultant *maximum tensile stress* is

$$(4) \qquad s_{t\ max} = \frac{s_t}{2} + \sqrt{\frac{s_t^2}{4} + s_s^2}.$$

Due to the term $s_t/2$, the maximum tensile stress is always greater than the maximum shear stress, since the root term is the same in both cases. However, it is always less than twice the maximum shear stress, except when s_s is zero. The shear stress is always the more serious one in ductile materials, since it is assumed that such materials are only half as strong in shear as in tension.

If we proportion a machine part of ductile material so that it will withstand the maximum shear stress without failing, it will always have sufficient strength. Shafts are usually subjected to both bending and torsional shear, and the standards of the American Society of Me-

* Philosophical Magazine, July, 1900.

chanical Engineers (A.S.M.E.) provide that the size of shafts made of ductile materials shall be determined by the maximum shear formulas. If made of hard and brittle materials, it may be satisfactory to compute them to withstand the maximum tension only.

It has been known, however, that hollow cylinders, for instance, have somewhat more strength than a maximum shear formula would indicate. Researches by Nadai, Lode, and others lead to the conclusions that sufficient safety will be obtained if ductile bodies are proportioned on the basis of the *deformation work* set up by the stresses, rather than for maximum shear. The deformation work per cubic inch for a material subjected to a stress s and a strain e is equal to $se/2$, or $s^2/(2E)$. It has been found that in ductile materials the *shear deformation* determines failure. These shears are equal to half the difference between the tensions (or compressions) on three planes at right angles, along which there are no shears. These tensions or compressions are called *principal* stresses. If we call the principal stresses s_1, s_2, and s_3, the condition for yielding of ductile materials is

$$(s_1 - s_2)^2 + (s_2 - s_3)^2 + (s_3 - s_1)^2 = 2s_0^2, \tag{5}$$

where s_0 is the *yield stress* in pure tension.

While it is important that students should know about this basic relation, the actual figuring usually reduces itself to the computation of a single stress. This stress is known as the *octahedral shear stress*. It is often transformed into an equivalent tensile or shear stress. For a tensile stress s_t and a shear stress s_s, the formula used is either one of the following:

$$\text{(6a)} \qquad s_{s\ \mathrm{def}} = \sqrt{\frac{s_t^2}{4} + \frac{3s_s^2}{4}} \qquad \text{or} \qquad \text{(6b)} \qquad s_{t\ \mathrm{def}} = \sqrt{s_t^2 + 3s_s^2}.$$

The computed $s_{s\ \mathrm{def}}$ or $s_{t\ \mathrm{def}}$ must not exceed the permissible working stresses in shear or in tension. It will be noticed that $s_{s\ \mathrm{def}}$ is always somewhat smaller than $s_{s\ \max}$; hence a saving in material results if one regards $s_{s\ \mathrm{def}}$ as determining failure.*

In writing formula (5) it was stated that half the difference between the principal stresses (in tension or compression) equaled the shear stresses. It is clear then that the *maximum shear stress* is half the difference between the largest and the smallest principal stresses. Since compressions are negative tensions, and a negative stress is always algebraically smaller than a positive stress, we have in the

* For further information about failure and particularly about the deformation theory see A. Nadai, *Theories of Strength*, Transactions A. S. M. E., July–September, 1933, APM–55–15, p. 111. The authors are under the greatest obligation to Nadai for personal advice in this matter.

case where both compressions and tensions are present a maximum shear stress equal to half the sum of numerical values of the largest tension and the largest compression.

Example 1. A brittle material is subjected to a tensile stress of 10,000 p.s.i. and a shear stress of 3000 p.s.i. If this material has a working strength in straight tension of 12,000 p.s.i., is it safe under the above stresses?

Directions. Compute the maximum tensile stress by formula (4). If it exceeds the 12,000 p.s.i., the material is unsafe.

Example 2. A ductile material is subjected to the stresses in Example 1 and has the same strength. Is it safe or unsafe?

Directions (a). If exceptional safety is required, or if it is decided to adhere to the directions of the A.S.M.E. shafting code, which requires the use of the maximum-shear theory, compute the maximum shear stress by formula (3). Since the material is assumed to be only half as strong in shear as in tension, the maximum shear stress so computed must not exceed 12,000/2 = 6000 p.s.i. As an alternative, we could also compute twice the value of the maximum shear stress. This value is $\sqrt{s_t^2 + 4s_s^2}$ and obviously must not exceed 12,000 p.s.i.

Directions (b). If greater economy of dimensions is desired, compute the equivalent deformation stress $\sqrt{s_t^2 + 3s_s^2}$. For safety this value must not exceed the allowable tensile stress of 12,000 p.s.i.

Example 3. A pressure of 1000 p.s.i. on the inside of a boiler shell causes a hoop tension of 10,000 p.s.i. and a longitudinal tension of 5000 p.s.i. What is the maximum shear stress at the inside of the shell?

Solution. The maximum shear stress is half the difference between the largest and the smallest stresses. In this case, the smallest stress is the compression of 1000 p.s.i., due to the pressure. This stress is negative, therefore

$$s_{s\,\max} = \frac{10,000 - (-1000)}{2} = 5500 \text{ p.s.i.}$$

Observe that if the pressure had been 8000 p.s.i., we should still have had to regard this compression as the smallest stress, since it is negative. The maximum shear stress would then have been (10,000 − (− 8000))/2 = 9000 p.s.i. If, without changing the hoop stress, there had been a tension of 5000 p.s.i., instead of the compression, then the maximum shear stress would have been (10,000 − 5000)/2 = 2500 p.s.i.

9. Strength Formulas for Shafts. If a solid circular shaft of diameter D is subjected to a bending moment M_b and a simultaneous torsion moment M_t, the tensile (or compressive) stress and the shear stress directly caused by these moments are $s_t = 32M_b/(\pi D^3)$ and $s_s = 16M_t/(\pi D^3)$. Substituting these values in formulas (3) and (6), respectively, we find

$$\pi\frac{D^3 s_{s\,\max}}{16} = \sqrt{M_b^2 + M_t^2}, \tag{7}$$

$$\pi\frac{D^3 s_{s\,\mathrm{def}}}{16} = \sqrt{M_b^2 + 0.75M_t^2}. \tag{8}$$

If we assume that the material is twice as strong in pure tension as in shear, and if we call the permissible tensile working stress s_t, we can also write under the maximum-shear theory,

$$\pi \frac{D^3 s_t}{32} = \sqrt{M_b^2 + M_t^2}, \tag{9}$$

and under the deformation theory,

$$\pi \frac{D^3 s_t}{32} = \sqrt{M_b^2 + 0.75 M_t^2}. \tag{10}$$

Formulas (7, 8, 9, 10) apply to ductile materials only. For brittle materials, the formula becomes

$$\pi \frac{D^3 s_t}{32} = \frac{M_b}{2} + \frac{1}{2}\sqrt{M_b^2 + M_t^2}. \tag{11}$$

Obviously, this formula gives a smaller value for D than formula (9), if the stress is the same in both cases. Brittle material, however, has a tensile strength not in excess of its shear strength; hence formula (11) applies. The student should check the derivation of formulas (9) to (11) and apply them to several materials to investigate the effect of different relative stresses.

10. Pure Shear and Transverse Shear. In the formulas so far presented, we have been dealing with pure shear. Such shear is defined as the stress caused in a body when two parallel planes A-A and B-B, some distance apart, are displaced relatively to one another without bending as shown in Fig. 9. If the displacing force is F, and displaced section is A, then the *pure shear stress* is F/A. The shear deformation e_s is defined as h/l, where h is the displacement and l the distance between the two planes. The ratio s_s/e_s is called either the modulus of elasticity in shear, or the *modulus of rigidity*, and is usually denoted by G. This relationship between stress and strain in shear is similar to the relationship in tension and compression expressed by Hooke's law. The stress-strain relationship for shear, within the limit of proportionality, is given by the equation

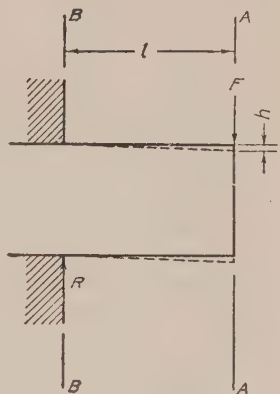


Fig. 9. Shear.

$$s_s = Ge_s. \tag{12}$$

Pure shear occurs only in torsion. In transverse shear, such as in a rivet (Fig. 10), there is bending in addition to the shear. In such cases, the shear stress is not evenly distributed over the section, but is a maximum at the *neutral axis*, where the bending stresses are zero. Since the average stress is F/A, the maximum stress is greater than F/A. For various common sections it is as follows:

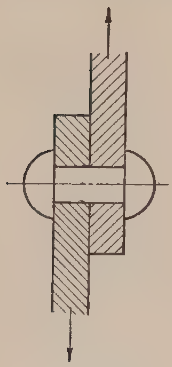


Fig. 10. Transverse Shear in a Rivet.

(13) For a rectangle, $$s_s = \frac{3F}{2A}.$$

(14) For a circle, $$s_s = \frac{4F}{3A}.$$

(15) For a thin circular ring, $$s_s = \frac{2F}{A}.$$

The general formula for the variation of the shear stress is

(16) $$s_{sy} = \frac{F}{Ix}\int_y^{y_1} yx\,dy,$$

in which s_{sy} is the shear stress at the distance y from the neutral axis, x is the width of the section at this distance, and y_1 is the distance to the extreme fiber. I is the moment of inertia of the section. It is evident that the integral gives the moment about the neutral axis of the area beyond the point considered. At the extreme fiber this area is zero; hence the moment and the stress are zero. If there is a massing of area near the extreme fiber, as in an I-beam, there may be a considerable shear stress near this extreme fiber, particularly at a point where the width of the section is suddenly reduced. This is true at the inside of the flange of an I-beam. Since the bending stress is also great at this section, it might be well to investigate the magnitude of the combined stress at this point. The actual distribution of the transverse shear stress in an ordinary rolled I-beam section is shown in Fig. 11 (after Morley). Such an investigation is rarely carried out in actual practice.

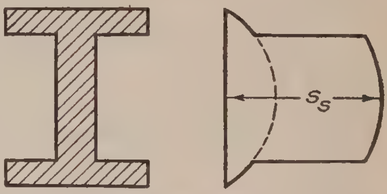


Fig. 11. Distribution of Transverse Shear Stress in an I-Beam.

11. Bending. The fundamental assumptions underlying standard bending formulas are that plane sections at right angles to the neutral axis remain plane in the flexed condition; that the material obeys Hooke's law; and that the modulus of elasticity is the same in tension and compression.

These assumptions are approximately true for steel and a number of other materials, but the last two assumptions are not true for cast iron. Cast iron does not obey Hooke's law (see Fig. 6), and the modulus of elasticity is higher in compression than in tension.

Referring to Fig. 12, plane sections may be passed through the neutral axis to represent the strain graphically. Under the assumption that plane sections remain plane as they rotate during the bending action, the strain in the material is linearly proportional to the distance from the neutral axis, and the trace $A'B'$ of the plane section in the flexed condition represents the strains graphically to some scale. In materials that obey Hooke's law, the same trace also represents the stresses, although of course to another scale, since the stresses are equal to a constant E times the strains. For equilibrium, the sum of the internal tensile forces must equal the sum of the internal compressive forces. This means that for a rectangular section the area AOA' must equal the area BOB'.

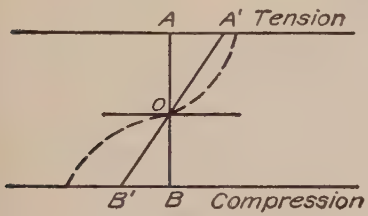


FIG. 12. PLANE SECTION IN BENDING.

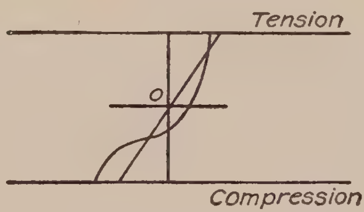


FIG. 13. DISTRIBUTION OF BENDING STRESS IN CAST IRON.

In the case of cast iron, the stresses will not be represented by a straight line, but by curves as in Fig. 13. For the same strain, the stresses on the compression side would be greater than those on the tension side and would be represented by the dotted line in Fig. 12. The sum of the compressive stresses would now be greater than the sum of the tensile stresses, and there will be equilibrium only if the neutral axis is shifted toward the compression side, as shown in Fig. 13. This, however, gives greater moment arms to the tensile stresses than they have in Fig. 12; hence these stresses need not be so great, for equilibrium, as they are in Fig. 12.

Conventional bending formulas applied to cast iron, therefore, give stresses which are greater on the tension side than those actually occurring in the material; hence it is permissable to allow greater computed stresses in bending than in pure tension. Ratios recommended by Bach are given in Table 2.

The standard bending formulas are

$$M_b = \frac{I}{c} s_b, \tag{17}$$

TABLE 2

RATIOS OF PERMISSIBLE COMPUTED BENDING STRESSES TO PERMISSIBLE TENSILE STRESSES IN CAST IRON

	Ratio	
	Finished Sections	Rough Sections
For circular sections	2.0	1.7
For rectangular sections	1.7	1.4
For average I-sections	1.4	1.2

and

$$\frac{EI}{\rho} = M_b. \tag{18}$$

In these formulas M_b is the bending moment, I the axial moment of inertia of the section, c the distance from the neutral axis to the extreme fiber, and ρ the radius of curvature to which the neutral axis is flexed. I/c is also called the *section modulus* and is denoted by Z.

Formula (18) is used for computing the bending stresses in wires and cables, bent to a comparatively small radius of curvature. In the ordinary bending action of machine parts, the flexure is slight and the radius of curvature is very great. The deflection in such cases is regularly computed by formulas derived from the approximate equation

$$EI\frac{d^2y}{dx^2} = M_b, \tag{19}$$

y being the coordinate at right angles to the neutral axis and x the coordinate along the axis.

The bending moment is usually computed from the reactions at the supports. These reactions are obtained by taking moments first about one support and then about the other.

In case of more than two supports, assumptions are usually made to simplify the computation. Thus for multi-throw crankshafts, Güldner, using researches of Ensslin, has found that safe, yet not excessive, moments are obtained by considering the shaft as though it were made of parts, hinged together at the bearings and freely supported.

In Table 3 are given the reactions, moments, and deflections for certain common cases of loading. In Table 4 are given the moments of inertia, the section moduli, and the radii of gyration for certain common sections. If the area of the section is A and the moment of

TABLE 3

REACTIONS, BENDING MOMENTS, AND DEFLECTIONS OF BEAMS

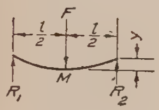
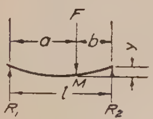
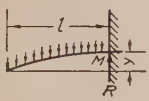
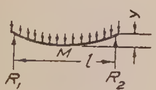
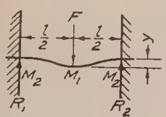
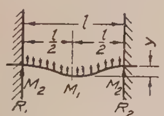
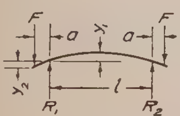
LOAD DIAGRAM	REACTION	MAX. MOMENT	MAX. DEFLECTION
	$R = F$	$M = Fl$	$y = \dfrac{Fl^3}{3EI}$
	$R_1 = R_2 = \dfrac{F}{2}$	$M = \dfrac{Fl}{4}$	$y = \dfrac{Fl^3}{48EI}$
	$R_1 = \dfrac{Fb}{l}$; $R_2 = \dfrac{Fa}{l}$	$M = \dfrac{Fab}{l}$	$y = \dfrac{Fb}{3EIl} \times \left[\dfrac{a(l+b)}{3}\right]^{3/2}$
	$R = wl$ Total Load $= wl$	$M = \dfrac{wl^2}{2}$	$y = \dfrac{wl^4}{8EI}$
	$R_1 = R_2 = \dfrac{wl}{2}$ Total Load $= wl$	$M = \dfrac{wl^2}{8}$	$y = \dfrac{5wl^4}{384EI}$
	$R_1 = R_2 = \dfrac{F}{2}$	$M_1 = M_2 = \dfrac{Fl}{8}$	$y = \dfrac{Fl^3}{192EI}$
	$R_1 = R_2 = \dfrac{wl}{2}$ Total Load $= wl$	$M_1 = \dfrac{wl^2}{24}$ $M_2 = \dfrac{wl^2}{12}$	$y = \dfrac{wl^4}{384EI}$
	$R_1 = R_2 = F$	$M =$ Constant between supports $= Fa$	$y_1 = \dfrac{Fal^2}{8EI}$ $y_2 = \dfrac{Fa^2}{3EI}\left(a + \dfrac{3l}{2}\right)$

TABLE 4

MOMENTS OF INERTIA, SECTION MODULI, AND RADII OF GYRATION OF COMMON SECTIONS

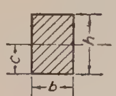
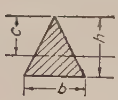
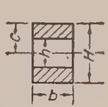
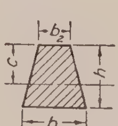
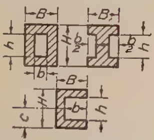
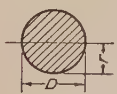
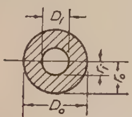
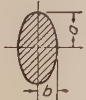
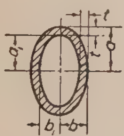
SECTION	MOMENT OF INERTIA I	SECTION MODULUS $\frac{I}{c} = Z$	RADIUS OF GYRATION $\sqrt{\frac{I}{A}} = k$
	$\frac{bh^3}{12}$	$\frac{bh^2}{6}$	$\frac{h}{\sqrt{12}} = 0.289h$
	$\frac{bh^3}{36}$	$\frac{bh^2}{24}$	$\frac{h}{\sqrt{18}} = 0.236h$
	$\frac{b}{12}(H^3 - h^3)$	$\frac{b}{6}\frac{H^3 - h^3}{H}$	$\sqrt{\frac{H^3 - h^3}{12(H - h)}}$
	$\frac{h^3(b_1^2 + 4b_1b_2 + b_2^2)}{36(b_1 + b_2)}$	$\frac{h^2(b_1^2 + 4b_1b_2 + b_2^2)}{12(2b_1 + b_2)}$	$\frac{h(b_1^2+4b_1b_2+b_2^2)}{3\sqrt{3}(b_1^2+2b_1b_2+b_2^2)}$

TABLE 4 (*Continued*)

MOMENTS OF INERTIA, SECTION MODULI, AND RADII OF GYRATION OF COMMON SECTIONS

SECTION	MOMENT OF INERTIA I	SECTION MODULUS $\frac{I}{c} = Z$	RADIUS OF GYRATION $\sqrt{\frac{I}{A}} = k$
	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$	$\sqrt{\frac{BH^3 - bh^3}{12(BH - bh)}}$
	$\frac{\pi D^4}{64} = \frac{\pi r^4}{4}$	$\frac{\pi D^3}{32} = \frac{\pi r^3}{4}$	$\frac{D}{4} = \frac{r}{2}$
	$\frac{\pi}{64}(D_0^4 - D_i^4)$ $\frac{A}{4}(r_0^2 + r_i^2)$	$\frac{\pi(D_0^4 - D_i^4)}{32D_0}$	$\frac{\sqrt{D_0^2 + D_i^2}}{4}$ $\frac{\sqrt{r_0^2 + r_i^2}}{2}$
	$\frac{\pi a^3 b}{4}$	$\frac{\pi a^2 b}{4}$	$\frac{a}{2}$
	$\frac{\pi}{4}(a^3b - a_1^3b_1)$ $\frac{\pi}{4}a^2(a + 3b)t$ (approx.)	$\frac{\pi}{4}a(a + 3b)t$ (approx.)	$\frac{a}{2}\sqrt{\frac{a + 3b}{a + b}}$ (approx.)

inertia is I, the *radius of gyration* is defined as

(20) $$k = \sqrt{\frac{I}{A}}\cdot \qquad \text{(Hence } I = k^2A.\text{)}$$

For particular sections not listed, the moment of inertia about any axis may be obtained from the relationship $I = \int y^2 dA$. Frequently it is more convenient to divide the section into simple sub-areas a_1, a_2, a_3, etc., and get the total moment of inertia by adding the moments of inertia of the sub-areas about the gravity axis of the whole section:

(21) $$I_{\text{total}_{x-x}} = I_{1_{x-x}} + I_{2_{x-x}} + I_{3_{x-x}}.$$

The moment of inertia of an area about an axis x-x parallel to its gravity axis is

(22) $$I_{x-x} = I_g + ad^2,$$

where I_g is the moment of inertia about the gravity axis of the sub-area a and d is the distance between the gravity axis of the sub-area and the gravity axis of the section.

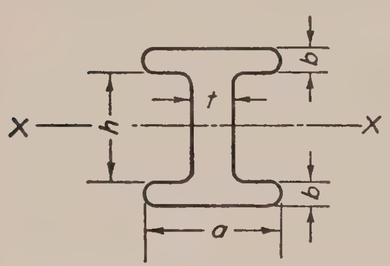


Fig. 14. Moment of Inertia of an I-Section.

Example. Derive the moment of inertia of an I-section such as in Fig. 14 about the gravity axis x-x.

Disregarding fillets and rounded corners, the complete expression for this moment of inertia is

$$\frac{th^3}{12} + 2\left[\frac{ab^3}{12} + ab\left(\frac{h+b}{2}\right)^2\right].$$

12. Torsion. All formulas for the shear stress s_s and the angle of twist ϕ, due to torsion, have the form

(23) $$s_s = \frac{M_t}{K_s},$$

(24) $$\phi = \frac{M_t l}{K_t G},$$

in which l is the length of the twisted member, if of uniform cross section, and K_s and K_t are constants for the section considered. For circular sections, K_s might be called the polar section modulus, and K_t is the polar moment of inertia.* Thus, for a solid circular shaft, K_s

* The symbol J is commonly used for the polar moment of inertia of circular sections.

is $\pi D^3/16$ and K_t is $\pi D^4/32$. For a hollow circular shaft of inner diameter D_i and outer diameter D_o,

$$K_s = \frac{J}{c} = \pi \frac{(D_o^4 - D_i^4)}{16D_o} \quad \text{and} \quad K_t = J = \pi \frac{(D_0^4 - D_1^4)}{32}.$$

In these cases, a plane section before twisting is assumed to remain a plane section when the member is twisted. Experiments on circular sections show that twists and stresses computed with this assumption agree well with those actually occurring. For non-circular sections, however, plane sections warp in the twisting and rather complicated conditions arise.

For thin-walled tubes of wall thickness t, whether of circular or non-circular section, the following formulas * may be used:

$$s_s = \frac{M_t}{2At}, \tag{25}$$

$$\phi = \frac{ms_sl}{2AG}, \tag{26}$$

where A is the area enclosed by the mean perimeter of the section, and m is the length of this perimeter. For the circular section $A = \pi(D_o + D_i)^2/16$ and $m = \pi(D_o + D_i)/2$. In this case, the stress is assumed to be uniformly distributed over the thin section. For non-circular sections, there will be a stress concentration at sharp corners.

For rectangular sections, the following formulas apply:

$$s_{s\,\max} = \frac{M_t}{\alpha bc^2}, \tag{27}$$

or approximately

$$s_{s\,\max} = \frac{M_t(3 + 1.8c/b)}{bc^2}, \tag{28}$$

$$\phi = \frac{M_tl}{\beta bc^3G}, \tag{29}$$

in which b is the longer and c the shorter side of the section. The coefficients α and β depend upon the ratio of b/c and have the values given in Table 5. The shearing stress reaches a maximum value at the middle of the longer side of the rectangular section and is zero at the corners.

* The formulas here given are based mainly on Timoshenko's *Strength of Materials* (Van Nostrand), supplemented by a number of other sources.

TABLE 5

TWISTING CONSTANTS α AND β FOR RECTANGULAR SECTIONS

b/c	1.00	1.50	2.00	2.50	3.00	4.00	6.00	8.00	10.00	∞
α	0.208	0.231	0.246	0.258	0.267	0.282	0.299	0.307	0.313	0.333
β	0.141	0.196	0.229	0.249	0.263	0.281	0.299	0.307	0.313	0.333
s_c/s_b	1.00	0.860	0.795	0.763	0.754	0.745	0.743	0.743	0.743	0.743

s_b and s_c are the torsional shear stresses at the middle of the long side b and short side c, respectively.

It is seen from the table that for very thin sections, formulas (27) and (29) become

$$s_s = \frac{3M_t}{bc^2}, \tag{30}$$

and

$$\phi = \frac{3M_t l}{bc^3 G}, \tag{31}$$

or

$$\phi = \frac{s_s l}{cG}. \tag{32}$$

These equations may be used not only for flat, thin strips, but also for an approximate solution of other thin sections, such as circular arcs, angles, or channels. In such cases, the value for b is taken equal to the mean developed length of the section. For instance, in the case of the channel in Fig. 15, we would substitute for bc^3 the value $b_1c_1^3 + 2b_2c_2^3$. The formula for the angle of twist then becomes

$$\phi = \frac{3M_t l}{(b_1c_1^3 + 2b_2c_2^3)G}. \tag{33}$$

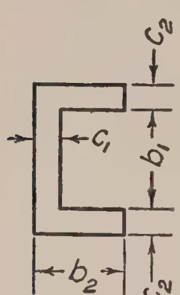


FIG. 15. CHANNEL SECTION.

The formula for the stresses at the middle of the sides is obtained by multiplying together the twist per unit length, the thickness of these sides, and G. For instance, the stress at the middle of the side b_2 is

$$s_s = \frac{3M_t c_2}{b_1c_1^3 + 2b_2c_2^3}. \tag{34}$$

Sharp re-entrant corners will cause stress concentrations, the magnitude of which is obtained by multiplying the stress given by formula (30) by a stress concentration factor. This matter will be discussed in § 31.

For any symmetrical section, the angle of twist, according to Morley, can be obtained with good approximation by the formula

$$\phi = \frac{40JM_t l}{A^4 G}, \tag{35}$$

where J is the polar moment of inertia and A is the cross-sectional area.

13. Keyways. In § 31, it is noted that serious stress concentrations may occur at the re-entrant corners of keyways, unless ample fillets are provided. According to the results of tests conducted by Moore,* the ratio of the presumable torsional strength of a solid circular shaft having an ordinary keyway to the strength of the same sized shaft without a keyway may be expressed by the formula

$$e = 1.0 - 0.2w - 1.1h, \tag{36}$$

in which e is the relative strength, w the ratio of the width of the keyway to the shaft diameter, and h the ratio of the depth of the keyway to the shaft diameter. The angle of twist is increased in the ratio c, expressed by the equation

$$c = 1.0 + 0.4w + 0.7h. \tag{37}$$

14. Columns. Euler's column formula was the first theoretical equation used in the design of columns. It has the general form

$$F = \frac{m\pi^2EI}{l^2} = \frac{m\pi^2EA}{(l/k)^2}, \tag{38}$$

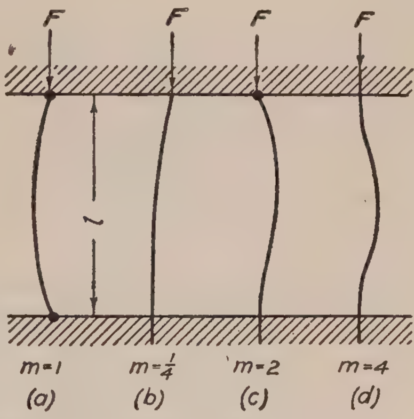


Fig. 16. Types of End Connections for Columns.

in which F = collapsing load on the column in lb., l = the length of the column in in., k = the least radius of gyration of the section = $\sqrt{I/A}$, I = the least moment of inertia of the section, A = the area of the section in sq. in., and m = a constant depending upon the end conditions of the column, as follows:

Both ends pivoted or hinged (Fig. 16*a*) . . . . $m = 1$.
One end fixed and other end free (Fig. 16*b*) . . . . $m = 1/4$.
One end fixed and other end pivoted (Fig. 16*c*) . . . . $m = 2$ (approx.).
Both ends fixed (Fig. 16*d*) . . . . $m = 4$.

Structural engineers assume such loadings as these and columns are designed accordingly. In machine design, however, it is usually

* Bulletin No. 42, Engineering Experiment Station, University of Illinois.

assumed that clamping is not very effective, and in the important cases of pitmans and connecting rods, the ends are of course pivoted. Euler's formula as generally used in the design of machine elements is, therefore, $F = \pi^2 EI/l^2$.

Euler's formula contains no allowance for compressive stress. It is, therefore, strictly applicable to long and slender columns only, in which the buckling action entirely predominates over the direct compression. This condition is found to be the case when the slenderness ratio l/k is roughly 100 or more. Euler's formula can be used for shorter columns if a very large factor of safety is introduced, but it is better in such cases to use formulas containing a direct allowance for compressive stress.

A safe formula of this type, and one that can always be used, is known as Rankine's formula:

$$F = \frac{s_c A}{1 + q(l/k)^2}, \tag{39}$$

where s_c is the compressive stress and q is a constant, which for steel should be taken equal to 0.0006. This value is based on investigations by Heldt and Bauschinger and allows for such unintentional eccentricities of loading as are apt to occur in practice.

For structural columns, the *straight-line* type of formula is in common use. The American Railway formula is representative of this group, the equation being

$$\frac{F}{A} = 16{,}000 - 70\frac{l}{k}, \tag{40}$$

in which F/A is the permissible load or stress per unit area, and is limited to a maximum value of 14,000 p.s.i.

15. Fatigue. If materials are subjected to a varying load for a period of time, they are found to break eventually at a stress lower than the ultimate static strength. Such a failure is commonly known as *fatigue* failure. The first comprehensive investigations of this matter were carried out by Wöhler and were published in Germany between 1860 and 1870. Wöhler used a testing apparatus that somewhat duplicated the action in pitmans or connecting rods and hence may have involved some impact. For this reason, it has been assumed that his test results show a somewhat greater strength reduction, due to fatigue, than would be caused by mere fluctuation of stress.

Moreover, it has been found that lack of surface finish reduces fatigue strength, and since only a highly polished surface can be specified and duplicated with some exactness, modern tests have been

largely carried out with polished specimens. Wöhler's specimens were not highly finished, and this condition tended to make the values of his fatigue strength low compared to those obtained under very favorable conditions. Wöhler's results are safe and the factors as given in Table 6 should be used for ordinary design conditions in which surfaces are not highly polished, or where some impact or corrosive influence may be present.

In the table are listed ratios of fatigue strength to static tensile strength considered safe under the conditions indicated. They compare closely with values recommended by Bach and are much used in design, particularly on the European continent.

TABLE 6

Ratio of Static Strength to Fatigue Strength

Material	Nature of Stress		
	Static	Released	Reversed
Ductile	1.0	1.8 to 2.0	3.0
Brittle	1.0	2.5	4.0

A "released" stress varies from zero to a maximum in one direction only. A "reversed" stress varies between equal values of tension and compression.

Fatigue rupture is caused by the spreading of a local defect or weakness, often resulting from a highly localized stress. Such a weakness is less serious in ductile materials than in brittle materials for the reason that a ductile material can flow or readjust itself around the weak spot, whereas a brittle material will break off sharply. Consequently, ductile materials under fatigue action are found to retain a greater percentage of their strength than brittle materials.

It is the range of stress variation that determines failure in fatigue, and this range grows less the higher the mean stress in the material becomes.* For instance, if the material is subjected to a mean stress equal to the static yield point, no range of variation at all is necessary to cause failure. On the other hand, if the mean stress is zero, that is, if the material is subjected to a stress varying between equal values in tension and compression, then the range which causes failure is the sum of the tension and compression stress at the point where breakdown in fatigue occurs. The rupturing stress in fatigue for fully reversed stress is now usually called the *endurance* strength of the material.

* See Timoshenko, *Strength of Materials*, vol. 2, p. 680 and p. 719.

At the present time, endurance strength is usually determined by subjecting rotating, polished test specimens to a transverse bending load.* Without shock or suddenness, the stress in the extreme fibers in such a specimen will gradually pass from a certain value in tension to an equal value in compression. The inner fibers are less highly stressed, however, and may lend supporting strength to the extreme fibers. The endurance strength under these conditions is commonly found to be about half of the ultimate static strength.

Since the adoption of the rotating beam type of endurance test, there has been a tendency to take the *endurance strength of well finished specimens at about one half the ultimate in all cases.*

According to tests by Haigh and others (Engineering, Sept. 21, 1917) a rough surface may reduce the endurance strength to about 40 per cent of the ultimate. If, however, a corroding agent such as salt water, ammonia, or hydrochloric acid is applied simultaneously with the alternating stress, the endurance strength is still further reduced, and sometimes in a very serious manner. Chromium added to steel reduces corrosion and also improves the endurance under the action of corrosive agents.

16. Impact and Sudden Stress. Impact for design purposes is measured as impact energy, $mv^2/2$. It can be absorbed only by deformation work. In this connection it is to be remembered that, within the limit of proportionality, if a material yields or deflects the amount y under the action of a force F, the average resisting force during the yielding is only $F/2$ and the deformation work is $yF/2$. If the material is unable to yield at all, it would have to exert an infinitely great force to resist even the smallest impact. It is therefore often better to absorb impacts by providing ductility and resiliency, rather than rigidity and strength. Rock crushers or ball mills may be lined either with manganese steel, which is extremely tough and ductile, or with rubber, which is very resilient.

A load *suddenly* applied, but without actual impact, will cause a momentary stress twice as high as the same load gradually applied. This is due to the fact that as the support first yields, the load acquires momentum, and overtravels the position of equilibrium before finally coming to rest.

Example. A hot ingot weighing 1000 lb. falls from a height of 8 ft. and strikes the floor. If the floor is to deflect only 0.1 in., within the limit of proportionality, what maximum resisting force will it have to exert? The ingot is in this case to be regarded as quite rigid.

* See Moore and Kommers, Illinois Bulletin No. 124; for instance, diagram on page 96.

Solution. The ingot in falling 8 ft. acquires a kinetic energy of $1000 \times 8 \times 12$ in. lb. The maximum resisting force F in the floor must be such that $0.1F/2$ will equal this energy. We have

$$F = \frac{2 \times 1000 \times 8 \times 12}{0.1} = 1{,}920{,}000 \text{ lb.}$$

It must be realized, however, that an ingot while hot is actually quite plastic. Hence, most of the deformation work necessary to absorb the impact may take place in the ingot rather than in the floor. The seriousness of impact is nevertheless made quite obvious by noting that if the ingot were non-yielding, a weight of 1000 lb. would produce a maximum flexing force almost 2000 times as large in striking a comparatively rigid floor from a height of 8 ft.

17. Working Stresses and Factors of Safety. The *working stress* for a material is the permissible stress used in the design of a machine member if it is to resist the loads applied and operate satisfactorily for a reasonable period of time. The length of this period may vary with the type of service. We do not expect an automobile or an agricultural tool to last forever. Technical improvements go on all the time and a machine may become *obsolete* in ten or twenty years. If this is the case, one would not be justified in going to any unnecessary expense of labor and material to make its service life extend indefinitely beyond any reasonable expectations.

It may be necessary at times to reduce weight or expense even though the durability is impaired. A dirigible of extraordinary strength might be too heavy to leave the ground; a farm tool costing thousands of dollars perhaps could never be sold, regardless of its merits. In such cases we may have to take chances on an ultimate breakdown in fatigue, if only this breakdown does not occur too soon. Furthermore, it may not be possible to provide safety against extraordinary carelessness or overload.

To arrive at the working stress, it is customary to divide the ultimate strength of the material by a number known as the *factor of safety*. In ordinary cases, the working stress must not exceed the yield point or the elastic limit, as the resultant deformations would make the macbine inoperative; and usually the endurance strength in fatigue must not be exceeded. Moreover, if the load is applied suddenly, there will be a momentary stress twice as high as the steady stress that occurs after equilibrium has been established. Our first problem, therefore, is to determine the stress that will barely cause failure under the particular conditions of loading. To this breakdown stress we must then apply a factor of safety in order to have a real *margin of safety*.

18. Determination of the Factor of Safety. If the factor of safety is applied to the ultimate strength, it should first contain two sub-factors a and b which determine the actual breakdown stress. The first factor a should give the elastic limit or yield point, if the load is steady, and the fatigue strength, if it varies. Very often in specifications for steel, an elastic limit of at least half of the ultimate strength is demanded. In such cases, the factor a should be taken equal to 2 for steady loads.

If we adhere to the idea that the endurance strength is half the ultimate strength, both for reversed and released loads on well finished specimens, we would make a equal 2 in these cases also. However, if we wish to be more conservative, we refer to Table 6, and under fatigue conditions give factor a the values listed in Table 7.

TABLE 7

VALUES OF SAFETY SUB-FACTOR a

	RELEASED LOAD	REVERSED LOAD
Ductile Materials	1.8–2.0	3.0
Brittle Materials	2.5	4.0

The second factor b depends upon the manner in which the load is applied. If the load is steady or very gradually applied, the factor b is 1. If the load comes on with absolute suddenness, b is 2. Intermediate conditions, such as the rise of pressure in engines, or the surging of a crowd on a bridge, may be allowed for according to the designer's judgment.

A third sub-factor c provides the actual margin of safety. It must also provide against such unknown factors as defective or unreliable materials, poor workmanship, careless maintenance, and unexpected overloads. The minimum value given to factor c is 1.25. For ordinary designs, the maximum value might be 1.75 to 2.00. The lower values may be taken for uniform, dependable materials and high grade workmanship, especially if a breakdown or a short life is not a very serious matter. If a breakdown is a serious matter, considered either from the point of view of danger to life or economic loss, then factor c should be given the higher values. In steel mill service, for instance, there are cases where a breakdown must be prevented at almost any cost, and values of c even as high as 3 or 4 may be found.

After the values of the sub-factors have been determined, the factor of safety is obtained as the product $a \times b \times c$.

19. General Considerations of the Factor of Safety. In practice, it is often customary to take the factors of safety without detailed analysis from actual designs which have functioned satisfactorily in

service. This is a safe procedure, if speeds, pressures, materials, etc. are somewhat similar, but it is not safe if conditions are radically different. The fatigue and impact conditions in a modern high-speed engine are far more serious than in an old style low-speed pumping engine. It is hardly possible to use as low working stresses in a solid-injection Diesel engine as in a medium pressure steam engine, because the dimensions of many parts would become unmanageable. It is often necessary to take chances in new developments, but in such cases there is particular need for analysis, so that eventual imperfections may be ascribed to the right causes.

It is also highly desirable, however, to submit a new machine to a period of experimentation and development before it is put into regular service. Unfortunately this cannot always be done. Thus a new type of battleship or airship may involve stress conditions that cannot be exactly pre-determined, yet the size and expense preclude experimentation with a full-sized model. This experimentation is entirely possible and is regularly done with any machinery or equipment which is to be manufactured on a production basis.

EXAMPLE 1. Determine the factor of safety to be used in the design of a connecting rod for a double-acting steam engine. The material is to be ductile, medium carbon steel. We may assume a equal to 3, b equal to 1.5, and c equal to 1.75. The factor of safety would then be $3 \times 1.5 \times 1.75 = 8$, approximately. For an ordinary, general drive engine, not intended for exceptionally heavy service, this factor might be entirely satisfactory. But for a blooming-mill reversing engine, where extreme impact and rough handling are certain to occur, and breakdown would mean great danger and economic loss, a designer might well use a factor of $3 \times 2 \times 3 = 18$. On the other hand, for an airplane engine, in which lightness is indispensable, a factor of safety of only $2 \times 1.5 \times 1.5 = 4.5$ might be attempted. In such an engine, inertia forces might cause an almost complete reversal of stress, even if the engine is single-acting. To justify a sub-factor $a = 2$, the rod should be carefully finished all over. It is a question whether the stress should not be regarded as sudden enough to demand a factor $b = 2$, and if there is any looseness or impact a sub-factor c higher than 1.5 might well be considered. Consequently, a factor of safety of only 4.5 demands a long period of experimentation before it can be regarded as fully justified.

This example shows the variation in working stresses that may occur quite legitimately. With steel having an ultimate strength of 80,000 p.s.i. in all cases, the working stress in the connecting rod of the blooming mill engine would be only 80,000/18 = 4450 p.s.i.; whereas in that of the airplane engine it might be as high as 80,000/4.5 = 17,800 p.s.i. Such an extreme difference may seem startling to the beginner, but it is entirely justified by the conditions in the two cases.

EXAMPLE 2. The Boiler Code of the American Society of Mechanical Engineers, which is now accepted as a legal basis for boiler design in most American states, demands a factor of safety of 5 in the boiler shell. The material used is a very ductile steel. In this case, whether we assume the load to be either released or static, the sub-factor a is 2. The load cannot be applied suddenly; therefore b is

equal to 1. These factors would make c equal 2.5, which would seem to be unnecessarily high, if the material retains its full strength at the temperature of operation. If heat is transmitted through the shell, great thickness is detrimental since it may lead to over-heating and corrosion on the outside. This high factor of safety in the past has been justified on the assumption that there might be a certain amount of burn-off and reduction of shell thickness in use. A uniform factor of safety for all thicknesses, however, makes the allowance for burn-off greater for a thick shell than for a thin shell, and this assumption may or may not be rational. Many designers have felt that a constant amount should be allowed for this burn-off for all thicknesses, and that the factor of safety for the residual thickness should be less than 5. In recent times, boiler temperatures have become so high that allowance must be made for a serious reduction in strength at the working temperature. This allowance, however, should not be made in a factor of safety referred to the strength at ordinary temperatures, but should be made by applying the factor of safety to the actual strength at the working temperature.

This example has been given to show that over-all factors of safety, even if based on successful experience in the past, are by no means always a safe guide in new design. Detailed analysis of the sub-factors, with intelligent allowances for actual working conditions, is always preferable.

20. Curved Beams. The stresses resulting from bending a straight beam are determinable from a simple relationship, $s = Mc/I$, derived upon the assumption that a plane cross-section originally normal to the neutral axis remains plane and normal after the axis is bent. On this basis the bending stress varies linearly according to the distance from the neutral axis, which coincides with the gravity axis. If the member is curved instead of straight with the gravity axis originally curved to a radius small in comparison with its radial depth, the relationship becomes more complicated. Even though the same assumption as to plane sections remaining plane is maintained and the total deformation at any point is still proportional to its distance from the neutral axis, the relationship becomes involved because the original fiber lengths are different at successive radii (thus affecting the unit elongation) and

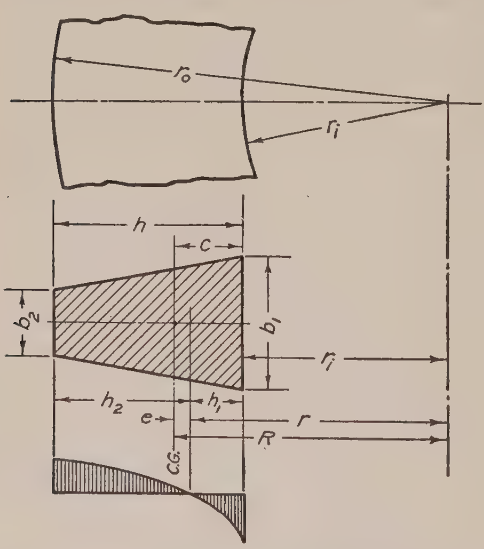


FIG. 17. TRAPEZOIDAL CROSS-SECTION OF A HOOK.

because the neutral axis no longer coincides with the gravity axis. The neutral axis shifts towards the center of curvature and the stress distribution is hyperbolic, as illustrated in Fig. 17. Note that the neutral axis always shifts towards the center and that for ordinary sections the greatest stress must always be at the inner radius in order that the total normal forces on the section shall balance.

As curved bars frequently occur in design, the formula as derived by Timoshenko * is given here. It is not simple to use and is justified only by the fact that calculations by the straight beam method are so inexact as to require extreme factors of safety, and because the formula indicates the effect of curvature. When the radius of curvature R of the gravity axis is less than 10 times the radial dimension of the cross-section, the curved beam formula should be used. As h/R decreases from 10 to 1, the per cent error in maximum stress obtained by assuming straight line stress distribution increases from 3.2 to 35. The stresses at the extreme fibers at the inner and outer radii are:

$$(41) \qquad s_i = \frac{Mh_1}{Aer_i}, \quad \text{and} \quad s_o = -\frac{Mh_2}{Aer_o}.$$

When the sense of the pure moment M is such as to increase the curvature, the negative sign indicates a compressive stress. The various symbols, with all distances in inches, have the following meaning:

s_i = stress at inner radius, r_i,
s_o = stress at outer radius, r_o,
M = applied moment in in. lb.,
h_1 = distance from neutral axis to inner radius,
h_2 = distance from neutral axis to outer radius,
A = area of cross-section,
e = distance from neutral axis to gravity axis of the section.

Although simple in appearance, the formula is somewhat difficult to use because, as we do not know immediately the location of the neutral axis e, the distances h_1 and h_2 are both unknown. It is first necessary to determine r, the radius of the neutral axis. In the general form,

$$r = \frac{A}{\int \frac{dA}{r - y}},$$

with particular values as shown in Fig. 18.

* Timoshenko, *Strength of Materials*, Part 2, p. 426.

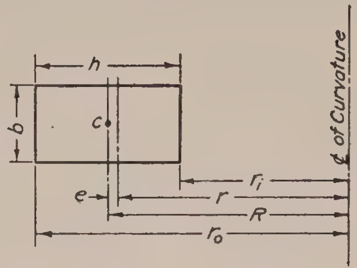
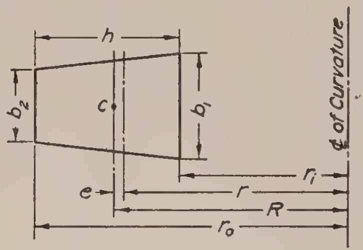
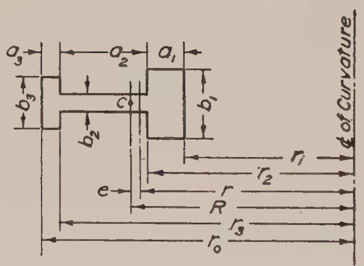
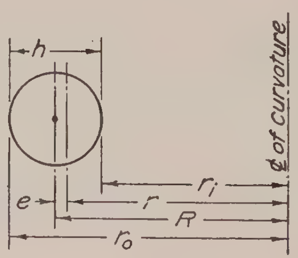
Section	Values for e or r
	$r = \dfrac{h}{\log_e \frac{r_o}{r_i}}$ $e = \dfrac{h}{12R}\left[1 + \dfrac{4}{15}\left(\dfrac{h}{2R}\right)^2\right]$
	$r = \dfrac{A}{\dfrac{b_1 r_o - b_2 r_i}{h} \log_e \dfrac{r_o}{r_i} - (b_1 - b_2)}$ *Note that when $b_1 = b_2$, this section reduces to a rectangle*
	$r = \dfrac{a_1 b_1 + a_2 b_2 + a_3 b_3}{b_1 \log_e \frac{r_2}{r_i} + b_2 \log_e \frac{r_3}{r_2} + b_3 \log_e \frac{r_o}{r_3}}$ *Note that when $b_3 = b_2$, this section reduces to a $\perp$ section*
	$e = R\,\dfrac{m}{1+m}$ $m = \dfrac{1}{4}\left(\dfrac{h}{2R}\right)^2 + \dfrac{1}{8}\left(\dfrac{h}{2R}\right)^4 + \dfrac{5}{64}\left(\dfrac{h}{2R}\right)^6 + \cdots$

FIG. 18. VALUES FOR LOCATION OF NEUTRAL AXIS FOR TYPICAL SECTIONS OF CURVED BEAMS.

EXAMPLE. Compute the stresses at the section A-A of a crane hook dimensioned as in Fig. 19, with the outline simplified to that of Fig. 17 for computation. The load is 20 tons.

Making $b_1 = 4$ in., $b_2 = 2\frac{25}{32}$ in., $h = 5.675$ in., $r_i = 3.4$ in., we have

$$\text{Area} = A = (1/2)(b_2 + b_1)h = 1/2 \times (2.781 + 4) \times 5.675 = 19.25 \text{ sq. in.}$$

$$\text{C. G. of the section from } r_i = c = \frac{h(2b_2 + b_1)}{3(b_2 + b_1)} = \frac{5.675(2 \times 2.781 + 4)}{3(2.781 + 4)} = 2.667 \text{ in.}$$

$$r = \frac{A}{\dfrac{b_1 r_o - b_2 r_i}{h} \log_e \dfrac{r_o}{r_i} - (b_1 - b_2)}$$

$$= \frac{19.25}{\dfrac{4 \times 9.075 - 2.781 \times 3.4}{5.675} \log_e \dfrac{9.075}{3.4} - (4 - 2.781)}$$

$$= 5.618 \text{ in.}$$

$$s_i = \frac{Mh_1}{Aer_i},$$

$$M = FR = 20 \times 2000 \times 6.067 = 242{,}680 \text{ in. lbs.},$$

where

$$R = r_i + c = 3.4 + 2.667 = 6.067 \text{ in.}$$

$$e = R - r = 6.067 - 5.618 = 0.449 \text{ in.}$$

$$h_1 = c - e = 2.667 - 0.449 = 2.218 \text{ in.}$$

$$s_i = \frac{242{,}680 \times 2.218}{19.25 \times 0.449 \times 3.4} = 18{,}130 \text{ p.s.i.}$$

$$s_o = -\frac{Mh_2}{Aer_o}, \quad h_2 = h - c + e$$

$$h_2 = 5.675 - 2.667 + 0.449 = 3.457 \text{ in.}$$

$$s_o = \frac{-242{,}680 \times 3.457}{19.25 \times 0.449 \times 9.075} = -10{,}700 \text{ p.s.i.}$$

$$\text{Tensile stress} = s_t = \frac{F}{A} = \frac{20 \times 2000}{19.25} = 2080 \text{ p.s.i.}$$

$$\text{Max. Total Stress} = s_i + s_t = 18{,}130 + 2080 = 20{,}210 \text{ p.s.i.}$$

$$\text{Min. Total Stress} = s_o + s_t = 8620 \text{ p.s.i.}$$

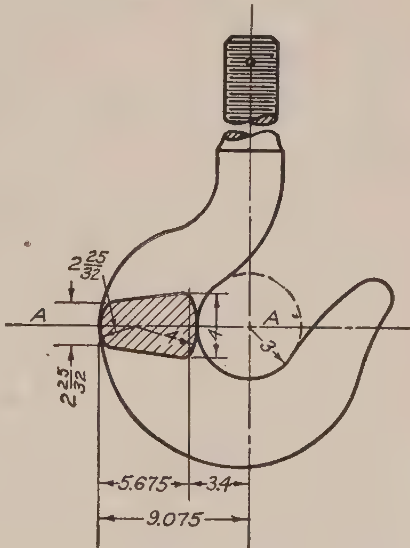


FIG. 19. TWENTY-TON CRANE HOOK.

21. Approximate Curved Beam Formula. Tables and approximate formulas published in Boyd's *Strength of Materials* give a multiplying factor to apply to the straight beam formula to obtain the maximum curved beam stress. For a curved beam of rectangular cross-section, where b is the width and h is the depth, the formula becomes

$$(42) \qquad s_i = \frac{6M}{bh^2}\left(1 + 0.25\frac{h}{r_i}\right),$$

and for one of circular cross-section of diameter d

$$s_i = \frac{32M}{\pi d^3}\left(1 + 0.3\frac{d}{r_i}\right). \tag{43}$$

On the basis of statements by Bach,* this ratio for the circular section may be applied safely to elliptical sections, where a is the radial axis of the ellipse and b the transverse axis; therefore

$$s_i = \frac{32M}{\pi a^2 b}\left(1 + 0.3\frac{a}{r_i}\right). \tag{44}$$

22. Stresses in Rings. In addition to the curved beam elements encountered in machine frames, levers, hooks, and other machine parts, the designer occasionally must check the stresses in rings and chain links. For a circular ring loaded at a point, as in Fig. 20, the maximum moment occurs at the point of application of the load and is equal to $-0.318Fr$. At a point 90 deg. away from the point of application is another maximum moment equal to $+0.182Fr$, at which section there is also a direct stress of $F/(2A)$, where A is the cross-sectional area of the ring. The bending stress may be obtained from the approximate formula (43) or (44), according to the section.

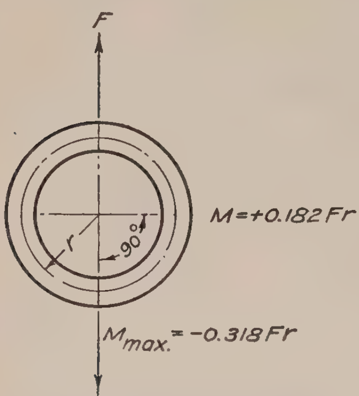


Fig. 20. Bending Moments in a Ring.

Links with a straight section of length l between the semi-circular ends (Fig. 21) need somewhat different treatment.† The moment, M_y, at the point of application is

$$M_y = \frac{Fr(2r + l)}{2(\pi r + l)}, \tag{45}$$

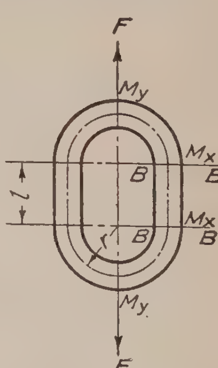


Fig. 21. Bending Moments in a Link.

and at the section 90 deg. away from the point of application

$$M_x = \frac{Fr(2r - \pi r)}{2(\pi r + l)}. \tag{46}$$

* Bach, *Maschinenelemente*, 12 ed., p. 43.
† Morley, *Strength of Materials*, Longmans, 1934, p. 409.

EXAMPLE. With a stress of 15,000 p.s.i., what load can a link of 1-in. round stock support, if the inside radius is 1 in., and the straight sides are 2 in. long?

We have

$$M_y = \frac{F \times 1.5(3 + 2)}{2(1.5\pi + 2)} = 0.56F = \frac{15{,}000 \times \pi \times 1^3}{32(1 + 0.3 \times 1/1)}$$

$$F = 2030 \text{ lb.}$$

$$M_x = \frac{2030 \times 1.5(3 - \pi \times 1.5)}{2(\pi \times 1.5 + 2)} = -388 \text{ in. lb.}$$

Hence the stress at the section *B-B* is

$$s = \frac{1.3 \times 388 \times 32}{\pi \times 1^3} + \frac{2030 \times 4}{2 \times \pi \times 1^2} = 6430 \text{ p.s.i.}$$

It will be seen that this stress is far less serious than that resulting from the bending moment at the point of load application.

23. Plates. The formulas here listed for solid plates either freely supported or clamped at the edges are those given by Timoshenko.* In Table 8 are given the maximum bending stresses at the edge and deflections at the center for circular plates under uniformly distributed load.

The shear stresses are usually neglected in practical engineering computations, but may be of noticeable amount in very thick plates. (See Timoshenko or Morley.)

TABLE 8

CIRCULAR PLATES—UNIFORMLY LOADED

CONDITION AT EDGE	BENDING STRESS s_b (p.s.i.)	DEFLECTION y (in.)
Clamped (Fig. 22a)	$\frac{3pr^2}{4t^2}$	$\frac{pr^4}{64K}$
Freely Supported (Fig. 22b)	$\frac{3(3+m)pr^2}{8t^2}$	$\frac{(5+m)pr^4}{64(1+m)K}$

Here t is the plate thickness in inches, p the intensity of uniform load in p.s.i., and m is Poisson's ratio, the average of which, according to Marks' *Mechanical Engineers Handbook*, is 0.270 for cast iron and 0.303 for steel. The quantity K is equal to $Et^3/12(1 - m^2)$, where E is the modulus of elasticity.

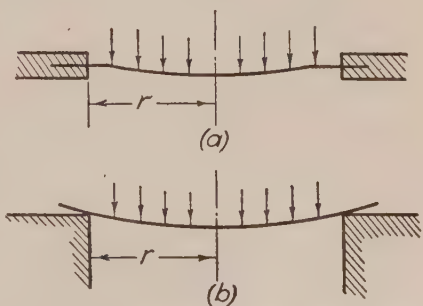
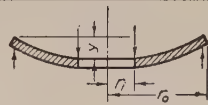


FIG. 22. BENDING OF CIRCULAR PLATES.

Concentrated forces occur at the center of plates, as in pistons and similar parts, but such plates usually have a hole in the middle. Theoretically, an absolutely con-

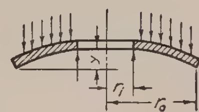
* *Strength of Materials*, vol. 2, p. 492, etc.

Fig. 23



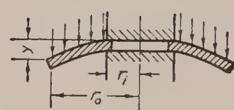
$s_b = K_1 \dfrac{F}{t^2}$ $\quad y = M_1 \dfrac{Fr_0^2}{Et^3}$

Fig. 24



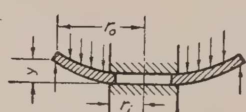
$s_b = K_2 \dfrac{pr_0^2}{t^2}$ $\quad y = M_2 \dfrac{pr_0^4}{Et^3}$

Fig. 25



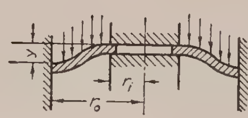
$s_b = K_3 \dfrac{pr_0^2}{t^2}$ $\quad y = M_3 \dfrac{pr_0^4}{Et^3}$

Fig. 26



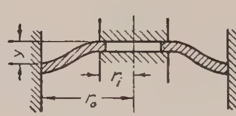
$s_b = K_4 \dfrac{pr_0^2}{t^2}$ $\quad y = M_4 \dfrac{pr_0^4}{Et^3}$

Fig. 27



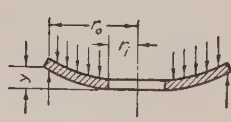
$s_b = K_5 \dfrac{pr_0^2}{t^2}$ $\quad y = M_5 \dfrac{pr_0^4}{Et^3}$

Fig. 28



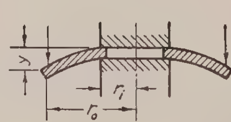
$s_b = K_6 \dfrac{F}{t^2}$ $\quad y = M_6 \dfrac{Fr_0^2}{Et^3}$

Fig. 29



$s_b = K_7 \dfrac{pr_0^2}{t^2}$ $\quad y = M_7 \dfrac{pr_0^4}{Et^3}$

Fig. 30



$s_b = K_8 \dfrac{F}{t^2}$ $\quad y = M_8 \dfrac{Fr_0^2}{Et^3}$

A. M. Wahl and G. Lobo, Jr.

Figs. 23–30. Stresses and Deflections in Flat Circular Plates with Central Holes.

centrated force on an unperforated plate causes an infinite stress in the center. Perforated plates can be dealt with most readily by means of formulas and constants furnished by Wahl and Lobo.*

The cases dealt with are shown in Figs. 23 to 30. The formulas are given with the illustrations, and the necessary constants in Table 9.

TABLE 9

Constants in Formulas of Wahl and Lobo for Circular Plates with Holes †

[a] α = 1.0		1.25	1.50	2.0	3.0	4.0	5.0
K_1	0.955	1.100	1.26	1.48	1.88	2.17	2.34
K_2	0	0.660	1.19	2.04	3.34	4.30	5.10
K_3	0	0.135	0.410	1.04	2.15	2.99	3.69
K_4	0	0.122	0.336	0.74	1.21	1.45	1.59
K_5	0	0.090	0.273	0.71	1.54	2.23	2.80
K_6	0	0.115	0.220	0.405	0.703	0.933	1.13
K_7	0	0.587	0.981	1.530	2.290	2.930	3.50
K_8	0	0.227	0.428	0.753	1.205	1.514	1.745
M_1	0	0.341	0.519	0.672	0.734	0.724	0.704
M_2	0	0.202	0.491	0.902	1.220	1.300	1.310
M_3	0	0.00231	0.0183	0.0938	0.2925	0.448	0.564
M_4	0	0.00343	0.0313	0.1250	0.2910	0.417	0.492
M_5	0	0.00077	0.00618	0.0329	0.1096	0.1792	0.2338
M_6	0	0.00129	0.00637	0.0237	0.0619	0.0923	0.114
M_7	0	0.18410	0.41390	0.6640	0.8237	0.8296	0.813
M_8	0	0.00510	0.02490	0.0877	0.2090	0.2930	0.350

[a] α = Outside Radius ÷ Hole Radius.

The notation is as follows:

r_o = outer radius of plate, in.,
r_i = inner radius of plate, in.,
$\alpha = r_o/r_i$,
F = total load concentrated around inner or outer edge, lb.,
p = uniformly distributed load, p.s.i.,
t = thickness of plate, in.,
E, s_b, and y are modulus of elasticity, bending stress, and deflection as usual.

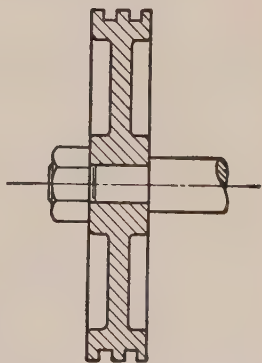


Fig. 31. Disc Type Piston.

The loading in Fig. 27 may be taken to apply to a simple disc piston ("Swedish" piston) such as is shown in Fig. 31. If greater exactness is wanted the pressure on the rim can be added as a concentrated load and an additional deflection and stress from this load can be computed.

* Transactions A.S.M.E., vol. 52, part 1, APM 52–3–p. 29.
† Transactions A.S.M.E., 1930, APM–52–3, p. 33.

A formula for a box piston as shown in Fig. 32 may be derived in the following manner. One wall is bent down by a distributed pressure as in Fig. 27, but is bent back by a concentrated resistance from the other wall as in Fig. 28. The latter wall is bent down only by the latter force. Since the deflection must be the same in both walls, we have

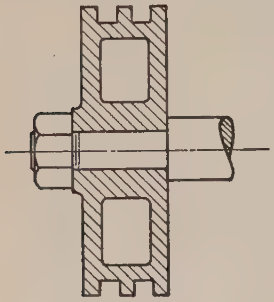


FIG. 32. BOX PISTON.

$$\frac{M_5 p r_o^4}{Et^3} - \frac{M_6 F r_o^2}{Et^3} = M_6 \frac{F r_o^2}{Et^3}.$$

Hence $M_5 p r_o^4 = 2M_6 F r_o^2$, or

$$F = \frac{M_5 p r_o^4}{2M_6 r_o^2} = \frac{M_5 p r_o^2}{2M_6}.$$

Further, from Fig. 28,

$$s_b = K_6 \frac{F}{t^2} = K_6 \frac{M_5 p r_o^2}{2M_6 t^2}. \tag{47}$$

This formula does not apply if the two walls are connected by ribs. In such case some approximate method must be used. For instance, the piston may be computed for bending on the (developed) section at the hub. If the piston is regarded simply as a coiled-up cantilever beam, the bending moment is

$$2\pi p \left(\frac{r_o^3}{3} - \frac{r_i r_o^2}{2} + \frac{r_i^3}{6} \right).$$

The section modulus at the root section is determined as for a series of I-beams standing side by side.

24. Elliptical Plates. The maximum bending stress in an elliptical plate uniformly loaded by the pressure p and freely supported at the edge may be taken, according to Morley,* approximately as

$$s_b = 1.25(2 - b/a)p \frac{b^2}{t^2}, \tag{48}$$

where a is the long and b the short half axes (Fig. 33). Morley suggests that clamping at the edge may never be quite dependable and that this formula be used also for clamped edges.

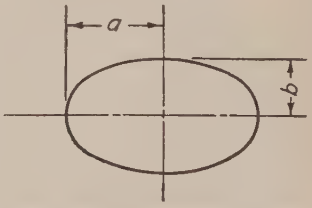


FIG. 33. ELLIPTICAL PLATE.

25. Square and Rectangular Plates (Uniformly Loaded). Timoshenko's values for the maximum bending moments per unit length of

* *Strength of Materials*, 1934, p. 455.

section lead to the following values for the bending stresses:

(49) For freely supported edges $s_b = K_1 \dfrac{pb^2}{t^2}$,

(50) For clamped edges $s_b = K_2 \dfrac{pb^2}{t^2}$,

where a is the long side and b is the short side of the plate, and t is the thickness.

The constants have the values in Table 10.

TABLE 10

CONSTANTS K_1 AND K_2 IN FORMULAS (49) AND (50)

a/b =	1.0	1.1	1.3	1.5	1.7	2.0	3.0	4.0	∞
K_1 =	0.288	0.332	0.416	0.487	0.545	0.610	0.714	0.740	0.750
K_2 =	0.308			0.455		0.498			0.500

Ribbed Plates may be computed, at least in first approximation, as beams of T- or I-section, disregarding the fact that they are supported at the ends, parallel to the webs.

26. Cylindrical and Spherical Pressure Vessels. Thin-walled Vessels under Internal Pressure. A thin-walled cylinder of diameter D and under an internal pressure p is subject to a tensile stress $s = pD/(2t)$ on a longitudinal section, if t is the wall thickness. On an annular transverse section the stress is $pD/(4t)$. The stress on the longitudinal section is twice as high as that on the transverse section. It is sufficiently accurate to regard the stresses as uniformly distributed over the section.

The stress in a thin-walled spherical vessel under internal pressure is uniform tangentially in all directions and is equal to $pD/(4t)$.

27. Thick-walled Vessels under Internal Pressure. In thick-walled vessels, the fact must be allowed for that the stress is not uniformly distributed over the section, but is much higher at the inside than at the outside. For brittle materials, experiments show that breakdown is caused by the maximum tensile stress. This stress is given by the formula of *Lamé:*

(51) $s = p\,\dfrac{k^2+1}{k^2-1}$, whence $k = \sqrt{\dfrac{1+p/s}{1-p/s}}$.

In this formula $k = D_o/D_i$, the ratio of the external diameter to the internal diameter.

For ductile materials the condition for yielding is best represented by a deformation work formula,* which takes the form

$$(52) \qquad s = \frac{\sqrt{3}k^2p}{k^2 - 1} = \frac{1.73k^2p}{k^2 - 1}, \qquad k = \sqrt{\frac{s}{s - 1.73p}}.$$

28. Thick Spherical Shells. Thick spherical shells of inside radius r_i and outside radius r_o under internal pressure p are subject to the stress

$$(53) \qquad s = \frac{p}{2}\left(\frac{r_o^3 + 2r_i^3}{r_o^3 - r_i^3}\right).$$

This stress will be found regularly less than the stress in a thin cylindrical shell of the same dimensions. Hence, if the bottom of a hydraulic cylinder be made hemi-spherical and of the same thickness as the adjoining cylinder wall, the bottom will be stronger than the wall. For casting reasons, however, it may be well to use the same thickness for bottom and wall. Flat bottoms should have very ample fillets in order to obtain a sound casting at the corners and to approach the spherical form as much as possible. Even with ample fillets it will be found as a rule that the flat bottom has to be thicker than the adjoining cylindrical wall.

29. Flanges. Flanges are usually computed as cantilevers built in at the root and loaded with a concentrated load at the bolt circle. If the thickness of the flange is t, the moment arm l, and the root diameter D_r, we have (see Fig. 34)

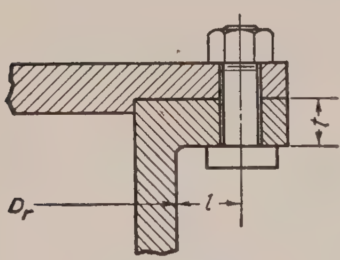


FIG. 34. FLANGE.

$$(54) \qquad t = \sqrt{\frac{6Fl}{\pi D_r s_b}},$$

where s_b is the permissible bending stress and F is the total force to be taken up by the bolts.

It should be understood that this formula may represent merely a conventional method of determining the flange thickness. The cylindrical wall to which the flange is anchored is not rigid and consequently will be subjected to distortions† and bending stresses which

* The authors are under great obligation to A. Nadai for these formulas. The underlying theory was given by Nadai in Transactions A.S.M.E., Applied Mechanics section, July–September, 1933, p. 111 (APM–55–15). Tests by Cook and Robertson, Engineering, Dec. 15, 1911, p. 786, indicate that formulas (52) are quite safe. According to their tests the stress would be only $1.67k^2p/(k^2 - 1)$.

† For some of the wall distortions occurring, see for instance Timoshenko, *Strength of Materials*, p. 525.

the formula entirely neglects. The main justification for formula (54) is that it has been widely used in engineering practice and has, in spite of its simplicity, led to dimensions which have been found safe and satisfactory.

30. Tubes under External Pressure. Morley * derives, for the collapsing pressure p_c of long thin tubes of thickness t and diameter D under external pressure, the formula

$$p_c = \frac{1}{1 - m^2} \times 2E\left(\frac{t}{D}\right)^3, \tag{55}$$

where m is Poisson's ratio, but states that experimentally the pressures have been found to be 25 to 30 per cent lower. This formula should then be used with caution, yet it makes possible the estimation of the collapsing pressures for tubes of materials which have not been experimentally investigated.

For values of t/D less than 0.025 the following experimental formulas have been found by Stewart and by Carman and Carr:

$$p_c = 50{,}000{,}000\left(\frac{t}{D}\right)^3 \quad \text{p.s.i. for steel tubes,} \tag{56}$$

$$p_c = 25{,}000{,}000\left(\frac{t}{D}\right)^3 \quad \text{p.s.i. for brass tubes.} \tag{57}$$

For a considerable range of t/D values greater than 0.03 the following formulas apply:

$$p_c = 95{,}520\frac{t}{D} - 2090 \quad \text{p.s.i. for seamless steel tubes.} \tag{58}$$

$$p_c = 83{,}270\frac{t}{D} - 1025 \quad \text{p.s.i. for lap-welded steel tubes.} \tag{59}$$

$$p_c = 93{,}365\frac{t}{D} - 2474 \quad \text{p.s.i. for brass tubes.} \tag{60}$$

To cover the whole range of thicknesses, Morley suggests the formula

$$p_c = \frac{2s_e t/D}{1 + \dfrac{s_e D^2}{Et^2}}, \tag{61}$$

where s_e is the elastic limit in compression. This collapsing pressure is somewhat lower than that obtained by a strictly theoretical derivation,

* Morley, *Strength of Materials*, 1934, p. 342.

but it may yet be somewhat higher than the actual one. This condition should be considered in selecting a factor of safety.

For short tubes supported at the ends, Morley tentatively accepts a formula proposed by Cook,

$$p_c = \frac{L}{l} p_1, \tag{62}$$

where p_1 is the collapsing pressure for a long tube, l is the actual length of the tube, and L has the value $1.73(D^3/t)^{1/2}$.

For the effect of stiffening rings, etc., the regulations of the Boiler Code of the American Society of Mechanical Engineers should be consulted. Some of these regulations are referred to in the chapter on pressure vessels.

31. Stress Concentration. Both theoretical and experimental investigations show that notches, holes, and indentations of various kinds cause local stresses much in excess of the average stress, which for tension or compression would be the load divided by the net cross-sectional area. At the edges m and n of a small hole in the middle of a flat plate as in Fig. 35, if the size of the hole is small compared to the width of the plate, the theoretical stress is found to be three times the average stress F/A, where F is the load and A is the cross-sectional area of the unperforated plate. For a plate notched at the sides as in Fig. 36, the stress at the notches is about twice the uniform stress in the unnotched plate. The stress at the sharp re-entrant corners of an ordinary key-seat is theoretically infinite; that at the root of an American Standard screw thread has been shown by photo-elastic determinations to be almost 6 times the average stress, and so forth.*

FIG. 35. STRESS CONCENTRATION IN A PERFORATED PLATE.

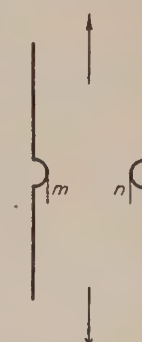


FIG. 36. STRESS CONCENTRATION IN A NOTCHED PLATE.

While this stress concentration is very serious, frequently it has been neglected in the past. Yet, no breakdowns have resulted. The tests of Moore on the effect of keyways in shafts (see page 25) show an over-all strength reduction not nearly so serious as the stress concentra-

* For stress concentration in general, see Timoshenko, *Strength of Materials* (Van Nostrand), p. 615, etc. For screw threads, see Moore and Henwood, Bulletin 264, Engineering Experiment Station, University of Illinois. For fillets at bosses of shafts and pins, see Garner, Product Engineering, June, 1931.

tion would lead one to expect. The reason is that under steady or very slowly varying load, the overstressed material can adjust itself plastically, so that the stress is transferred to adjacent, less severely stressed parts. This condition, however, holds true only for *ductile* materials. For brittle materials stress concentration is always serious; and *for ductile materials under varying stress*, it is likewise serious. Many serious breakdowns in modern high-speed machinery may be explained by stress concentration.

It is well to explain, however, that sometimes machine parts may take up varying loads and yet not be subject to varying stress. Thus bolts and tie rods that hold machine parts together are usually pulled up so tightly that they will not stretch and allow separations to form when the load is applied. Hence, even if the load varies, they are under a steady stress greater than that which would be caused by the direct application of the load. If it were not for this condition, it is very probable that there would be fatigue failure of bolts much oftener than has been actually observed.

In any case, the provision of ample fillets has become a matter of first importance in the design of modern machinery. It is now usually demanded that the draftsman actually specify the size of fillet radii on drawings. Moore and Henwood (see footnote, page 44) found that a Whitworth thread, which is rounded at the root, has a considerably higher endurance strength than an American Standard thread of the same pitch.

32. Vibratory Stresses. A very serious stress increase may occur if an oscillatory load happens to have a period that agrees with the natural elastic period of the machine part to which it is applied. The general theory underlying vibratory stresses and several of the fundamental formulas will be discussed in a separate chapter devoted to the subject.

33. Temperature Stresses. Suppose that a pipe line was held rigidly to a trench wall at freezing temperature and that a fluid at a temperature of 432° F. was admitted to the pipe. The pipe would attempt to expand to its free length at this latter temperature, but would be restrained by the wall to the length corresponding to the temperature of 32° F. The stress induced in the pipe would be that corresponding to a compression from the former length to the latter. The formula for this temperature stress will be

$$s = E\alpha(T_1 - T_2), \tag{63}$$

where E is the modulus of elasticity, α the coefficient of expansion per

degree Fahr. temperature difference, T_1 and T_2 the final and original temperatures, respectively. The coefficient for iron is 0.065/10,000. In the above example we would have

$$s = \frac{30{,}000{,}000 \times 0.065 \times 400}{10{,}000} = 78{,}000 \text{ p.s.i.}$$

This example gives an idea of the enormous stresses which may readily be caused by temperature differences quite commonly occurring in heat engines, chemical apparatus, and other types of machinery. It is always a mistake to attempt to take up these stresses by large dimensions. Such dimensions simply increase the forces due to the temperature differences and the final stress remains the same. The proper procedure is to introduce a yielding and flexible design which can be distorted somewhat without failure. In pipes, expansion joints or gooseneck bends are used; in combustion engines, sleeves with one end free to slide, or thin, curved walls are methods of relieving this stress (see Figs. 124 and 402).

Usually no attempt is made to compute or estimate the magnitude of the temperature stresses. Designs are simply adopted which will render them innocuous. Sometimes, however, they may have to be computed. Such may be the case, for instance, with turbine discs, stressed by the heat expansion of the rim at starting; or in the case of railway tracks, welded together for the conduction of electricity.

34. Working Stresses at Elevated Temperatures. In the usual strength computations, the working stresses as ordinarily used have been based on data obtained by standard short-time strength tests performed at ordinary temperatures. Short-time tests performed at somewhat elevated temperatures, for instance by Spring and Bregovsky,* showed that the strength of cast iron was maintained quite well up to 1000° F., and for steel at least up to 600° F.

Cast iron, however, shows the phenomenon of growth, that is, increasing in size at temperatures in excess of a few hundred degrees Fahr. This phenomenon becomes quite serious at beginning red heat, particularly if there is a variation of temperature. For this reason it has been necessary to use semi-steel or steel for valves and other parts subjected to highly superheated steam. Valve seats have been made of Monel metal and perhaps pure nickel may be even better, although bronze seats, particularly aluminum bronze, have been used in combustion engines.

With steadily increased working temperatures in steam power plant operation, it has been observed that the ordinary short-time tests

* Published in pamphlet form by the Crane Company in Chicago.

do not give dependable strength data. The elongation of the part continues under load and may reach serious values resulting in ultimate rupture, if sufficient time elapses. Since this condition, called *creep*, may continue for years, the designer is confronted at the present time by the extremely serious fact that tests have not progressed sufficiently to determine the rupturing stresses at elevated temperatures for even very common materials. The practice at present is to limit the stress by permitting a certain maximum elongation in a certain length of time. This procedure does not provide absolute safety, but it does lead to a considerable reduction in permissible stresses. Thus, the yield point for a certain high-grade stainless steel established from short-time tests is given as 28,000 p.s.i. at 1000° F. At this temperature, however, it has been found to stretch 1 per cent in 10,000 hours under a stress of only 13,000 p.s.i. Since 10,000 hours are equivalent to a little more than a year of service of 24 hours a day, and a stretch of 1 per cent is certainly not negligible, it is evident that a stress of 13,000 p.s.i. is a very serious one for this material.

It is the practice of a prominent manufacturer of high-temperature apparatus to base design on stresses which produce a stretch of 1 per cent in 100,000 hours. To meet these conditions it appears necessary to limit the stress for a rather wide range of carbon steels to about 3000 p.s.i. at 1000° F. This value is less than 28 per cent of a stress of 11,000 p.s.i. held permissible in boiler shells by the Boiler Construction Code of the American Society of Mechanical Engineers.

A great deal of work on the creep of metals is being done at the present time. H. J. Tapsell * published a book on the subject in 1931. Timoshenko deals with creep in his *Strength of Materials*. J. J. Kanter and L. W. Spring have published the results of numerous tests undertaken by the Crane Company in Chicago. Yet, at this writing, no standard practice has been adopted regarding the determination of working stresses at elevated temperatures; when such recommendations are presented, no doubt they will be subject to modification for a considerable length of time.

PROBLEMS

1. At a certain point in a shaft there is a tensile stress of 10,000 p.s.i. caused by bending and a pure shear stress of 5000 p.s.i. caused by torsion. Compute the maximum tensile stress, the maximum shear stress, and the tensile and shear stresses derived from the conception that the deformation work determines failure.

2. The principal stresses at a certain point in a material are a tension of 10,000 p.s.i., a tension of 5000 p.s.i., and a tension of 2000 p.s.i.; at another point the stresses

* Oxford University Press, England.

are a tension of 10,000 p.s.i., a compression of 5000 p.s.i., and a tension of 3000 p.s.i. What are the maximum shear stresses at the two points?

3. A shaft is subject to a torque of 5000 in. lb. and a bending moment of 8000 in. lb. Assume the permissible tensile stress to be 10,000 p.s.i. and the permissible shear stress to be 5000 p.s.i. What would be the necessary diameter of this shaft on the basis of the maximum shear theory and on the deformation-work theory, if the shaft is of ductile material; and on the basis of the maximum direct-stress theory, if it is of a brittle material? (By "direct stress" is meant tension or compression.)

4. A circular pin having a cross-sectional area of 1.5 sq. in. is stressed in transverse shear by a force of 15,000 lb. How great is the shear stress at the neutral axis of the section? If the pin were square, how much would it be?

5. In a cast-iron crank shaft the bending stress as computed by standard bending formulas based on Hooke's law is 10,000 p.s.i. On the basis of Bach's ratio figures what might we assume the actual tensile stress in the shaft to be, if the computed stress occurs

(a) in a finished circular part?
(b) in a rough circular part?
(c) in a rough rectangular part?
(d) in a finished rectangular part?

6. Supposing that the permissible actual tensile stress in cast iron is 5000 p.s.i., what stress might we use in applying standard bending formulas based on Hooke's law in the four cases listed in Problem 5?

7. The moment of inertia of rectangular sections of width b and depth h about the neutral axis perpendicular to h is $bh^3/12$. Compute the moment of inertia about the axis through the center of gravity of an I-section of the following dimensions: Flange width 2 in., flange thickness 0.5 in., web height 6 in., web thickness 0.4 in.

8. What is the torsional shear stress and what is the angle of twist in a circular shaft 3 ft. long, 3 in. diameter, subject to a twisting moment of 30,000 in. lb.? What would the stress and the angle of twist be if the shaft were square with 3 in. side, or rectangular with a section 2 in. thick and 4 in. wide? The shaft is of steel with a modulus of rigidity of 12,000,000 p.s.i.

9. A round shaft of 2.5 in. diameter can stand a torque of 18,000 in. lb., if it is not weakened by a keyway. On the basis of Moore's formula, what torque could it stand if it had a keyway 0.6 in. wide and 0.[illegible] in. deep, and in what ratio would the twist be increased by this keyway?

10. (a) Figure by means of Euler's formula and for a factor of safety of 20 the permissible buckling load on a pin-ended circular steel strut 90 in. long and of a diameter of 2 in. Compute by Rankine's formula the compressive stress in this strut, assuming the factor q to be 0.0006. (b) Conversely, supposing the load to be 2000 lb., what diameter of the strut is necessary to give a factor of safety of 20 with Euler's formula; and again, what is the compressive stress as computed by Rankine's?

11. Compute (a) for the conditions of Problem 10b the cross-sectional dimensions and the stress for a square section, (b) for a rectangular section of side ratio 3 : 1.

12. Select a standard structural angle with equal legs for the conditions of Problem 10. Assume a certain leg size, obtain from a handbook the approximate minimum radius of gyration, which does not vary much with the thickness, and then compute the necessary cross-sectional area and the thickness. If the angle comes out unreasonably thin, or unreasonably thick, a different leg length will have to be

selected. In applying Euler's formula assume both ends of the column to be guided in line, but not clamped.

13. A cylindrical steel rod 60 in. long and with a diameter of 3 in. is made of 0.4 per cent carbon steel, which may be regarded as a ductile material. If, when used as a strut in a building, it could barely support without breaking a steady load of 120,000 lb., what, in view of Wöhler's findings, would be its breaking load, if it were used

(a) as a connecting rod in a single acting compressor, inertia forces being neglected?

(b) as a connecting rod in a double-acting steam engine?

Impact effects and buckling action, as well as inertia forces, may be neglected.

14. An automobile, traveling at a speed of 10 ft. per sec., runs into a telegraph pole, which may be regarded as perfectly rigid. The bumper deflects 3 in. If this deflection is within the limit of proportionality, and if the car weighs 3000 lb., what was the maximum force exerted on the bumper?

15. What would be suitable factors of safety in a steam-turbine disc, if made of a brittle material like cast iron, and if made of a ductile material like steel? The disc is stressed mainly in tension from centrifugal force. Overspeed is guarded against by dependable mechanisms; materials and workmanship are of the highest grade; and it is essential to go to the very limit of permissible stress to provide the required circumferential speed. Give sub-factors *a*, *b*, and *c*.

16. A shaft supports a heavy rotating gyroscope disc. The torque supplying the motion is negligible, so that the shaft is stressed almost exclusively by the bending moment from the weight of the disc. Consider how the stresses vary in the extreme fibers of this shaft and indicate a suitable factor of safety, giving the sub-factors *a*, *b*, and *c*.

17. Determine the factor of safety for the pitman of a rock crusher of jaw type. (See illustration, Fig. 149.) The jaws are pulled apart by a spring, and the pitman may be regarded as stressed in compression only. The stresses are of uncertain magnitude and serious impacts and sudden loads must be allowed for. Give sub-factors *a*, *b*, and *c*.

18. Circular No. 16 of the Engineering Experiment Station of the University of Illinois proposes the following formula for the stresses in curved beams:

$$s = s_1 + K(Mc/I).$$

s_1 is the direct stress resulting from pull or compressive load, and K has the approximate value

$$K = 1.00 + m\frac{I}{bc^2}\left(\frac{1}{R-c} + \frac{1}{R}\right).$$

I is the moment of inertia of the section as used in straight-beam formulas, in.

b = maximum breadth of section, in.

c = distance from centroidal axis (center-of-gravity line of the sections) to inside fiber, i.e., to the extreme fiber nearest the center of curvature, in.

R = radius of curvature of centroidal axis of beam, in.

m = 1.05 for circular and elliptical sections, 0.5 for other sections.

Use this formula to determine the stress under a load of 250 lb. in a circular link of 0.5 in. inside radius, made of 0.5 in. round steel. The link is closed up, but not welded, so that the whole stress comes on one side only. The desired stress is on the section at right angles to the pull. Check the stress so computed against the stresses computed by means of the formulas in the text.

19. Compute the thickness of the wall and the flat bottom of a hydraulic cylinder having an inside diameter of 8 in. The pressure is 3000 p.s.i. Compute first the thickness for cast iron with a permissible stress of 6000 p.s.i., then for steel with a permissible stress of 15,000 p.s.i. The bottom is regarded as a flat plate clamped at the inside of the cylinder. Steel is taken as a ductile, cast iron as a brittle material.

20. Compute the necessary thickness of a hemispherical bottom for the cylinder in Problem 19, using the same stresses.

21. Suppose the cylinder in Problem 19 to have an outside diameter of 10 in. and that a head is held on by bolts arranged on 12-in. bolt circle. If the cylinder is made of steel and the permissible bending stress is 15,000 p.s.i., how thick would the flange have to be?

22. What is the outside collapsing pressure of a long steel tube of 6 in. inside diameter and a wall thickness of 0.5 in. Observe that the formulas contain outside diameter. If the tube is only 30 in. long, and is firmly supported and stiffened at the ends, what would be the collapsing pressure?

CHAPTER 3

ENGINEERING MATERIALS

35. Selection. In the design of machinery in general, a vast variety of materials of both organic and inorganic origin is utilized. We generally think of metals as the usual materials of design, but, although used to a lesser degree, such materials as wood, leather, rubber, and other plastics have widespread use, and others, such as fabrics, cork, special minerals, etc., have limited use. The designer should know about the availability and characteristics of materials in order that he may intelligently select and fully utilize their possibilities by suitable preparation and fabrication. In our discussion here, we must confine ourselves to a brief review of only the most common materials in general use.*

In making a selection of a material we must first decide what constitutes a "proper material." A proper material may be defined as one which best performs the functions required with the least total cost. This does not mean that the material having the lowest unit cost is best, because a more expensive material may permit reduction of weight, easier heat treatment or fabrication, or it may possess other advantages that make the final result less costly. And at times, of course, luxury, appearance, or extreme safety is desired even at great expense. Such rare materials as silver, platinum, diamonds, etc., are necessary for certain purposes. Demand for an expensive material may stimulate its preparation to such an extent that the cost is drastically reduced. Aluminum is a familiar example of such development.

Designers are interested principally in the physical properties and the cost of the finished part and only incidently in the chemical constituents and methods of preparation from the raw material. Exceptions to this lack of immediate concern are the adaptability to fabrication (machinability and weldability of metals) and a knowledge of the heat-treatment of metals to indicate how certain properties may be obtained.

* Information regarding developments in the field of materials is published in a variety of periodicals including Machine Design, Product Engineering, and other publications intended more particularly for the designer. The American Society for Testing Materials (A.S.T.M.), the Society of Automotive Engineers (S.A.E.), and the American Society for Metals (A.S.M.), through their year books and handbooks are particularly active in giving information on materials. The American Foundrymen's Association gives a wealth of information regarding cast materials in its Cast Metals Handbook. Publications in the fields of Welding, Chemical Engineering, and others, of course, furnish contributions to the knowledge of materials.

A knowledge of the characteristics and various physical properties of a material is necessary for practical design.

The physical properties of most importance are strength, rigidity, resistance to corrosion and to fatigue failure, and in some cases, weight. Other properties that may be of importance are hardness, impact resistance, heat and electrical conductivity, wear resistance, low friction, machinability, and weldability. When several of these characteristics are desired simultaneously, selection of the most suitable and the most economical material is sometimes difficult. Steam turbine blades, for instance, must have exceptional resistance to corrosion and erosion and at the same time possess sufficient strength to withstand the stresses induced by the steam pressure and centrifugal force. Steel containing 11½ to 12½ per cent chromium is widely used for this purpose, but is not wholly successful.

36. Classification. In general we may say that certain groups of materials are used mainly because they are abundantly available and cheap. This condition is particularly true of the *ferrous* group of metals.

A second group is resorted to when the corrosion resistance of the ferrous group is not sufficient. *Copper* has been the basic material in this group. For most purposes it is alloyed with zinc, tin, nickel, or aluminum. *Zinc* alone is also used for corrosion resistance and as a cheap material in die-casting alloys. *Cadmium*, another material of the zinc group, is gaining increasing acceptance as an excellent plating material for rust resistance, and *chromium* must also be mentioned for this same purpose. Iron alloys with *chromium* and *nickel* as the minority constituents, either alone or in combination, are now being used quite extensively for resistance to oxidation and heat. A surface impregnation of iron with aluminum, known under the trade name of *calorizing*, or *calite*, is also of importance for resistance to high temperature under oxidizing conditions.

A third group of materials is used for *lightness*. The most important material in this group is *aluminum*, but *magnesium*, a lighter material than aluminum, is gaining increasing attention. *Beryllium* is a metal, lighter than aluminum, though heavier than magnesium, with a higher melting point and a greater strength than either. At the present time it is entirely too rare and expensive to be used alone or as the basic material in an alloy for widespread application. It is used mainly as an alloying element to strengthen copper.

The enormously high strength that may be imparted to ferrous metals by alloying and heat treatment renders them capable of com-

peting with light-weight metals in light-weight design. Wood, on account of its low weight, is also an important material for light-weight design, especially for experimental or temporary structures.

Cheapness, corrosion resistance, and lightness are of course not the only guiding properties in the selection of materials. As previously mentioned, wear resistance, machinability, casting properties, even appearance have to be considered, and at times even control the selection of the material. *Manganese steel*, for instance, is a relatively inexpensive material used on account of extreme toughness and wear resistance, although it is not suitable for most applications where other materials may be used, because it cannot be machined by cutting tools and can be finished only by grinding. *Duriron*, a high-silicon cast iron, finds frequent application because of its resistance to a number of acids, but its use is limited by its low tensile strength and by the fact that it is machinable only by grinding. Certain rust resistant coatings, for instance, the phosphate type, give quite good protection for many purposes, but their somewhat unattractive appearance often limits their use where painting is not desired.

Since there is considerable overlapping and uncertainty in any attempted classification strictly according to use, we shall here review the materials in three arbitrary groups, based mainly on composition, namely: *ferrous* metals, *non-ferrous* metals, and *non-metals*.

Ferrous Metals

37. Ferrous Metals. Ferrous metals are the most commonly used, and, with proper alloying and treatment, they may be adapted to almost all simple needs. Special requirements, however, such as light weight, corrosion resistance, and electrical conductivity may make the use of some other material more desirable. The advantages of iron as a base metal, in addition to its abundance and low cost, are its strength and its adaptability to fabrication. It may be readily cast, forged, machined, and welded or brazed. Principal limitations are its weight (0.26 to 0.28 lb. per cu. in.) and its susceptibility to corrosion.

38. Cast Iron. Weight for weight, the cheapest metal to use is cast iron. It is composed of pure iron alloyed with 2 to 4 per cent of carbon, 0.25 to 3.00 per cent of silicon, and varying small percentages of manganese, sulphur, and phosphorus, the last two being impurities in most cases. Cast iron is also defined as an iron containing so much carbon or its equivalent that it is not malleable as cast. In soft *gray iron* most of the carbon occurs as free graphite flakes imbedded in the pure iron and iron carbide; this weakens it in tension and in shear. Ordinary

cast iron has an average strength of about 20,000 p.s.i. in tension, 80,000 p.s.i. in compression, and 30,000 p.s.i. in shear. The modulus of elasticity is not constant for cast iron but is usually taken as 15,000,000 p.s.i. Cast iron has no definite yield point, does not obey Hooke's law (its limit of proportionality being zero), but its elastic limit is high, in fact close to the ultimate strength. The permissible bending stress for cast iron is higher than the tensile stress for reasons set forth on page 17.

White cast iron has all the carbon chemically combined with the iron in the form of iron carbide. It has a white appearing fracture and is hard and brittle. Whereas gray iron usually contains over 1.25 per cent of silicon, white iron usually has less than 1.25. *Chilled cast iron* is gray iron with a white iron surface produced by rapid cooling in the mold. To accomplish this result the mold is fitted with cast iron liners (*chills*) to dissipate the heat rapidly.

Malleable iron is made by oxidizing or merely annealing white iron. If the casting is packed in an oxidizing or neutral material and held at a temperature of about 1650° F. for several days, the carbon is either oxidized away or precipitated in amorphous form. Malleable iron has a tensile strength of up to 50,000 p.s.i., and even more, with an elongation of 10 to 25 per cent, and is especially resistant to shock. It is easily machined if it is not of too great toughness.

The characteristics of cast iron are very much dependent upon the constituents and the foundry control. Special high strength is obtainable in *Pearlitic iron*, where the carbon is precipitated out in fine particles, uniformly distributed in a pearlitic matrix. By keeping the combined carbon low, these strong irons machine satisfactorily, although not so well as soft gray iron, because they contain low total carbon. Perlitic iron may be obtained by superheating the iron before casting or by controlled slow cooling. Tensile strengths of 40,000 to 50,000 p.s.i. are obtained. The addition of high percentages of steel scrap, resulting in the so-called *semi-steels*, gives a tensile strength of about 55,000 p.s.i.

39. Alloyed Cast Iron. The addition of *nickel* will refine the grain and increase the strength of cast iron. *Molybdenum* up to 1.5 per cent is the most effective means of increasing the tensile strength. Deoxidizing cast iron by the addition of calcium silicide at the ladle produces *Meehanite*, a strong, uniform iron with a tensile strength from 35,000 to over 50,000 p.s.i. and a modulus of elasticity of 15,000,000 to 20,000,000 p.s.i. as cast. Heat treatment increases the tensile strength to 110,000 p.s.i. and the modulus of elasticity to 26,000,000 p.s.i. Uniform structure is an important attribute of these high test irons.

High resistance to corrosion by acids (especially nitric acid) may be obtained by increasing the silicon content of white iron to 14.5 per cent, as in *Duriron*. Although its tensile strength is only about 10,000 p.s.i., it is very hard and is machinable only by grinding. Corrosion resistance, although to different acids, is a property also of *Ni-resist*, a nickel-chromium cast iron often containing other alloying ingredients. It has a tensile strength between 20,000 to 30,000 p.s.i. and is machinable with cutting tools.

40. Cast Steel. Cast steel normally contains not over 0.45 per cent of carbon and is usually poured from a melt made in an open hearth, or electric furnace. It is more difficult to pour than cast iron, having a higher pouring temperature, greater shrinkage, and greater susceptibility to blow holes from gas. Careful design and modern foundry practice produce reliable steel castings with ultimate strengths between 60,000 and 80,000 p.s.i. Addition of alloying elements, such as nickel, chromium, and manganese, improves the properties. Steel castings should be annealed to remove internal stress. Cast steel is more resistant to shock than cast iron, and it is frequently used in place of steel forgings.

41. Wrought Iron. Wrought iron is an iron resulting from the working of a mass of highly refined iron and slag. Its chief characteristic is the inclusion of fine particles of slag fibers distributed through the section. Wrought iron is the oldest of the ferrous metals and was originally produced directly from the ore by reduction with charcoal. Later it was produced from pig iron in puddling furnaces by burning out the carbon by means of iron oxide (mill scale or ore). Recently quantity production methods have been developed in which the slag is introduced artificially.*

Good wrought iron has a carbon content of 0.02 to 0.03 per cent, although it may be as high as 0.10 per cent. The physical properties are:

Tensile Strength	48,000 p.s.i.
Yield Point	30,000 p.s.i.
Elongation in 8 in.	25 per cent
Modulus of Elasticity	29,000,000

The resistance of wrought iron to corrosion is the reason it is used in sheet form or for pipes and containers. It is generally used in the rolled condition, although forgings are also used. It has good fatigue

* In the Byers process, pig iron is decarburized in a Bessemer furnace to low-carbon steel. This steel is poured into a ladle containing a molten silicate slag produced from iron oxide and silicious material. The slag is at a lower temperature than the steel and the steel solidifies in a fine granular form which sinks to the bottom of the ladle, forming a wrought-iron ball.

resistance. The metal may be welded, formed, or machined without difficulty.

42. Plain Carbon Steels. Steel differs from cast iron in that it has no carbon in the free state. The percentage of carbon varies from 0.08 to 1.5 with consequent difference in properties. Steel is classified according to carbon content approximately as follows: "Very mild," with carbon below 0.15; "mild," or "low carbon," with carbon from 0.15 to 0.30; "medium carbon," with carbon from 0.30 to 0.60; and "high carbon," or "hard," with carbon above 0.60. Up to 0.85 per cent carbon the tensile strength in the annealed condition increases directly and the ductility decreases with the carbon content. Both low and medium carbon steels are used generally for machine parts, whereas high carbon steels are used for springs or tools. Low carbon steels are readily welded and forged since they are plastic over an extensive temperature range. They are very ductile and hence are resistant to shock and impact, but are not responsive to heat treatment by quenching. Medium carbon steels are more difficult to forge and weld, but tensile strength and elastic limit can be increased considerably by quenching at the expense of lessened ductility. High carbon steels are difficult to forge and weld but may be hardened to a good cutting edge by quenching.

43. Heat-Treatment. The susceptibility to heat-treatment is a particularly important property of steels. Heat-treatment has for its object the improvement of strength, ductility, hardness, or general dependability. There are many variations of the process, the most important of which are:

1. *Annealing:* Heating above the critical temperature range (to about 1550 to 1700° F. depending upon the carbon content), holding at that point for some time, and then cooling slowly through that range in the furnace, or other control medium. This process causes a softening of the steel and a release of the internal stresses. Parts are annealed to (a) restore the crystalline structure disturbed by cold working, (b) remove internal strains produced by heat-treatment, (c) refine the grain size to produce ductility, and (d) soften for machining.

Normalizing: A form of annealing where the cooling takes place in still air at ordinary temperature. It is used to release internal strains caused by cold working.

2. *Quenching:* Rapid cooling by immersion in a cooling medium, after heating above the critical point.* The object is to arrest the structural change and bring down to ordinary temperatures a composi-

* Temperature at which the structure changes rather abruptly.

tion which gives strength and hardness. Quenching in water or brine is more drastic than quenching in oil.

3. *Tempering* (also called "drawing"): Reheating a previously hardened piece to a point below the critical temperature and cooling

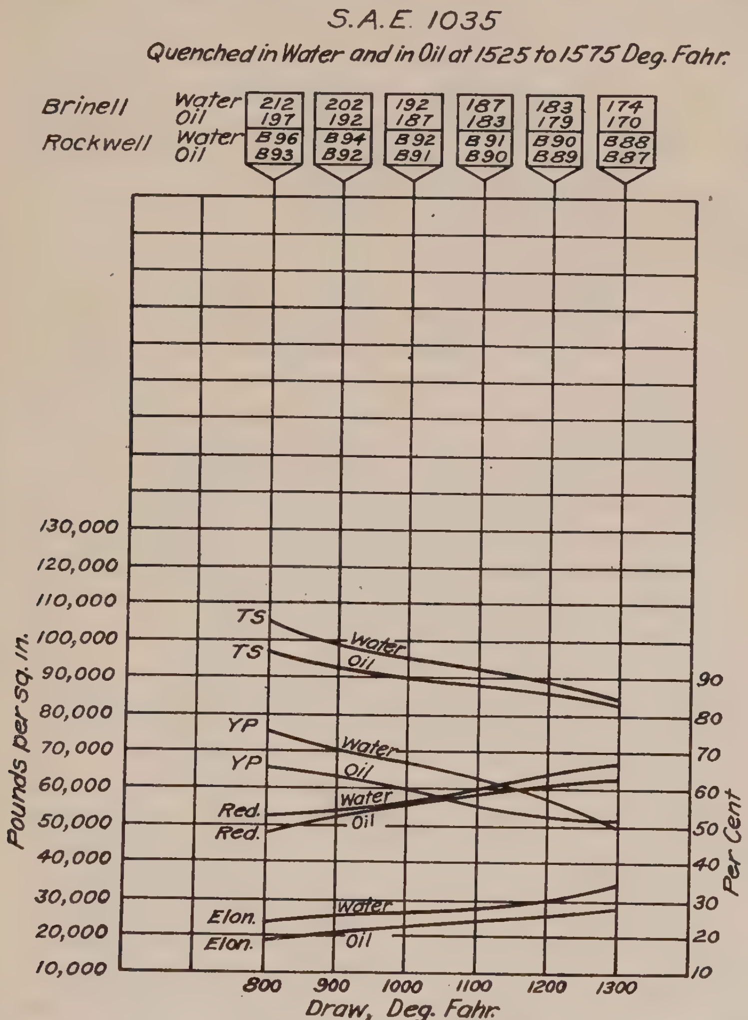


FIG. 37. CHART SHOWING THE EFFECT OF HEAT-TREATMENT ON S.A.E. 1035 STEEL.

at the desired rate. In heat-treating, the first quenching usually leaves the material harder and more brittle than desired. Then, in order to increase the ductility without too great a sacrifice of strength, the

material is heated to a temperature below the critical point, thus allowing a slower transition of molecular structure with consequent better control and a release of internal stress. The higher the drawing temperature, the greater the increase in ductility and the decrease in strength. The Handbook of the Society of Automotive Engineers gives charts showing the effect of heat-treatment on various materials, one of which is shown in Fig. 37.

4. *Case Hardening:* Producing a hard surface, or case, by carburizing and heat-treating or by nitriding. *Carburizing* gives a hard layer of high carbon steel over a tough core of low or medium carbon steel. This process of adding carbon to the ferrous alloys consists of heating the parts to a point below melting while they are packed in a carbonaceous material or exposed to a carboniferous gas. Carbon is absorbed to a considerable depth; this forms a layer of high carbon steel which is then hardened by heat-treatment. *Nitriding* is the process of exposing the ferrous alloy to ammonia vapor at a temperature of about 950° F. This converts the surface, at least partly, to a nitride, thus forming a hard case which needs no further heat-treatment. The high temperatures that accompany carburization and subsequent quenching tend to cause warping of the piece and the loss of surface smoothness. With nitriding, all heat-treatment and machining may be done previously to nitriding and the lower nitriding temperature causes no appreciable distortion or surface roughening. The surface resulting from each method is very hard and wear resistant. The nitrided surface is excellent for wear resistance, but because of the thinness of the case it is not so satisfactory as a carburized surface for sustaining concentrated loads such as occur, for instance, in gears. Nitriding is most effective on special medium carbon steel containing aluminum, chromium, and molybdenum.

44. Alloy Steels. When metals are dissolved in each other and then solidified, an *alloy* results. Alloy steel is obtained when the other elements added to the iron and carbon are in sufficient quantities to influence the physical properties. Practically all alloy steels must undergo special heat-treatment to obtain the properties desired. The steel alloying elements are effective, in general, by (a) influencing the amount of carbon in solid solution, (b) affecting the grain size, (c) removing impurities, (d) changing the critical point, and (e) retarding the rate of structural change on cooling. The last two effects greatly facilitate heat-treating.

The alloying elements generally used in steel and the effects they produce are as follows:

1. *Nickel* refines the grain, and gives increased strength and hardness without a proportionate decrease of ductility.

2. *Silicon* increases the strength when used in amounts up to 2 per cent.

3. *Chromium*, up to 2 per cent, retards the rate of structural change and produces a fine grain structure. It is used to give high strength, and particularly high elastic limit, as well as hardness, with some, but not serious, loss of ductility.

4. *Vanadium*, a scavenger for removing oxygen, produces a fine grain with considerable toughness. It also intensifies the effect of chromium.

5. *Tungsten* raises the critical point and makes steel air hardening; *molybdenum* has a similar effect, but also improves both strength and ductility, as well as machinability of alloy steels.

6. *Manganese* retards the rate of structural change and at 14 per cent produces a steel which after quenching from 1800° F. is very ductile and has great resistance to abrasion.

7. *Copper* is used for corrosion resistance and for high-strength low-alloy steels. Such steels are cheaper than those with higher percentages of expensive alloying elements.

Several of these elements may be used simultaneously to obtain special physical properties when given a double heat-treatment. High elastic limit with ample ductility, hard wear-resisting surfaces combined with high core strength and toughness, high impact and fatigue resistance, are some of the properties that are readily attainable.

45. S.A.E. Steel Classification. Specifications and designation numbers for alloy steels developed by the Society of Automotive Engineers have come into widespread use. The designation number generally contains four figures, the first figure indicates the class of the steel, the second the approximate percentage of the principal alloying element, and the last two the "points" of carbon. (A "point" is one hundredth of one per cent.) The basic figures for the various classes are:

1. Plain carbon steels.
2. Nickel steels.
3. Nickel-chromium steels.
4. Molybdenum steels.
5. Chromium steels.
6. Chromium-vanadium steels.
7. Tungsten steels.
9. Silico-manganese steels.

S.A.E. steel No. 1035, for which the chart is shown on page 57, is a plain carbon steel (indicated by the "10— —") containing 0.35 per cent carbon (indicated by the "— —35"). Similarly S.A.E. 2330 is a nickel steel of approximately 3 per cent nickel (3.25 to 3.75) and approximately 0.30 per cent carbon (0.25 to 0.35). The chart

for this steel is shown in Fig. 38. It will be noted that in its soft condition the 3½ per cent nickel steel is not superior in characteristics to the plain carbon steel. Heat-treatment, however, greatly increases its strength.

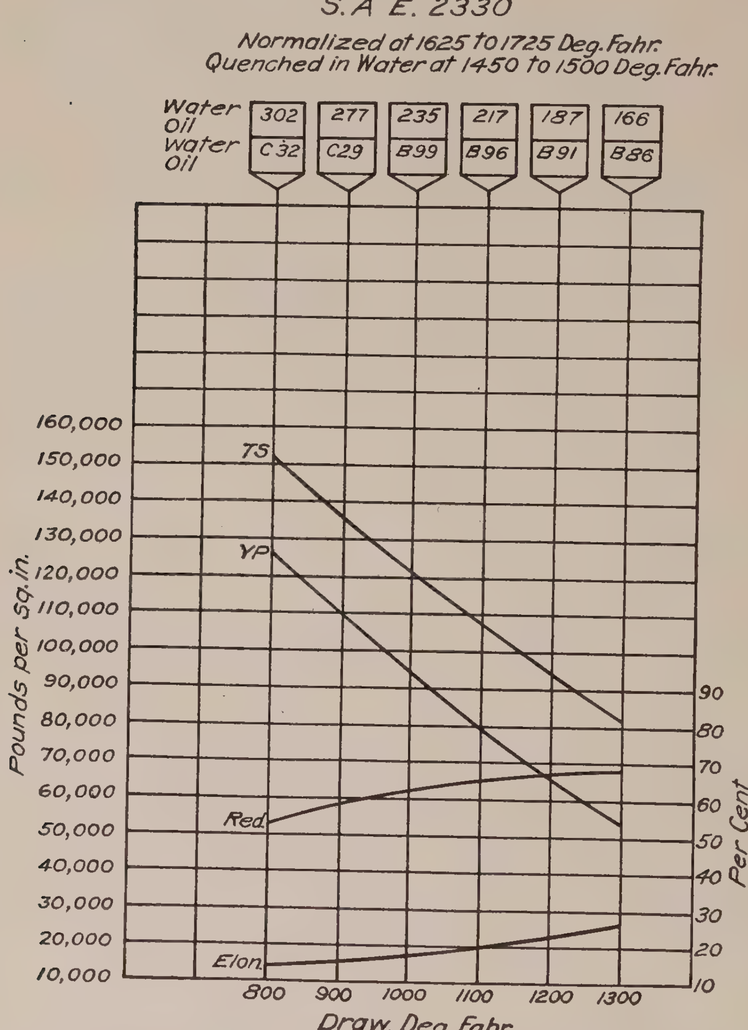


FIG. 38. CHART SHOWING THE EFFECT OF HEAT-TREATMENT ON S.A.E. 2330 STEEL.

46. Survey of Alloy Steels. The alloy steels most widely used for machine parts at the present time are 3½ per cent nickel steel, nickel-chromium steel, and chromium-vanadium steel. Nickel-chromium steel is extensively used in the automotive industry. Chromium-

vanadium steels have similar properties and are also used for springs, as are silico-manganese steels. On account of their very high elastic limit, plain chromium steels have been widely used for balls, rollers, and races of ball and roller bearings, although molybdenum steels are now often preferred. Plain vanadium steels, containing 0.2 per cent vanadium, are very tough and dependable, having in the annealed state an ultimate strength of about 80,000 p.s.i. Manganese steel with about 14 per cent manganese, when quenched in water from 1800° F., will have an elongation of 50 per cent with an elastic limit of 50,000 p.s.i. It is extremely tough and becomes very hard when cold-worked. As it cannot be machined, it must be finished by grinding. It is used for rock crushers, steam shovel jaws, and other applications where high wear and shock resistance are required.

The following steels are selected from the S.A.E. classifications to indicate the recommended uses:

No. 1020 (*plain carbon steel*) machine steel. Forges and machines well but is not screw-machine stock. Can be used for a variety of forged, machined, and case-hardened parts where strength is not important. Strength not much affected by heat-treatment.

Nos. 1025 and 1030 (*plain carbon steel*). Responds to heat-treatment better than No. 1020 and can be used for forged, machined, or cold-worked parts requiring more strength.

Nos. 1035 and 1040 (*plain medium carbon steel*). Good machining and heat-treating properties. Suitable for small and medium size forgings possessing fairly high physical properties.

Nos. 1112 and 1120 (*carbon steel with 0.075 to 0.15 per cent sulphur*). For general screw-machine parts that do not demand special properties. The addition of sulphur makes the material free-cutting.

No. 2315 (*3½ per cent nickel steel*). Primarily for case-hardening. Used for gears. Responds to various heat-treatments.

Nos. 2330 and 2335 (*3½ per cent nickel steel*). For heat-treated parts requiring considerable strength and toughness, such as axles and shafts. Should not be case-hardened.

No. 2512 (*5 per cent nickel steel*). For case-hardened parts requiring an exceptionally tough core after heat-treatment.

Nos. 3115 and 3120 (*nickel-chromium steels*). For case-hardening. Used for gears of high accuracy and strength.

Nos. 3125 and 3130 (*nickel-chromium steels*). Used interchangeably with 2330 and 2335 for greater strength and toughness than obtainable with plain carbon steels. Should not be case-hardened.

No. 3230 (*nickel-chromium steel*). For heat-treated, machined, and forged parts used for severe service conditions that demand higher physical properties than are obtainable with 2330, 2335, 3125, or 3130. Responds to heat-treatment in large sections better than those just mentioned.

47. Low Alloy Steels. "Low alloy" steels are the steels containing relatively low percentages of the expensive alloying elements, but

possessing a yield point and ultimate strength greater than plain carbon steels. These steels generally are copper bearing and possess good resistance to corrosion as well as good machinability and weldability. They are very desirable for structural purposes where high strength and light weight are essential requirements. Development of these steels is still in the early stages and the future should see their extensive use. Several mills now supply these materials in the form of sheets, strips, plates, structural shapes, and tubes at about twice the cost of plain carbon steel. They are sold under trade names, such as Cromansil, Yoloy, Hi-Strength, Corten, etc. The following is a typical composition:

Element	Per Cent
Carbon	0.10 to 0.30
Manganese	0.50 to 1.00
Phosphorus	low to 0.12
Sulphur	low
Nickel	0.40 to 0.80
Copper	0.50 to 1.50
Silicon or Molybdenum	up to 0.20

Yield point	60,000 to 75,000 p.s.i.
Tensile strength	70,000 to 90,000 p.s.i.
Elongation in 8 in.	20 per cent.

Heating to a temperature of 900 to 1000° F. will raise the yield point to about 90,000 p.s.i. When copper is present, heat-treating causes precipitation hardening (hardening in storage or use). These steels are also suitable for torch hardening. Rapid quenching after heating with a torch flame gives a surface hardness dependent upon the carbon content of the steel. This process is used for hardening gear teeth and causes very little distortion.

48. Stainless Steels. The *stainless steels* are another group of alloy steels, used principally for their resistance to corrosion. Stainless steels may be classified into three groups according to the chromium content, which determines the cost as well as the amount of corrosion resistance:

(a) "18–8," containing 18 to 30 per cent chromium and 8 to 12 per cent nickel, gives best resistance.

(b) 12 to 20 per cent chromium without other alloying element includes the original stainless steel with 13 per cent chromium and gives good resistance, particularly when polished.

(c) 4 to 7 per cent chromium, either with or without additional alloying elements, such as molybdenum and tungsten, gives mild resistance.

The cheaper alloy with the lower chromium percentage is satisfactory for use in contact with high temperature oil. The resistance to corrosion in the atmosphere and in the presence of various organic acids makes these alloys well adapted to food storage.

Good strength at high temperatures is another valuable characteristic of this class of steel. "Ascaloy" 55 of the Allegheny Steel Company is recommended for service up to 2150° F. Great progress is being made in improving the characteristics of these steels. Proper selection allows forging, spinning, deep drawing, soldering, and welding.

Nickel promotes the formation of so-called *Austenitic* steel which cannot be hardened to a cutting edge, but requires only grinding or pickling to be corrosion resistant. *Chromium* promotes the formation of so-called martensitic steel which takes a good cutting edge, but requires polishing to be corrosion resistant. Responsiveness to heat treatment ceases if the chromium exceeds 16 per cent.

Non-Ferrous Metals

49. Corrosion Resistant. The non-ferrous metals are usually more expensive than the ferrous metals; hence they are used only when some particular feature not possessed by iron or steel is required. Resistance to corrosion, so lacking in the ordinary steels, is a requirement that frequently justifies the use of the more expensive non-ferrous metals. Other special characteristics, such as electrical conductivity, heat conductivity, non-magnetic properties, good machinability, may influence their use.

Lead is much used as a liner for acid containers and for piping, but it possesses so little strength that it cannot be used alone to sustain stress. It is used in bearing alloys, principally to give plasticity and a smooth surface.

Zinc is a soft metal used in the unalloyed state generally in the form of sheets and strips, and for steel coating. Its principal use in the alloyed form is in die castings. Well known alloys of zinc are those containing about 93 per cent zinc, 4 per cent aluminum, 3 per cent copper or less, and not more than 0.1 per cent magnesium. The two alloys recognized by the A.S.T.M., the S.A.E., and the Cast Metals Handbook are designated according to the last named reference as F-11, which contains 3 per cent copper, and F-12, which does not contain copper. These alloys have a tensile strength of from 40,000 to 45,000 p.s.i. or better. Alloy F-12 is weaker, but it is more ductile and impact resistant, especially if comparison is made after prolonged exposure to water vapor at 95° C. The compressive strength of F-11 is about twice the tensile strength.

Copper and its alloys are most widely used to resist corrosion. Copper tubing is extensively used for condenser tubes, water lines, and oil lines. Being very malleable, copper may be rolled, drawn, and spun easily. It is difficult to cast, but castings may be produced if boron is

added. As copper is an excellent conductor of heat, it is not readily weldable. As a commercial conductor of electricity it has no equal. The basic copper alloys are the bronzes and brasses.

Machinery bronze is 90 per cent or more of copper with tin. *Standard brass* is about 67 per cent copper with zinc. Deoxidizers are used to give increased strength and sometimes denote the characteristic name of the alloy. *Phosphor bronze* in the rolled form is as strong and ductile as medium carbon steel. *Manganese bronze*, made from 40 per cent zinc brass, is as strong as steel in the cast, forged, or rolled condition. *Aluminum bronze* is 90 per cent copper and 10 per cent aluminum and has the strength of mild steel. It has good wear resistance and is much used for worm gears.

Because of their corrosion resistance and ease of forming, bronze and brass are still widely used in valve and carburetor parts and similar applications. Bronze is generally used in the cast form, while brass is cast or rolled. Bronze is used as a material for bearings and gears for reasons other than its corrosion resistance. Bronze consists of hard particles imbedded in a softer matrix. In a bearing, the hard particles support the load, the softer matrix allowing the hard particles to adjust themselves to the surface of the shaft. As the softer material wears away, small spaces are left for the retention of the lubricant; this leaves the mating surface, generally steel, unimpaired. Bronze has a high compressive strength, machines to a smooth surface, and is resistant to the corrosive action of any acids in lubricating oils. Since the bearing surface must be replaced after a certain amount of wear has occurred, the bronze is applied in the form of replaceable shells or liners in order to limit the expense of the material. For gears, particularly worm gears, bronze is used for the larger gear and steel for the smaller gear or worm. To limit the cost, a bronze rim may be bolted to a cast iron hub and spider.

The Society of Automotive Engineers has standardized various bronze and brass alloys as shown in the typical specifications below:

Specification No. 62, Hard Cast Bronze

Element	Per Cent	Properties	
Copper	86.00 to 89.00	Ult. tensile strength	30,000 p.s.i.
Tin	9.00 to 11.00	Yield point	15,000 p.s.i.
Lead, max.	0.20	Elongation in 2 in.	14 per cent.
Iron, max	0.06		
Zinc	1.00 to 3.00		

No. 62 is a strong, general utility bronze suited for both gears and bearings. *Leaded gun metal*, especially suited for bushings, is obtained if lead is substituted for zinc. (See Chapter on *Plain Bearings* for other bearing metals and their properties.)

Specification No. 41, Yellow Brass

Element	Per Cent	Properties	
Copper	62.00 to 65.00	Ult. tensile strength	25,000 p.s.i.
Lead	2.00 to 4.00	Yield point	12,000 p.s.i.
Zinc	31.00 to 36.00	Elongation in 2 in.	20 per cent.
Tin	1.00		
Traces of iron, etc.			

This brass gives good commercial castings, is cheap and easily machined.

Specification No. 43, Manganese Bronze

Element	Per Cent	Properties—Cast	
Copper	53.00 to 62.00	Ult. tensile strength	60,000 p.s.i.
Zinc	38.00 to 47.00	Yield point	30,000 p.s.i.
Lead, max.	0.15	Elongation in 2 in.	15.0 per cent.
Manganese	trace		

This high strength brass is not a true bronze. It is deoxidized by manganese. The addition of iron and other metals will raise the strength still higher. *Tobin bronze* and *naval bronze* are similar alloys.

These various copper alloys give strong and neat castings and may be used instead of steel forgings where the die expense would be prohibitive. They may be forged and rolled into rods, and as such are used for valve stems, subaqueous shafting, etc. Special shapes may be extruded and some of the copper alloys may be die cast. Die pressed parts may be obtained with an ultimate strength of 45,000 p.s.i. and a yield point of 18,000 p.s.i. These materials machine easily and cause but little wear on the dies. Because of their corrosion resistance properties they are used for various machine parts such as screw propellers, pump impellers, turbine runners, etc.

Everdure, a copper alloy with 3 per cent silicon and 1 per cent manganese, gives an ultimate strength of 90,000 p.s.i. for die pressed parts. It has high corrosion resistance and can be welded to steel and non-ferrous metals.

Beryllium copper is a copper alloy of great strength and fatigue resistance obtained by the addition of quantities up to 3 per cent of beryllium, an expensive, light weight metal. In the wrought condition the alloy is very well adapted for springs and electrical parts subjected to a large number of stress repetitions. Hard wearing surfaces and high corrosion resistance are obtainable. One alloy can be heat-treated to give a tensile strength of 82,000 p.s.i. with an elongation of about 20 per cent. Being precipitation hardening, castings may be machined in the annealed condition and then easily hardened by heat-treatment. A 2.25 per cent beryllium-copper alloy after precipitation hardening gave a tensile strength of 120,000 p.s.i., yield point of

90,000 p.s.i., Brinell hardness of 400, and electrical conductivity 35 per cent of standard annealed copper. With more beryllium (up to 3 per cent) even higher values have been attained. Strong non-sparking tools have been made of beryliium copper. "Be-Cu" has been used for marine propellers, cams, steam fittings, and valve and injector parts. Small quantities of beryllium are very effective in deoxidizing copper castings. Die cast parts have higher strength than sand castings.

Monel metal is a natural alloy of 68 per cent nickel, 29 per cent copper, with iron, manganese, and other minor ingredients. Monel is stronger and tougher than the common steels, and is more ductile than a steel of equal strength. Ultimate tensile strengths of over 100,000 p.s.i. and yield points of over 80,000 p.s.i. are obtainable with an elongation of 25 per cent in 2 in. for rolled or forged parts. The strength of castings is around 70,000 p.s.i. Addition of a small amount of aluminum gives *K Monel* having an ultimate strength of over 160,000 p.s.i. and a yield point of about 100,000 p.s.i. with an elongation of 15 per cent in 2 in. Strength properties are retained up to 400° F. Heat-treating is effective with this alloy.

High strength at elevated temperatures and resistance to corrosion against most agents are valuable properties of Monel metal. It is widely used in marine applications. Monel can be welded and brazed, and is machinable but tough. It can be forged, drawn, and spun quite as readily as steel.

Ordinary Aluminum is resistant to ordinary atmospheric corrosion and certain acids but is not suitable for use near salt water. Its corrosion resistance is increased by anodic treatment of the surface. Very pure aluminum may be rolled over the surface of aluminum alloys and produce corrosion resistance even against salt water. The properties of aluminum are discussed more fully under the heading of light-weight high-strength materials.

50. Surface Treatments. Various surface treatments are in extensive use to protect metals, particularly ferrous metals, from corrosion. The processes commonly used for preventing the rusting of steel are *galvanizing*, *sherardizing*, *plating*, and *parkerizing*, or similar processes. *Galvanizing* is fundamentally a metal coating by means of electro-deposition out of a solution, but in common trade parlance it usually refers to zinc coating only. The best zinc coatings are produced by electro-deposition, and galvanizing by this method is receiving increasing application. The usual way to galvanize, however, is to dip the object in molten zinc, and this method is used to galvanize bulk materials like sheets and pipes. *Sherardizing* consists in heating

objects in powdered zinc, which sublimates and penetrates the pores of the surface to be treated. It produces a tough, wear resisting cover for castings and forgings, and is usually applied to smaller parts.

Plating is the electro-deposition of the protective metal on the surface to be protected. Zinc, cadmium, copper, nickel, and chromium are metals currently used for plating. Zinc is the cheapest of the group and is extensively used for application on bulk goods, as already explained. It is being replaced to a considerable extent by *cadmium* on shaped articles, and particularly on finished articles, the dimensions of which should not change perceptibly in the plating process. Cadmium deposits extremely well and prevents the spread of rust even if the coating is thin with perhaps slight imperfections. *Nickel*, and nickel on copper, have been standard platings for a long time. Nickel, however, has some tendency to flake and tarnish. *Chromium* has a higher luster than any of the other plates. It is hard, resists scratching, and wears very well. It can be applied on gages and other precision measuring devices, increasing their life materially, even if the deposit is very thin. Under-size gages may be restored to size by chromium plating and refinishing. Chromium does not adhere well to steel, and it is so porous that it gives but little protection against rusting unless it is deposited on an underlying layer of nickel (which in turn may be deposited on copper). Moreover, it deposits with difficulty in corners and recesses, and it may be necessary to have electrodes shaped like these recesses to make it deposit satisfactorily.

The type of protection of which *parkerizing* is a well known representative is obtained by dipping the metal object in a solution which converts the surface into a salt, such as a phosphate or a chromate. Parkerizing gives a mat gray or black phosphate surface, which may be used without further coating, and has the advantage that it does not noticeably change finished dimensions. *Bonderizing* produces a surface that is porous and permits paint to adhere particularly well. *Electro-granodizing* consists in electro-plating with zinc or cadmium, and then by a dipping process converting the surface into a phosphate of the plating metal. The chromodine * process converts the surface into a chromate and is said to furnish superior protection when the object is apt to be dented or bent.

51. High-Strength Light-Weight Materials. The materials used mainly for their lightness are aluminum and magnesium and their alloys, wood, and the plastic resins. High-strength steel, however, on account of its superior strength, gives a very good ratio of strength to

* See article *New rustproofing processes*, Automotive Industries, September 21, 1935, p. 390.

weight, and in many cases there is a tendency to use it instead of the light-weight metals. Where corrosion resistance is necessary, high-chromium or nickel-chromium steels may be used. The low alloy steels will give a weight reduction of one-third over plain carbon steels. The accompanying table gives the strength-weight factor for some of the common materials and is obtained by dividing the ultimate strength in pounds per square inch by the specific gravity.

TABLE 11

Material	Strength-Weight Ratio/1000
Music wire	400,000 ÷ 7.85 = 51
High alloy steel	150,000 to 200,000 ÷ 7.85 = 19 to 25
Duralumin	58,000 ÷ 2.85 = 20
Magnesium dowmetal	20,000 to 40,000 ÷ 1.80 = 11 to 22
Low alloy steel	90,000 ÷ 7.85 = 11.5
Mild steel—normalized	55,000 ÷ 7.85 = 7.0
Aluminum 2S (castings alloy)	13,000 to 24,000 ÷ 2.70 = 4.8 to 9
Monel metal	85,000 to 140,000 ÷ 8.8 = 9.7 to 16
Wood (compression)	(dependent upon kind, age, condition) 10 to 18

52. Aluminum. Pure aluminum is weak and its use is confined to rolled, pressed, and drawn articles, such as kitchen utensils. Parts requiring strength are made of aluminum alloys, which may be cast, rolled, forged, and extruded. Certain aluminum alloys, such as duralumin, are of great value where high strength-weight ratio is required, as in parts where dynamic stresses are important and where the supported weight must be kept low.

There is at present quite a bewildering array of cast aluminum alloys. Some are intended for sand casting, others for die castings, and still others for casting in permanent molds. Parts may be used in the original cast condition, or they may be subjected to a subsequent heat treatment to bring out their physical properties. These alloys are designated by number symbols, among which possibly those of the Aluminum Company of America and those of the Society of Automotive Engineers are best known. But the United States Army, the American Society for Testing Materials, the American Foundrymen's Association, and no doubt other organizations have numbers of their own.

In this country, the main alloying elements used to increase strength are copper, silicon, and magnesium, while in Europe zinc has also been used. Copper improves strength, but reduces ductility very sharply. The basic *copper* alloy (S.A.E. 30, Al. Co. 12) contains about 8 per cent copper. More modern alloys contain also small percentages of silicon, whereas the die casting alloys contain also zinc and nickel. The copper alloys, sand cast, have a tensile strength of 18,000 to 22,000 p.s.i., a yield point (at 0.2 per cent permanent set) of about 14,000 p.s.i.,

an elongation of about 1 to 2 per cent, and a specific gravity of about 2.85. The zinc alloy has somewhat higher values for both strength and elongation than these, but is less resistant to corrosion and heat.

The basic *silicon* alloys (S.A.E. 35 and 37, Al. Co. 43 and 47) contain from .5 to 12 per cent silicon; higher silicon gives higher strength and ductility; lower silicon has excellent casting properties. Both alloys possess good corrosion resistance, especially after the anodizing treatment. These alloys have a tensile strength of 19,000 to 26,000 p.s.i., a yield point of 9000 to 11,000 p.s.i., an elongation of 4 to 8 per cent, and a specific gravity of 2.65 to 2.66. They are somewhat difficult to machine.

The basic *aluminum-magnesium* alloy (S.A.E. 320, Al. Co. 214) contains 3.25 to 4.25 per cent magnesium. The aluminum-magnesium alloys have low specific gravity, excellent combination of strength and ductility, machinability, and tarnish resistance. They have a tensile strength of about 25,000 p.s.i., a yield point of about 12,000 p.s.i., an elongation, sand cast, of about 9 per cent, and a specific gravity of about 2.64.

The property of strength increase by heat treatment is imparted to the aluminum castings alloys by variations in amount of the main alloying element and the addition of small quantities of iron, nickel, etc.

The basic *wrought* aluminum alloy is *duralumin* (S.A.E. 26, Al. Co. 17 S) invented in 1903 by Wilm and of far-reaching importance in the development of aircraft of all kinds. It contains a minimum of 92 per cent aluminum, with 3.5 to 4.5 per cent copper, 0.2 to 0.75 per cent magnesium, and 0.4 to 0.1 per cent manganese. Its properties are developed by a rather complicated process of hot working, quenching, cold working, and *aging*. The properties attained by the last three processes are lost by renewed hot working, or by annealing. This characteristic is a disadvantage that has led to intensive efforts to develop more stable alloys. A very important one in this respect is S.A.E. 27, Al. Co. 25 S, which has excellent hot working properties. It contains some silicon, but no magnesium. Fully heat-treated, it has a tensile strength of 55,000 to 63,000 p.s.i., a yield point of 30,000 to 43,000 p.s.i., and an elongation of 9 to 22 per cent. In comparison, duralumin has a tensile strength of 50,000 to 55,000 p.s.i., a yield point of about 30,000 p.s.i., and an elongation of 16 to 18 per cent. (All data from the S.A.E. Handbook of 1936.)

The Aluminum Company of America, which occupies a dominating position in the aluminum industry, has introduced for its alloys a nomenclature that is finding increasing acceptance, particularly for wrought materials. The alloy carries a designation number which, in the wrought condition, is followed by the letter *S*. Heat

treatment is indicated by the letter *T*, which without modification indicates maximum strength. *O* designates annealing, *W* cold working, and so forth.

At the present time duralumin (17 S) is used mainly for bulk goods, such as plates, tubes, rods, rivets, etc.; alloy 25 S on the other hand is used for forgings, such as automotive connecting rods and other similar parts.

Pure aluminum is very resistant to corrosion. Alloys are less so, particularly if they have been subjected to "aging" at elevated temperature to bring out their strength ("precipitation heat treatment"). The alloys containing manganese are most resistant to corrosion, and since duralumin contains manganese and is aged at ordinary temperature it resists corrosion comparatively well. This resistance can be materially increased, however, if the surface of duralumin is coated with a thin sheet of pure aluminum. The product so obtained is called "Alclad."

53. Magnesium. Lighter than aluminum, magnesium and its alloys are used where extreme lightness is desirable. A specific gravity of 1.8 is attainable, and this means a material one-third lighter than aluminum. Magnesium alloys are used most extensively in aircraft construction and also for large parts that must be moved by hand. Magnesium is resistant to ordinary atmospheric corrosion, and magnesium alloys are being used, particularly in Germany, for combustion engine pistons. However, these materials are sensitive to salty air and salt water. Magnesium and its alloys are resistant to nearly all alkalies, oils, and many organic hydrocarbons, but are attacked by most acids and by chloride solutions.

The Dow Chemical Company occupies a leading position in magnesium production in this country and calls its magnesium alloys "Dowmetal." The various grades of Dowmetal contain from 85 to 95 per cent magnesium, up to 12 per cent of aluminum, and some manganese. In one alloy, copper and cadmium are added to give high thermal conductivity. Dowmetal may be sand or die cast; forged, rolled into sheet and plate; also extruded into shapes, bars, rods, and tubes. It is not readily blanked, drawn, or pressed. It is easily machined and may be gas and spot welded, although special precautions are necessary in each case. Generous fillets must be used in the design of all parts.

The Dow Chemical Company designates the different alloys by a letter. Dowmetal F, used for forgings, rolled and extruded parts, has a tensile strength of about 40,000 p.s.i., a yield point of about 30,000 p.s.i., and an elongation of about 15 per cent in 2 in. Dowmetal A has a

tensile strength of over 24,000 p.s.i., a yield point of over 10,000 p.s.i., an elongation of about 10 per cent in 2 in. It is used for unheat-treated sand castings, which have a fair degree of toughness. Proper heat-treatment will raise the tensile strength to 37,000 p.s.i. and the yield point to 16,000 p.s.i. Heat-treatment is of the "solution" and "aging" type, such as used in aluminum. Wrought Dowmetal A has a tensile strength of almost 50,000 p.s.i. and a yield point of 37,000 p.s.i., with an elongation of 15 per cent in 2 in.

Beryllium is a metal lighter than aluminum with a high yield point and a high melting point. At present its high production cost limits its use. Its importance as an alloying element with copper has been mentioned.

Non-Metals

54. Non-Metals. *Wood* is an excellent high-strength, light-weight material, but its use is reduced by its inflammability and its tendency to warp and rot. Plywood construction minimizes the tendency to warp, and has many applications for strong light structures. Since it is composed of thin sheets of wood glued together, cross-grained, its life and effectiveness are dependent upon the glue binder.

In addition to wood, we have many natural and synthetic materials that are used in the construction of machine parts. Gears and pulleys are made of compressed *paper*, *rawhide*, vulcanized *fiber*, and laminated *phenol plastics*, principally to absorb shock and for quiet operation. Vulcanized fiber is used also for gaskets, electrical insulation, and wearing surfaces. *Bakelite*, a phenol plastic, has had considerable success as a bearing material for certain purposes and is also used in the molded state for many parts of instruments, electrical devices, etc. Other synthetic plastics have been developed for special uses, and at present a very active development is taking place in this field.

Belts are made of *leather*, *cotton*, or other fibers, some of which may be impregnated with resinous substances. Rubber is used extensively to vulcanize together the cotton layers of textile belts.

Rubber can be made in various degrees of hardness and elasticity. In the hard state it is used for handles, electrical insulators, linings for containers, valves, and pumps exposed to acids. Soft resilient rubber impregnated with such materials as carbon or zinc oxide resists wear very effectively and is used as a lining for surfaces subjected to abrasion. Bearings of rubber are effective in gritty water. Rubber is very resistant to repeated flexings and impacts. It is therefore extensively used for bumpers, cushions, tires, etc. It is an excellent material to reduce noise and absorb vibration, preferably carrying the load in

shear. Rubber can be vulcanized directly to metal. Its greatest defects, oxidation and deterioration with age, sensitivity to oil and gasoline, have been overcome with synthetic rubber-like materials such

TABLE 12

PHYSICAL PROPERTIES OF MATERIALS

Material	Tensile Strength	Compressive Strength	Yield Point	Modulus of Elasticity $\times 10^{-6}$	Elongation in 2 in.
Gray cast iron, avg.	20,000	80,000		15	
Gray cast iron, S.A.E. 111	30,000			15	
Nickel cast iron	38,000			15	
Alloy cast iron, S.A.E. 122	45,000			15	
Meehanite iron	35,000	135,000		15	
Malleable iron	57,000		37,500	25	22
Wrought iron	48,000	same as tensile	26,000	28.5	
Cast steel (hard)	80,000		36,000	30	17
Cast steel (med.)	70,000		31,500	30	20
Cast steel (soft)	60,000		27,000	30	24
Nickel steel cast					
norm. and drawn	85,000		51,000	30	29
quenched	146,000		128,000	30	20
14% manganese cast steel	140,000		50,000	30	50
Wrought carbon steel					
S.A.E. 1010	58,000		43,000	30	40
S.A.E. 1020	62,000		45,000	30	35
S.A.E. 1025	70,000		46,000	30	32
S.A.E. 1035	75,000		48,000	30	30
Low alloy steels					
Cromansil	90,000		62,000	30	25
Yoloy	92,000		73,000	30	29
Corten	74,000		50,000	30	25
High alloy steels					
Nickel, S.A.E. 2330	82,000 to 152,000		57,000 to 126,000	30	14 to 26
Ni-Cr S.A.E. 3130	92,000 to 168,000		68,000 to 141,000	30	10 to 29
Moly. S.A.E. 4140	110,000 to 180,000		85,000 to 155,000	30	10 to 21
Cr-Va S.A.E. 6140	108,000 to 180,000		87,000 to 160,000	30	13 to 21
Stainless steel					
13% Cr, 0.10% C.	65,000 to 125,000		35,000 to 100,000	30	20 to 35
18% Ni, 8% Cr (KA-2)	85,000 to 95,000		30,000 to 40,000	30	55
Aluminum					
Pure, soft	13,000		4,000	10.3	35
Pure, cold worked	24,000		21,000	10.3	10
Duralumin, soft (17S-O)	25,000 to 35,000		7,000 to 10,000	10.3	14 to 22
Duralumin, heat treated (17S-RT)	55,000 to 63,000		30,000 to 40,000	10.3	10 to 25

TABLE 12—*Continued*

Material	Tensile Strength	Compressive Strength	Yield Point	Modulus of Elasticity $\times 10^{-6}$	Elongation in 2 in.
Cast aluminum alloys					
Copper, S.A.E. 30	18,000 to 22,000		14,000	10.3	1 to 3
Silicon, S.A.E. 35 & 37	19,000 to 26,000		9,000 to 11,000	10.3	4 to 8
Magnesium alloy					
SAE 320, Al. Co. 214	25,000		12,000	10.3	9
Permanent mold, Cu. or Si, heat-tr.	31,000 to 45,000		16,000 to 40,000	10.3	0.5 to 8
Al. Co. of A. 12 and 212	22,000	38,000	14,000	10.3	2
195-T4	31,000	43,000	16,000	10.3	6
195-T62	40,000	56,000	27,000	10.3	14
Magnesium alloys					
Dowmetal (as cast)	24,000	44,000	10,000	6.5	3
Dowmetal (wrought)	38,000	57,000	27,000	6.5	14
Red brass, cast, 85% Cu., 5% Pb, 5% Sn	26,000		12,000	13	15
Yellow brass, cast, 65% Cu, 2% Pb, 1% Sn	20,000			13	15
Brass, cast, 65% Cu, no Pb	45,000	60,000		13	35 in 5 in.
Brass, wrought, 67% Cu, no Pb, some Sn	50,000 to 60,000		22,000 to 31,000	13	25 to 30
Manganese bronze, sand cast (SAE, ASTM)	65,000			15.5	25
Bronze, 10% Sn	30,000		15,000	15.5	14
Leaded gun metal, 10% Sn, 2% Pb	30,000		12,000	15.5	10
Leaded phosphor bearing bronze, 10% Sn., 10% Pb	25,000		12,000	15.5	8
Phosphor gear bronze	35,000		20,000	15.5	10
Aluminum bronze, as cast	65,000		25,000	15.5	15
Aluminum bronze, drawn	80,000		50,000	15.3	4
Brass rod	45,000		18,000	13	25
Brass wire	100,000			13	
Soft copper wire					
large dia	36,000				35 in 10 in.
small dia	40,000				20 in 10 in.
Beryllium-copper	135,000				
Monel	70,000 to 100,000		30,000 to 70,000	25	2 to 20
Monel K	120,000		60,000	25	30
Everdur, soft	55,000		25,000		48
Everdur, hard	113,000		75,000		5

Modulus of elasticity in shear, $G = \frac{mE}{2(m+1)} \cong \frac{3}{8} E$ for steels at ordinary temperatures. m is the reciprocal of Poisson's ratio.

TABLE 12—*Continued*

Material	Tensile Strength	Compressive Strength	Yield Point	Modulus of Elasticity $\times 10^{-6}$	Elongation in 2 in.
Bronzes, wrought					
Manganese, Tobin, Aluminum, Phosphor					
hard.	70,000 to 100,000		50,000 to 60,000	15.5	25
soft.	60,000 to 75,000		25,000 to 30,000	15.5	40
Zinc die casting alloys					
SAE 903 or AFA F-12. .	36,000	60,500		12 to 15	5
SAE 921 or AFA F-11. .	44,000	93,000		12 to 15	9
Rubber, soft					
ordinary quality.	1,000 to 2,000				
highest quality.	3,500 to 4,500				
Rubber, hard.	1,000 to 10,000				
Vulcanized fiber.	16,000				
Bakelite.	6,000 to 11,000			0.01 to 0.025	
Oak					
parallel to grain.	8,000				
perpendicular to grain. .	1,400				
White pine.					
parallel to grain.	2,700				
perpendicular to grain. .	300				

Note No. 1. The values of all strength and elastic properties vary according to the exact composition and the details of the heat-treatment.

Note No. 2. The transverse shear strength for materials of equal strength in tension and compression may be taken as 75 per cent of the tensile strength. For cast iron the Cast Metals Handbook of the American Foundrymen's Association gives the ratio of transverse shear strength to the tensile strength as varying from 1.6 for irons of 20,000 p.s.i. tensile strength and less, 1.4 for somewhat stronger irons, down to about 1.00 for irons having tensile strengths of from 55,000 to 60,000 p.s.i.

Note No. 3. The torsional shear strength is conservatively assumed to be 50 per cent of the tensile strength for materials having the same strength in tension and compression. For cast iron it varies with the shape of the section. According to Bach, as quoted in the Cast Metals Handbook, the factor varies from 0.82 for a hollow ring to about 1.0 for a cylindrical section, to 1.4 and more for rectangular and triangular solid sections.

as *Neoprene* * and *Thiokol.* The coefficient of friction of these materials when wet is low, so that they provide a good oil seal for rotating shafts.

Cork is used as a gasket material and for the isolation of noise and vibration. For the latter purpose it is sometimes mixed with rubber. Cork is generally used in the ground state and is held together by a

* The Du Pont Co. advocates the use of *neoprene* as a generic term for the type of synthetic rubber they formerly called *Duprene.*

binder which may be used to regulate the characteristics of the product. Cork is a good friction material and has been used for clutch facings. *Felt* is also used to deaden vibrations and is extensively used as an oil seal.

Asbestos as a base is an important material for clutch facings and brake linings. Various *refractory* materials are used where electrical resistance and resistance to high temperatures are necessary. *Glass* is also used as an insulator.

Oils of animal, vegetable, and predominantly mineral base, supply our lubricants. Oils and synthetic materials are used for paint and protective coatings.

Any material which serves its purpose satisfactorily may be used as an engineering material. Many useful ones have not been mentioned and new ones are being developed constantly.

PROBLEMS

1. List all the factors and properties that you think should be considered in selecting a material for design use.

2. What influences the permissible bending stress for cast iron and how is it related to the tensile strength?

3. What advantages has cast iron as a construction material?

4. What advantages are there in using alloy cast irons?

5. For what particular parts is malleable iron used? Wrought iron?

6. What are the so-called "semi-steels"? Why are some automotive crank shafts made of semi-steel?

7. Distinguish between "normalizing" and "annealing." What is the purpose of each?

8. Distinguish between "case hardening" and "cyaniding."

9. How are alloying elements effective in changing the properties of steels?

10. What is the system used in designating a steel in the S.A.E. numbering? What carbon percentage would you expect in S.A.E. 9250, silico-manganese steel?

11. What are "low alloy steels"? For what service are they especially suited?

12. What are the three types of stainless steel and for what type of service is each used?

13. What materials are successfully used for commercial die-castings? What are the physical characteristics of the die-cast metal having the most extensive use?

14. Define: Brass, bronze, alloy.

15. For what types of service are the brasses most used? The bronzes?

16. What are the constituents and physical properties of Monel metal?

17. What are the advantages and disadvantages of aluminum alloy as a material for an automobile cylinder head; an automotive piston?

18. How are metal surfaces protected from corrosion?

19. What heat treatments may be given to aluminum alloys? To magnesium alloys?

20. What factors would you consider in selecting the material for the body of a barbers' hair clipper if the following were available: hard rubber, fiber, phenol plastic (such as Bakelite), zinc alloy die-casting? Give reasons.

21. A roller chain may be used for various applications. List the different properties required for each of the four component parts and select the materials to be used.

22. List the important characteristics necessary in the material selected for use in each of the following and tell what material you would use:

(a) Connecting rod.
(b) Cam shaft.
(c) Handle for a screw driver.
(d) Steam turbine blade.
(e) Sluice gate.
(f) Electrical control panel.
(g) Mine pump piston.
(h) Lining for a ball tumbling mill.

23. (a) Why is bronze a good bearing metal and brass a poor one?
(b) Can copper be forged? Welded?
(c) List some uses of Monel metal. How does its cost compare to steel?
(d) List four common industrial uses of rubber.

24. Select a material for each of the following:

(a) Container for hydrofluoric acid.
(b) Pump impeller for sulphuric acid.
(c) Exposed bearing in a ditch pump.
(d) Edge surface for a road scraper.
(e) Sheath for telephone cable.
(f) Vats in a canning factory.
(g) Transporting tanks for milk.
(h) Propeller shaft for outboard motor.

25. Select a material for each of the following:

(a) Small intricate case containing electrical wiring and contacts.
(b) Bearing material for main bearing on Diesel engine.
(c) Bearing plates for supporting the roller on the free end of a bridge.
(d) Contact points on an electric relay.
(e) Railway car coupling.
(f) Lawn mower wheel.
(g) Cylinder for high pressure hydraulic pump.
(h) Locomotive hydrostatic lubricator body.
(i) Speedometer housing.

26. Select surface finishes for each of the following:

(a) Automobile instrument panel.
(b) Laboratory microscope.
(c) Power plant steam flow meter.
(d) Box for assorted small tools.
(e) Precision bench lathe.
(f) Desk lamp for office workers.
(g) Typewriter.
(h) Astronomical telescope.

CHAPTER 4

MANUFACTURING PROCESSES

55. Introduction. Manufacturing processes may be divided into two general groups, namely, primary and secondary form-giving processes. The primary form-giving processes convert the raw material into the general shape required and include *casting*, *forging*, *rolling*, *drawing*, *extruding*, *stamping*, and *welding*. The secondary, or finishing, processes give the piece the exact dimensions required and impart to it the surface conditions necessary for its proper functioning. In this group are included *planing*, *shaping*, *turning*, *milling*, *drilling*, *boring* and *reaming*, *broaching*, *grinding*, *honing* and *lapping*, and *polishing*. There are additional machining processes such as *screw-cutting*, *tapping*, *thread-milling*, *gear-cutting*, etc., that are special adaptations of processes already mentioned.

Materials such as wood, rubber, textiles, fiber, plastics, and others have a wide application in machinery. These materials generally require form-giving processes of a special nature, and in many cases the designer is concerned only with the properties of the material in its commercial forms.

56. Casting. Castings are produced by pouring molten metal into a mold or die. The metal in cooling solidifies to the form outlined in the mold. The mold usually consists of two or more sections and is formed by packing molding sand around a wood pattern, although metal patterns are commonly used where the production is large. Holes, special shapes, and internal passages are produced by means of baked sand cores, which are inserted in the mold and fit in recesses prepared for them by the pattern. The mold must be parted to permit the withdrawal of the pattern and the insertion of cores. The molten metal is poured into the mold through passages leading from the top and flows by gravity into the impression. Impurities float to the top, leaving the casting uniform and dense in structure. The sand must be porous to allow liberated gases to escape, and the impression must be of such form that the molten metal will flow freely into all portions of the mold. Thin sections should be avoided as the metal might solidify before the mold is filled.

Castings are made of iron, steel, various brasses and bronzes, aluminum and its alloys, and the various white metal alloys. Cast

iron is the cheapest of the sand castings and should be used where its properties are adequate. Castings may be made in permanent molds of metal when the quantity to be produced justifies the cost of the molds. A metal mold or metal inserts in a sand mold are used to chill a white iron casting, producing very hard surfaces resistant to wear. Dense castings may be produced by utilizing the centrifugal pressure created by a rotating mold.

57. Design of Castings. The following precautions should be observed in designing parts that are to be cast:

(a) All walls should be sufficiently thick to allow the molten metal to flow freely into all corners. The greatest thicknesses are required for steel; some foundries specify 3/4 in. as a minimum. Iron can be cast as thin as 1/8 in. when necessary, but for commercial castings of some size, wall thicknesses of 3/8 in. and greater are preferred. Sand castings of iron may be made within 1/16 in. of the specified dimensions. The most sharply defined castings are obtained from the brasses and bronzes, the strongest of which are those of manganese bronze.

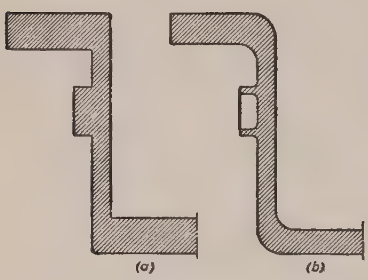


FIG. 39. REDESIGN OF CYLINDER CASTING. IMPROVED DESIGN SHOWN AT (b).

(b) All walls should be made the same thickness, if possible, but where different thicknesses are necessary, a gradual transition should be made from one to the other. Otherwise, a thin wall will solidify first, and, when a heavier adjoining section cools later, it will exert a heavy pull on the thin wall. The different rates of contraction during cooling and the orientation of the resultant crystals develop internal stresses and points of weakness which may result in cracks. Figure 39 illustrates how a thick portion may be reduced to minimize these casting stresses. Greater strength and rigidity for the same weight will be obtained by using box type sections or I or T shapes. As shown in Fig. 40, these sections are made up of relatively thin walls instead of the equivalent solid sections. Solid sections of great thickness are liable to develop blow-holes and other interior defects.

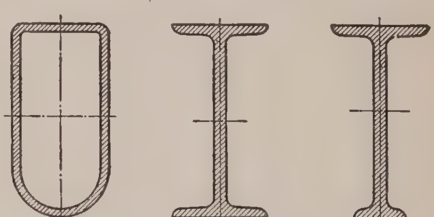


FIG. 40. CAST BOX, I, AND T SECTIONS.

(c) Large *fillets* should be used in all corners. The crystals do not interlace regularly when solidifying at sharp corners and lines of weakness are developed. This condition is doubly serious because sharp corners are points of severe stress concentration under load.

(d) Parts should be designed so that patterns may be drawn readily from the molds. In Fig. 41a the boss interferes with the withdrawal of the pattern. By an arrangement like that shown in Fig. 41b, the pattern can be withdrawn, leaving the bosses in the sand from which they can be removed separately. The use of such a pattern, however, adds considerably to the cost of the casting. Extremely complicated castings can be produced by the use of cores, but core boxes are an added expense and complication. An automobile cylinder, for instance, is made with an elaborate arrangement of cores. If holes are not too deep and sufficient draft is provided for drawing the pattern, as in Fig. 42, holes can be cast without cores.

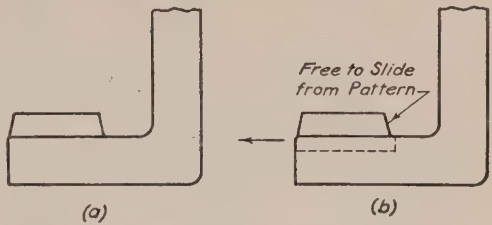


FIG. 41. PATTERN PROVIDED WITH REMOVABLE BOSS.

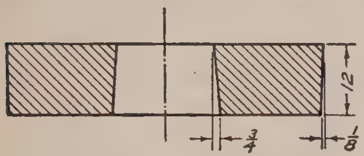


FIG. 42. PATTERN PROVIDED WITH DRAFT TO PERMIT WITHDRAWAL.

58. Die-Casting. When a metal mold is used and the molten metal is introduced under pressure, the process is called *die-casting.* Zinc, tin, and aluminum base alloys are the metals commonly used for die-casting because the pouring temperatures are not destructive to the die or mold. Brass may be die-cast; iron and steel have been die-cast experimentally. Zinc, tin, and aluminum alloys may be cast with a tolerance of $\pm$ 0.0005 in., if necessary, while a limit of 0.002 to 0.003 in. is easily maintained on many types of castings. Holes are usually cored; inserts of other metal may be used, and, where desirable, complicated parts may be made by using removable cores. In some cases, die-cast parts are the most economical form of production even when relatively few parts are produced. Even though an expensive die is required, machining operations are eliminated or are greatly reduced, and the finished casting has a good appearance. The production of die-castings is a very specialized industry and the engineering advice of the manufacturers should be obtained in the design of such castings.

59. Forging and Hot Pressing. Raw material for forgings is usually in the shape of bars, billets, or plates. While in a heated condition, the stock is transformed into the desired shape by bending, slitting, drawing (thinning), and upsetting (thickening). Naturally, this process is not so flexible as casting, but a skilled blacksmith can shape a piece of metal into surprisingly intricate forms. The agitator screw shown in Fig. 43 is an example of hand forging. One blade is first drawn out from the side of the original hub section. The other side of the hub is left larger and is split, whereupon the remaining blades are formed by spreading and drawing them out from the split part.

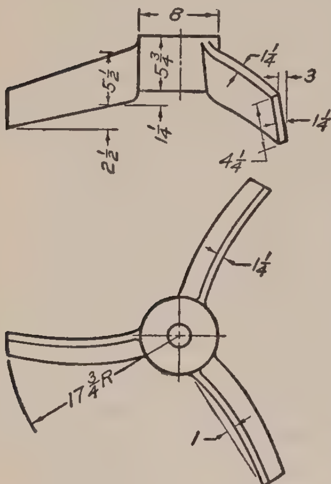


Fig. 43. Agitator Screw.

Drop forging utilizes the energy stored in a heavy falling hammer to forge metal to shape while hot. The hammer is raised either by friction or steam. The impact force of the hammer may be delicately regulated by the operator to produce complicated forms. Forging dies are used in conjunction with these types of hammers; hence the part to be forged must be designed with a draft of at least 7 deg. to enable its withdrawal from the dies.

Machine forging, or upsetting, is a process by which metal is forced to shape in a die by heavy pressure. Hot bar stock is successively passed through a series of dies where it is upset, formed, pierced, and finally worked into the desired shape. Bolts, gear blanks, gear clusters, and other work requiring a variety of diameters on one piece, are suitable for this type of forging.

Hot pressing consists of forming metal to shape in a very rigid type of power press. A hot piece of metal is pressed and extruded in suitable dies into a smoothly finished piece to accurate dimensions. This process is used in forming automobile valves. Parts may also be finished to accurate dimensions in a similar press by cold forming, or "coining."

60. Rolling, Drawing, and Extruding. *Rolling* is fundamentally a process for producing bulk articles such as plates, bars, rods, beams, etc. Hot-rolled steel has a black oxidized surface and requires machining operations for smoothness and accuracy of dimensions. Cold-rolled steel has a bright finish and is rolled to size with sufficient

accuracy to permit its use without further finishing for such parts as shafting, screw machine products, etc. Rolled articles must be of a cross-section which enables them to clear themselves from the rolls, and undercuts are impossible. Automobile front axles may be rolled as well as forged.

Drawing is a process by which the cross-section of the metal is diminished by pulling it through an accurately formed hole in a drawing die. The operation is performed cold and only the simpler forms can be produced without excessive resistance and tearing. Sections are usually in the form of small wires, thin rods, strips, etc.

Extruding consists of forcing a material in its plastic state through a specially shaped aperture under the application of pressure. It affords an excellent method of producing special shapes in quantities without the necessity of subsequent machining operations. The process is applicable to brass and certain alloys of tin, lead, and other soft metals. Representative extruded sections are illustrated in Fig. 44.

FIG. 44. EXTRUDED SECTIONS.

61. Stamping, Punching, and Spinning. *Stamping* is the cold shaping and forming of sheet metal by pressing between dies. By progressive drawing operations in suitable dies, accompanied by intervening annealings, sheet metal can be formed into shells of considerable depth, and edges and flanges can be bent even to a comparatively small radius. Straight-sided holes are preferable to tapered holes. Undercut recesses may be formed by the use of special devices, but such operations should be avoided if possible.

Blanking is the process of shearing out shapes from plates by means of punches and dies, whereas *piercing* is the operation of forming holes by the same method. Frequently a number of these operations, such as blanking, drawing, piercing, and trimming, may be performed by a single die. Punched holes may be of various shapes, but in general the diameter of the hole should not be less than the thickness of the plate.

On account of the die cost, stamping is ordinarily a quantity production process. Where the demand for sufficient quantities warrants the expense of the dies, stamping, as compared with casting, is a rapid and accurate process and frequently saves much in weight and cost. By welding together stamped parts, objects of considerable

intricacy may be built up, possessing strength and lightness. Automobile bodies are constructed by assembling and welding together a number of stamped parts. The technique of stamping is highly empirical and requires a knowledge of the plastic flowing of metal, gained mostly from experience.

Metal spinning is a process applicable to the forming of sheet metal when the part involves a surface of revolution. The metal is spun to shape over a "former" of wood or metal attached to the face plate of a speed lathe. Pressure is applied to the sheet by means of a blunt nosed tool, which presses it against the "former." Spinning is an economical method of forming parts if the quantities are small. If the metal is heated and sufficient power is applied to the operation, spinning may be applied to heavy articles, such as boiler heads.

It should be remembered that success in cold forming metals as described above depends to a large extent upon having metals with the proper metallurgical characteristics. Since sheet metal forming is confined entirely to the region beyond the elastic limit, rigid specifications must be established to obtain a suitable, uniform material; otherwise, ruptures and cracks will occur. The thickness of the metal that can be stamped or formed depends largely upon the power available.

62. Finishing Processes. In the processes of die-casting, cold-rolling, extruding, stamping, and, in some cases, hot-rolling, sufficient accuracy and smoothness may be obtained so that in many instances machine parts produced by these methods require no further finishing to operate satisfactorily. However, ordinary sand castings, forgings, rolled and welded parts do not have the accuracy of dimension or smoothness of surface necessary for satisfactory fits and for smooth interaction of moving parts. They must, therefore, be subjected to further finishing processes, but, in the interests of economy, such finishing should in general be carried out only when and where it is really required.

In this connection, however, it should be remembered that there may also be considerations other than those of accuracy of fit and smoothness of operation that render finishing desirable. Fatigue strength is increased and the corrosion resistance of some materials is improved by a careful finish of the surface. Where maximum lightness is required, as in airplane engines, finish may be desirable merely to reduce all excess weight. Finally, attractive appearance, even some suggestion of luxury, may induce pride of ownership; may promote sales which may or may not be legitimate from the engineering point of view, and may promote better care and upkeep, which always are desirable from the engineering point of view.

Finishing may be accomplished by cutting tools, or by abrasives; the latter are finding more extended use all the time. The finishing processes with cutting tools comprise *planing*, *shaping*, *slotting*, *milling*, *turning*, *boring*, *drilling*, *reaming*, and *broaching*. Hand scraping is often employed to eliminate high spots from finished surfaces when greater accuracy is desired, as in fitting bearings.

63. Planing, Shaping, Slotting. All three of these processes are adapted to the finishing of plane surfaces. The particular method chosen depends upon the special character and the size of the piece to be machined.

Planing is a high-grade finishing process applicable to both jobbing and production work. The planer has a reciprocating platen, or table, to which the work is fastened and by which it is carried under one or more stationary tools. For large production operations, a number of

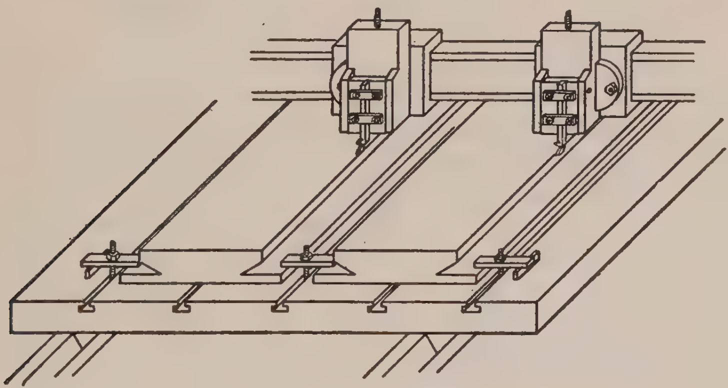


FIG. 45. PLANING.

parts may be mounted on the planer table at the same time and several tools in parallel may be used, as in Fig. 45. In designing parts to be planed, the method of supporting and clamping them on the planer table should be considered. Special bosses may have to be provided on the casting to prevent it from yielding while being clamped and machined. When clamps fastened over the edge of the piece would interfere with finishing the top completely, special bosses for clamping can be provided. Planers are especially adapted to the surfacing of large machine parts.

Shapers and *slotters* are particularly suited for machining small parts and are much used in die and tool making. The *shaper* is smaller than the planer and differs from it in that the tool is carried by a

reciprocating ram over the stationary work. The *slotter* carries a tool which reciprocates vertically, the work being mounted upon a table which can be rotated into different positions. It is particularly suited for key-seating the hubs of wheels and for making dies and intricate shapes (see Fig. 46).

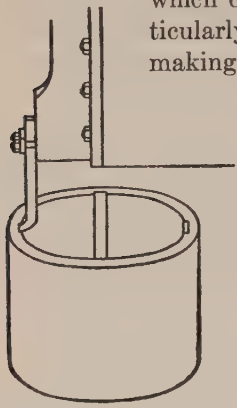


FIG. 46. VERTICAL SLOTTING.

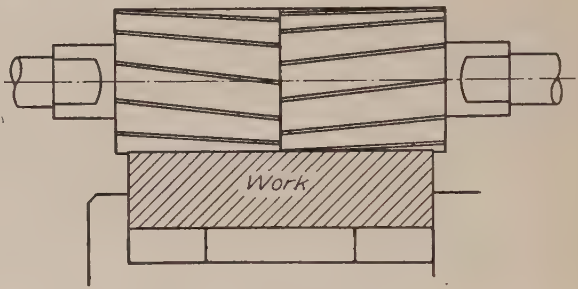


FIG. 47. SLAB MILLING.

64. Milling. *Milling* is performed by means of a rotating cutter having a multiplicity of cutting edges. These cutters are of numerous types and only the principal types will be mentioned here. *Plain cutters* with teeth on the curved surface of a cylinder are used for machining plane surfaces, or slab milling (Fig. 47). *Side cutters* with teeth on the curved surface and ends of the cylinder are used for cutting simultaneously surfaces perpendicular to each other, or two parallel edges, as in a slot (Fig. 48). The width of the slot may be governed by separating two half side cutters by means of a spacer. Parallel surfaces may be accurately spaced by controlling the distance between the inside cutting edges, as in straddle milling (Fig. 49). Thin cutters are used as *slitting saws*. *End mills* are cylindrical cutters with teeth on the curved surface and on one end and with a shank for driving on the other end (Fig. 50). These cutters are used for cutting keyways and slots, as well as for machining plane surfaces. *Form cutters,* which have teeth ground to a special contour, are used for machining unusual shapes, such as flutes in taps, gear and sprocket teeth, etc. (Fig. 51). Milling cutters are much more expensive than the single-point tools used in planing and shaping and are also more difficult to grind.

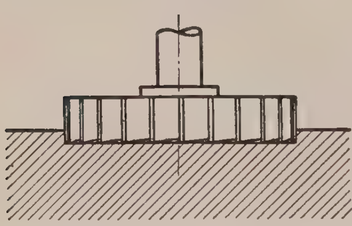


FIG. 48. SIDE MILLING.

The quality of work obtainable by milling or planing is comparable, although the accuracy of planing depends only upon the condition of the machine, tool, and work-holding device. The accuracy of milling, on the other hand, is influenced also by the form of the cutter, which reproduces its own eventual imperfections on the work. Milling machines have been displacing planers principally because they are better adapted to mass production methods. The reciprocating action of the planer or shaper results in a certain amount of inactive time, and the production of formed contours requires skill, or resetting.

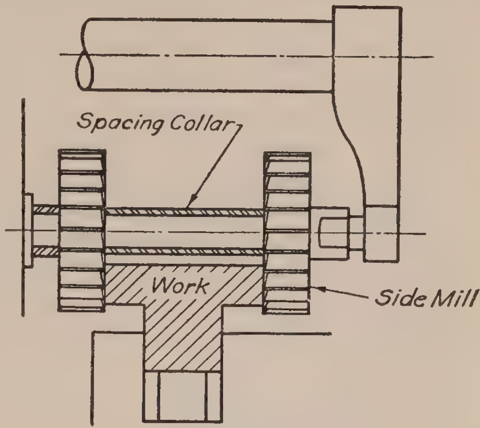


FIG. 49. STRADDLE MILLING.

Milling machines have undergone considerable development, often to a point of high specialization, as for instance in gear hobbing machines. *Milling planers* are the result of mounting milling heads and planing heads on a planer frame, so that in applying the two methods simultaneously some of the advantages of each may be derived.

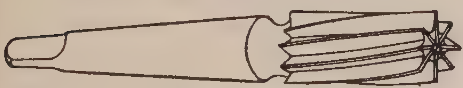


FIG. 50. TAPERED SHANK END MILL.

65. Turning and Boring. The common lathe with head and tail stock, carriage and lead screw, is a most familiar machine tool. As applied to modern specialized and quantity production methods, it has undergone so many modifications and developments that a complete description of the various types of lathes and attachments would fill books. Suffice it to say that from simple fixing and chucking arrangements there have been developed a variety of means for feeding the work into the lathe automatically, or for holding it with simple quick-acting devices. From a carriage with one or two tool rests

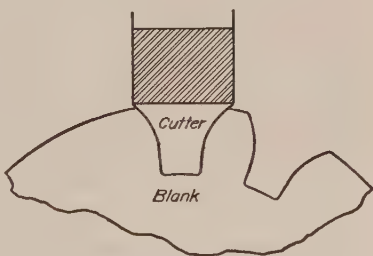


FIG. 51. FORM MILLING GEAR TEETH.

there have been developed turret lathes with a whole series of tools that are progressively brought into operating position, and automatic lathes, or screw machines, where chucks which hold the work are rotated from one position to another so as to engage a new set of tools at each index point. The simple hand feeds have been replaced by cam attachments which not only feed but also produce new settings. In contrast with former days when a skilled operator was proud of handling a single lathe, we now have a whole battery of machines tended by a man without particular skill. On the other hand, it was not unusual in former days for one man to finish a product completely, while now we use a whole organization of tool designers, tool makers, operators, and inspectors to complete the same product. The classical general-purpose lathe, however, is still extensively used for jobbing purposes and for large work, as well as in the manufacture of special tools, dies, jigs, and fixtures.

The *vertical boring mill* is very convenient for turning and boring large pieces which are difficult to hold in an ordinary lathe. In a machine of this type, the work is fastened to a revolving horizontal table while the tools are given vertical and transverse feeds. Vertical boring mills have been built to turn diameters of thirty feet or more.

The *horizontal boring mill* has a strong revolving horizontal boring bar which can be adjusted vertically as well as axially. The work is mounted on a stationary table which has a transverse feed and is also adjustable along the axis of the boring bar. This type of machine is especially convenient for locating and boring holes in large and irregular pieces.

66. Drilling and Reaming. An ordinary drill press, or drilling machine, is not a tool of high accuracy either for locating a hole or for forming it to size. Deviations of 0.010 in. are easily possible. Large holes may be located best by first drilling a small hole and then enlarging it to size. If further improvement in size and finish is required, the hole may be finished by reaming. Straight and tapered holes can be reamed. A *reamer* has a number of cutting edges which are formed straight or helical relative to the pitch surface. These edges remove a comparatively small amount of metal from the previously formed hole, ordinarily about 1/64 of an inch on the diameter.

Where particular accuracy is required the holes may be located and bored by specially evolved methods, or they may be bored in *jig-borers* which fix the location to within 0.0001 in., or even 0.00005 in. if necessary. When drill presses are used on production work, the location of holes is governed by the guiding action of hardened steel drill bushings in a device known as a *drill jig*.

67. Broaching. Broaching is a machining process that has recently undergone great development. Metal is removed by pulling or pushing over the surface of the work a long tool (Fig. 52) which brings into action successively a number of cutting teeth, stepped in size to remove successive layers of metal. The operation may be applied to internal or external surfaces. Internal keyways, splines, and non-circular holes are frequently machined by broaching. External broaching is applied to parts with surfaces which will allow a free passage of the broach. Examples are wrench jaws, connecting rod bosses, cylinder heads, etc.

The broach itself may combine roughing tool, finishing tool, and burnishing tool, all in one, thus completing the whole machining operation in one pass. Unlike the reamer, the broach, having many

Fig. 52. Pull Type Broach.

teeth, has a small cutting load per tooth. A considerable amount of metal can be removed in one stroke, depending upon the size and length of the broach, the profile of the broached surface, the kind of material being cut, and other factors. Burnishing is accomplished by having the edges of the final teeth rounded-off, thus producing a compressing action on the finished surface. The broaching machine is a prolific production tool.

68. Grinding, Honing, and Lapping. Grinding is performed by passing over the surface of the work a grinding wheel rotating with a high peripheral speed, usually about 6000 ft. per min. The wheel consists of particles of hard, fine abrasive materials, bonded together in a supporting matrix of sufficient strength to hold the cutting particles until they are dulled. The abrasives are of different materials and are graded as to fineness. The matrix bond composition varies with different wheels. The wheel selected depends upon the material being ground, and the finish desired. Grinding is used as a production method for removing material as well as a means of obtaining a smooth finish and accurate dimensions. Time can be saved in quantity production by eliminating the finishing tool cuts, and finish grinding directly from a coarse roughing cut. This method is applicable to flat and profiled parts, but is especially effective for cylindrical parts. For such parts, the rough turning is carried out to within 0.010 to 0.015 in. of the final diameter and the part is then finish ground to size. Cylindrical surfaces are finished either by wheels that are traversed over the length, or for lengths up to 12 in., by wide wheels that are fed into the work radially. Profiled surfaces are ground by wheels fed radially.

The curvature of the profile must be smooth and not too deep. Form tools, shell noses, and crowns of pulleys are examples of work suitable for profile grinding.

Flat surfaces may be finished either by edge grinding as in Fig. 53, or face grinding (Fig. 54), the former being more accurate. The latter

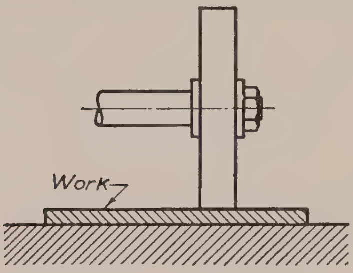


FIG. 53. EDGE GRINDING.

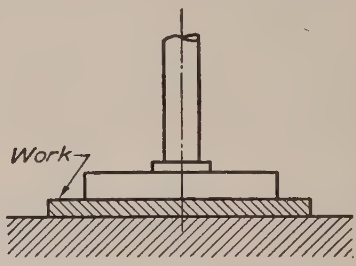
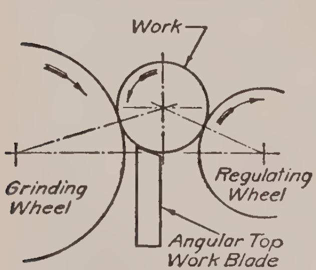


FIG. 54. FACE GRINDING.

method is used for finishing the flat surfaces of heads and covers instead of by planing or milling.

In grinding cams, the work is held between centers while the grinding wheel, which is guided by a master cam, is given a reciprocating motion across the work. Internal cylindrical grinding is performed by having the grinding wheel rotate on a fixed axis, or the wheel may be given a planetary motion as it rotates. Fixed axes are used when the work can be revolved to produce the internal cylindrical surface. Planetary grinding is used for cylinder blocks and other work which cannot be rotated.

Centerless grinding is a comparatively recent development in modern industry. In Fig. 55 the piece to be ground is supported on a rest between a *regulating* wheel, rotating at a relatively slow speed, and the grinding wheel rotating at high speed. Parts without shoulders may be fed through from the side between the wheels, the feed being obtained by mounting the regulating wheel with its axis at a small angle to the axis of the grinding wheel. Pieces with shoulders or tapers are fed down between the wheels from above. Centerless grinding may be used for small quantities, although it is especially suitable for mass production. This method of grinding entirely eliminates set-up time, which, with transfer time, is an item to be con-



Cincinnati Grinders Inc., Cincinnati, O.

FIG. 55. CENTERLESS GRINDING.

sidered when grinding between centers or in finishing by combined turning and grinding.

Honing is in wide use for the final finishing of cylinder bores. The operation is performed by simultaneously rotating and reciprocating an adjustable holder containing a number of abrasive stone strips. It is claimed that honing produces an excellent finish that is more economical than grinding.

Lapping consists in charging a "lap," made of cast iron, brass, or other soft metal, with a fine abrasive suspended in oil and rubbing it on the surface to be finished. By this means the highest of accuracies and fits are obtainable. Gage blocks that are accurate within one millionth of an inch under proper temperature and contact conditions are produced by a precision lapping operation. Oil pump plungers that operate under pressures of several thousand pounds per square inch are lapped and require no packing, yet are free to move in their cylinders. Such fits are of course destroyed by even slight surface imperfections.

69. Polishing. Polishing is not an operation for securing exactness of dimensions, but merely a means of obtaining a smooth finish, a high luster, and a more pleasing appearance.

70. Accuracy. Because of the tendency of engineering departments to specify finished dimensions with closer limits than is necessary, it must be emphasized that the cost of production increases considerably with a decrease in tolerance. The permissible variation in the dimensions of parts should be as great as the proper assembly and functioning of the parts will allow. When tolerances are set, the condition of the machines available for producing the parts must be taken into consideration. The accuracy of a machine tool depends upon the rigidity of the machine, and this is determined by the type, the particular construction, and the working condition of the machine; upon the rigidity of the tool and tool holder; and upon the work-holding fixture. The condition of the tool and the stiffness of the work also influence the results obtained.

Except for the drilling machine, all of the machine tools of the cutting type give about the same degree of accuracy under ordinary conditions. Dimensions may be held to about 0.0005 in., and even closer when necessary. Ordinarily, however, the machines employing cutting tools should not be expected to give production accuracy closer than 0.001 to 0.002 of an inch. Many companies have printed on their drawings, "Dimensions between finished surfaces are to be held within plus or minus 0.010 in. unless otherwise specified," or a similar statement.

If close tolerances are necessary for interchangeable manufacture, jigs and fixtures may be carefully prepared so that hole locations and other dimensions may be duplicated on any number of parts. By this method, the skill of an expert tool maker and the results obtainable from expensive, accurate machines may be transferred to a mediocre workman operating one or more ordinary drilling machines. Special devices called fixtures for holding the work, simplify and regulate set-ups, and are frequently combined with jigs, which guide the tool.

PROBLEMS

1. Sketch a cross-sectional view of a pulley pattern that will have minimum shrinkage stress.
2. Sketch a cast iron wall bracket with proper regard to casting problems and stress concentration.
3. Why is the design of die-castings different from the design of sand castings? Should die-cast letters be raised or depressed from the surface?
4. What are the advantages of using extruded parts?
5. What are the reasons for giving a smooth finish to machine parts?
6. By what processes are keyways formed? How are T-slots machined?
7. What factors determine whether a plane surface is to be machined by planing, milling, or broaching?
8. When is a hole drilled, when bored, when broached, when cored?
9. What is "lapping"? When and why is it used?
10. Explain the process of centerless grinding.

CHAPTER 5

RIVETED PRESSURE VESSELS AND RIVETED JOINTS

71. Pressure Vessels. A pressure vessel is a container for liquids and gases under pressure. Such vessels may be cast in one piece, or made of separate preformed plates welded or riveted together. Although welding is of increasing importance, riveting still continues to be a widely used method of constructing large tanks, many boilers, oil stills, and other drums.

The greatest included volume for a given enveloping surface is obtained in a sphere. Furthermore, a spherical shell under pressure is uniformly stressed in all directions, thus permitting the most economical use of the material. While spherical shells are used as containers for gases and volatile liquids, cylinders are more generally used because the plates are more easily formed to shape, and cylinders are easier to support than are spheres. These advantages of the cylindrical construction usually offset its disadvantages, namely, that a greater quantity of plate must be used than for a sphere of equal volume, and that the plates are not stressed so efficiently.

72. Shell Stresses. In the usual construction of pressure vessels, the thickness of the shell is small compared to the diameter, so that the stress induced in the shell may be considered as uniform across the wall thickness. The expression for this stress may be determined by equating the force tending to tear the shell apart to the resistance the shell offers to rupture. Thus for the sphere, the total pressure tending to burst the shell apart on a diametral section is resisted by the stress acting on a circumferential ring. We have

$$(1) \qquad \frac{\pi D^2}{4} p = \pi D t s_t, \qquad \text{or} \qquad s_t = \frac{Dp}{4t},$$

where D is the internal diameter, p the internal gage pressure, t the shell thickness, and s_t the tensile stress. These formulas are rational formulas and hold good for any consistent units of measurement. In countries where the English system of units is in use, D and t would usually be given in inches, p and s_t in pounds per square inch.

The stress on a circumferential section of a cylindrical shell, due to the pressure acting on the ends of the cylinder (Fig. 56), is identical to the stress in a spherical shell. The stress on a longitudinal section of a

cylinder is obtained by considering the pressure acting on a cylindrical ring (Fig. 57) of length l. Then

$$(2) \qquad Dlp = 2tls_t, \qquad \text{or} \qquad s_t = \frac{Dp}{2t}.$$

It is to be noted that the stress on a longitudinal section is twice as great as on a circumferential section. Consequently the girth or

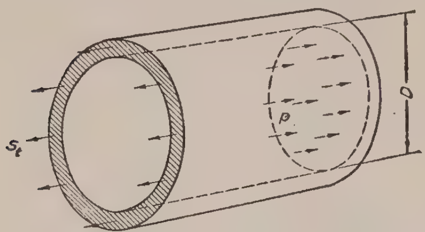


Fig. 56. Stress on a Circumferential Section of a Cylinder.

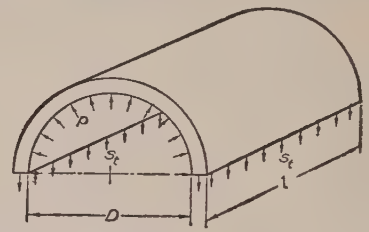


Fig. 57. Stress on a Longitudinal Section of a Cylinder.

circumferential joints need be only one-half as strong as the longitudinal joints. The longitudinal joint should be made as strong as possible without unreasonable labor cost or complication. The girth joint need never exceed one-half the strength of the solid plate.

When two plates are riveted together, the strength of the resultant joint, because of the holes in the plates, is obviously less than that of the original plate. Thus the permissible pressure in a shell is governed by the strength of the riveted joints. Referring to formula (2), if s_t is the allowable stress, the maximum permissible pressure in a cylinder without joints is $p = 2ts_t/D$. If the shell contains a longitudinal joint of efficiency e, the permissible pressure is only $p = 2ts_te/D$, which gives a necessary shell thickness of

$$(3) \qquad t = \frac{pD}{2s_te}.$$

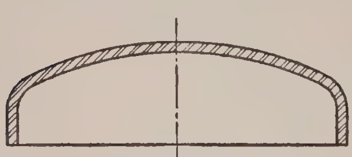


Fig. 58. Ellipsoidal Head for Pressure Vessel.

The heads of cylindrical pressure vessels are preferably made of elliptical section, as in Fig. 58, with the minor axis equal to one-half the radius of the shell. With these proportions the stress in the head will not exceed the stress in the cylindrical portion and consequently the head need not be computed separately.*

* For a discussion of the stresses in heads see particularly W. M. Coates, Transactions of the A.S.M.E. (APM–52–12, p. 117, 1930). The design of boilers and pressure vessels is governed to a great extent by legal requirements due to safety regulations. *The Boiler Construction Code* and the *Code for Unfired Pressure Vessels* of the American Society of Mechanical Engineers contain rules which have been sanctioned by the majority of states

73. Rivets. A rivet is a permanent fastening consisting of a short round bar with a head formed at each end (Fig. 59). It is used to unite and hold together tightly two or more plates by the pressure exerted between the heads. Rivets are regularly furnished with a head at one end, the other head being formed under pressure after the rivet is in place. This head may be formed either hot or cold, depending upon the rivet size, under the force from an ordinary hammer, pneumatic hammer, or hydraulic pressure die. During the heading, whatever the method, the rivet must be held in place by a hammer or counter-die in contact with the preformed head (Fig. 60).

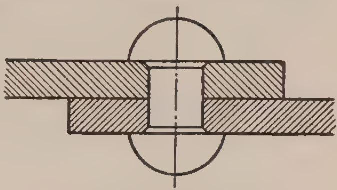


FIG. 59. RIVET.

In the usual method of calculating the strength of riveted joints, it is assumed that each rivet carries its proportionate share of the total load and that the rivet transmits its load by shearing resistance only. A rivet may be in single shear, as in Fig. 61, or in double shear, as in Fig. 62. Actually, a properly headed rivet should hold the pieces so firmly together that friction prevents the plates from slipping, since, if slipping does occur, the joint may leak. With riveted joints designed for sufficient uniformly distributed shear strength, serious slipping does not occur, even though the frictional resistance between the plates is neglected in the computation.

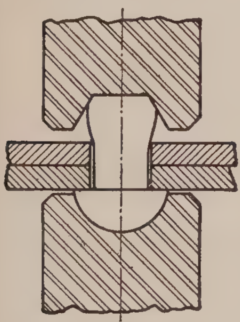


FIG. 60. COUNTER-DIE USED IN HEADING RIVET.

Rivets are made of dead soft steel or wrought iron. The material must be of such quality that while cold, the shank can be bent back on itself through 180 deg. without cracking; and after being heated to not less than 1200° F. and quenched, it must

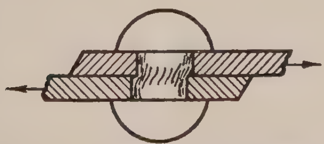


FIG. 61. RIVET IN SINGLE SHEAR.

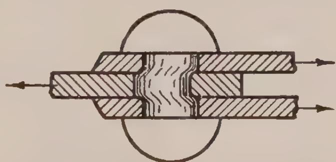


FIG. 62. RIVET IN DOUBLE SHEAR.

pass this same test. The rivet must also flatten when hot, without cracking, to a diameter of 2½ times the diameter of the shank.

A variety of head shapes are available for use under different conditions. The most commonly used is the button head (Fig. 63a). The cone head (Fig. 63b) is stronger, but as it takes more power to form this head, it is not suitable for hand riveting. The countersunk head (Fig. 63c) is used where smooth surfaces must be maintained, but the conical hole necessary causes an additional weakening of the plate.

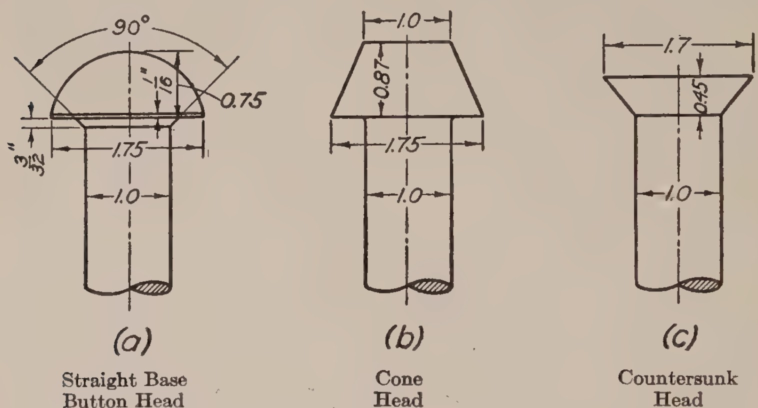


Fig. 63. Typical Forms of Rivet Heads.

Rivet holes for high grade joints should be drilled. Rivet holes may be punched, but as this operation renders the plate hard and brittle at the hole edges, punched holes are forbidden by the Boiler Construction Code of the A.S.M.E., unless the brittle portion is reamed out after punching. For boiler riveting the hole diameter is usually made 1/16 in. larger than the diameter of the cold rivet shank.

74. Calking. Riveted joints may be made tight against leakage of gases and liquids by *calking,* which consists in driving the edge of one plate down into close contact with the plate beneath it. The calking is done by hammering a blunt nosed tool against the edge, which should be prepared by chipping or planing off to a bevel at an angle somewhat greater than 70 deg. (Fig. 64).

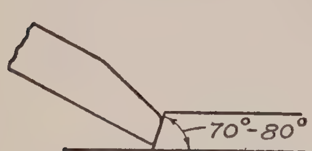


Fig. 64. Calking Tool.

In order that calking be effective, the rivets at the calked edge must be fairly close together to prevent the plate from springing out of contact. A good practical rule is to make the rivet spacing at the calked edge not more than eight times the thickness of the plate to be calked. This rule is taken from German marine practice. Somewhat

larger pitches are occasionally found in American practice, but no generally accepted formula governing the maximum length of these pitches can be given.

75. Types of Riveted Joints. Riveted joints may be *lap* joints or *butt* joints. Lap joints may be single riveted, as in Fig. 65, or double

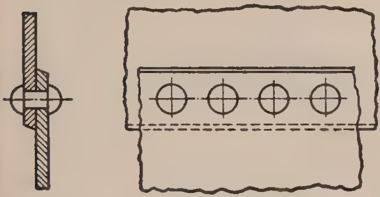


FIG. 65. SINGLE-RIVETED LAP JOINT.

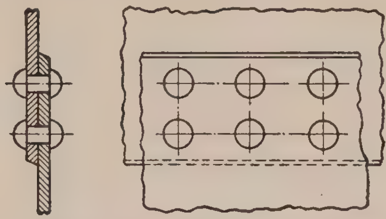


FIG. 66. DOUBLE-RIVETED LAP JOINT.

riveted, as in Fig. 66. More than two rows of rivets are seldom used in a lap joint in a pressure vessel, because lap joints are principally used as girth joints where the strength need not exceed one-half the strength of the solid plate. When more than one row of rivets is used, the rivets are usually staggered.

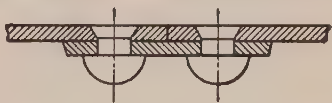


FIG. 67. SINGLE STRAP BUTT JOINT.

Butt joints may have a single strap, or cover plate (Fig. 67), or a double strap, as in Figs. 68 and 69. The single strap joint is evidently no stronger than a lap joint and requires twice as many rivets. There is also the same objection to it that there is to a lap joint, namely, that the pulling forces acting on the plates are not in line and form a couple which causes a bending action in the plate. This effect is illustrated in Fig. 70. A single strap butt joint is therefore used only when one side of the joint must be smooth and is accompanied by the use of countersunk rivet heads (Fig. 67).

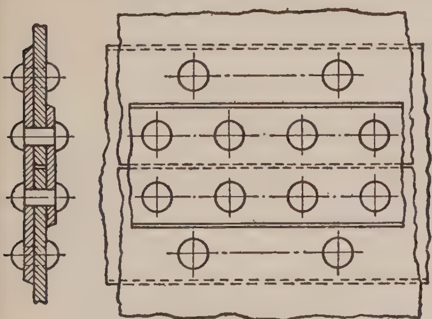


FIG. 68. DOUBLE-RIVETED BUTT JOINT.

For highest efficiency, butt joints are designed with cover plates of unequal width. The spacing of the rivets along the narrow plate (usually the outside cover) is governed by calking requirements and is less than would be desirable for high efficiency. If the other

plate is extended and an additional row of rivets is added, but with a spacing of two or three times that already used, the joint is strengthened. The resistance to tearing the plate across the outer row is now greater because of the fewer holes across the plate, and the resistance to shearing and crushing is also greater because additional rivet area has been added. The narrow plate is then the only one calked. Butt joints with straps of unequal width are standard for longitudinal joints of pressure vessels as these joints are subject to the maximum pull.

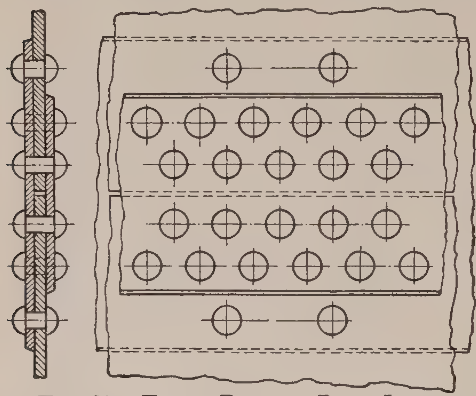


FIG. 69. TRIPLE-RIVETED BUTT JOINT.

To facilitate calking, boilers have the narrow strap outside. It has been found, however, that boiler water containing alkalies may cause embrittlement of the plate, if it enters the joint. For this reason joints should be calked on the inside to protect the metal at the joint, and cover plates of equal width are used with some sacrifice of strength. This loss of strength may not be important as a highly efficient joint is not always necessary. The drums of water tube boilers, for instance, are perforated with tube holes of large diameter along the side, with a consequent weakening of the plate below that obtained with a low efficiency joint.

FIG. 70. EFFECT OF PULLING A BUTT JOINT WITH A SINGLE STRAP.

Present practice for boiler drums is to use longitudinal joints with equal cover plates for drums with tube holes, unless welding, X-ray inspected, is used. (See page 100 for a welded joint in a boiler drum.) For boiler and pressure vessels without tube holes in the sides, the longitudinal joint is still made with cover plates of unequal width.

76. Strength and Efficiency of Joints. The efficiency of a riveted joint is the ratio of the least strength to the strength of the solid (unpunched) plate. The efficiency is determined by computing the load at which the joint will fail and then dividing this load by the strength of the solid plate. To determine the load at which failure will occur, it is necessary to list the various ways in which failure is possible. The methods of possible failure are limited by the conditions

of the joint and may be determined by a simple analytical inspection. All failures are from one or a combination of the three following causes:

1. *Tearing of the plate.* The main plate may tear longitudinally between the rivets in the same row. Tearing of the plate diagonally between the rivets in different rows or tearing or shearing from the outer rivets to the edge of the plate is prevented by spacing the rows of rivets at a sufficient distance from each other, and by allowing sufficient material at the edge of the plate. The *back pitch* is made $2d$ or more,* and the distance from the edge of the plate to rivet center (*margin*) between $1\frac{1}{2}$ to $1\frac{3}{4}d$, where d is the diameter of the rivet hole.

2. *Shearing the rivets.* The rivets may fail in either single shear or double shear. The shear area is twice as large in double shear as in single shear and the A.S.M.E. Boiler Code assumes that steel rivets are twice as strong in double shear as in single shear. Laws of some states assume that the strength in double shear is only 85 per cent more than in single shear. In determining the strength of the rivet in shear the cross-section of the driven rivet, that is, the area of the hole, should be used.

3. *Crushing the plate back of the rivets.* Crushing is a flowing out of the plate away from the rivet, as illustrated in Fig. 71. Tests at the Watertown Arsenal have shown that the resistance to this kind of failure is proportional to the projected area of contact of the rivet with the plate (hole diameter times plate thickness).

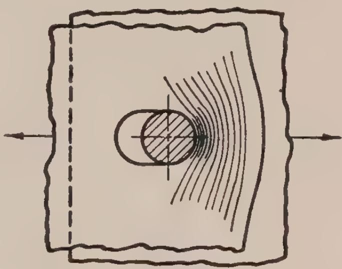


FIG. 71. FAILURE FROM CRUSHING.

77. Proportion of Joints. Since riveting has been the standard method of building boiler shells, it has been necessary for boiler inspectors and insurance representatives to scrutinize the strength of joints very carefully. The Hartford Steam Boiler Inspection and Insurance Company has developed a series of typical joints, the dimension and efficiencies of which may be regarded as standard. Consequently, one of these designs should be used unless some special consideration makes it impracticable to do so.† The essentials of these joints are given in Tables 13–16.

* The exact rule is found in the boiler construction code of the A.S.M.E.

† For further condensed information about these joints see, for example, Marks, *Mechanical Engineers' Handbook*, or C. A. Norman, *Principles of Machine Design;* for more complete information, S. F. Jeter, *Riveted Boiler Joints*, or publications of the Hartford Steam Boiler Inspection and Insurance Company.

TABLE 13

STRENGTH DATA FOR RIVETED JOINTS
(A.S.M.E. Boiler Code)

Factor of safety = 5
Tensile strength of steel plate....... s_t = 55,000 p.s.i.
Resistance to crushing of steel plate s_c = 95,000 p.s.i.
Strength of iron rivets in single shear s_s = 38,300 p.s.i.
Strength of iron rivets in double shear s_s = 76,000 p.s.i.
Strength of steel rivets in single shear s_s = 44,000 p.s.i.
Strength of steel rivets in double shear s_s = 88,000 p.s.i.

TABLE 14

DATA FOR DOUBLE-RIVETED JOINT (FIG. 68)

Plate Thickness	Strap Thickness	Diam. Rivet Hole	Eff.	Long Pitch
$\frac{1}{4}$	$\frac{1}{4}$	$1\frac{1}{16}$	82.8	4
$\frac{5}{16}$	$\frac{9}{32}$	$1\frac{3}{16}$	81.9	$4\frac{1}{2}$
$\frac{3}{8}$	$\frac{5}{16}$	$1\frac{3}{16}$	81.9	$4\frac{1}{2}$
$\frac{7}{16}$	$\frac{3}{8}$	$1\frac{5}{16}$	81.3	5
$\frac{1}{2}$	$\frac{7}{16}$	$1\frac{5}{16}$	81.3	5

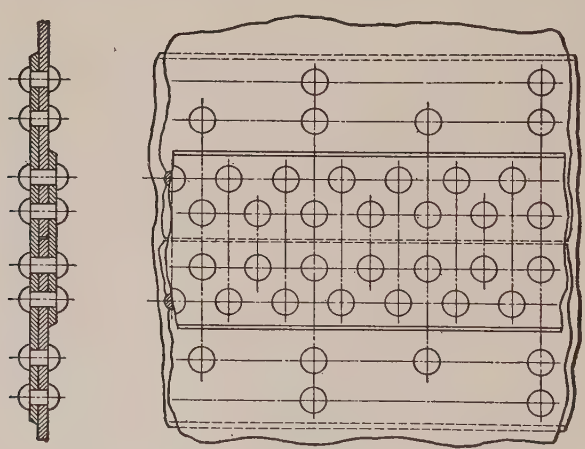


FIG. 72. QUADRUPLE-RIVETED BUTT JOINT.

It is very instructive, however, to carry out the design of such a joint in detail, even though many of the dimensions are taken from a table. The principles are best illustrated in the design of the juncture of a circumferential lap joint with a longitudinal butt joint with straps of unequal width.

TABLE 15

DATA FOR TRIPLE-RIVETED JOINT (FIG. 69)

PLATE THICKNESS	STRAP THICKNESS	DIAM. RIVET HOLE	EFF.	LONG PITCH
1/4	1/4	11/16	87.5	5½
5/16	9/32	13/16	87.5	6½
3/8	5/16	13/16	88.4	7
7/16	3/8	15/16	87.9	7¾
1/2	7/16	15/16	88.3	8
9/16	7/16	1 1/16	86.7	8
5/8	1/2	1 1/16	86.7	8
11/16	1/2	1 3/16	85.6	8¼
3/4	1/2	1 3/16	85.5	8¼
13/16	9/16	1 5/16	84.6	8½
7/8	5/8	1 5/16	84.1	8¾
15/16	11/16	1 5/16	83.7	9
1	3/4	1 7/16	83.4	9½

TABLE 16

DATA FOR QUADRUPLE-RIVETED JOINT (FIG. 72)

PLATE THICKNESS	STRAP THICKNESS	DIAM. RIVET HOLE	EFF.	LONG PITCH
1/4	1/4	11/16	93.8	11
5/16	9/32	13/16	93.8	13
3/8	5/16	13/16	94.2	14
7/16	3/8	15/16	94.0	15½
1/2	7/16	15/16	94.1	16
9/16	7/16	15/16	94.1	16
5/8	1/2	1 1/16	93.4	16
11/16	1/2	1 3/16	92.8	16½
3/4	1/2	1 3/16	92.7	16½
13/16	9/16	1 5/16	92.3	17
7/8	5/8	1 5/16	91.2	17½
15/16	11/16	1 5/16	90.1	18
1	3/4	1 7/16	90.2	19
1 1/16	3/4	1 7/16	89.0	19
1 1/8	3/4	1 7/16	88.0	19
1 3/16	13/16	1 7/16	87.7	20
1 1/4	7/8	1 7/16	86.8	20

EXAMPLE. Assume that the problem involves the design of the joints of a boile shell having a diameter of 65 in. to withstand an internal pressure of 150 p.s.i.

1. We will assume, tentatively, a double-riveted butt joint for the longitudinal joint. From Table 14, the average efficiency e for such a joint is about 82 per cent. In order to determine the plate thickness, the working stress must be established. The Boiler Code of the A.S.M.E. specifies the ultimate strengths listed in Table 13 and applies a factor of safety of 5. Thus $s_t = 11{,}000$ p.s.i. Then, from formula (3),

$$t = \frac{pD}{2s_te} = \frac{150 \times 65}{2 \times 11{,}000 \times 0.82} = 0.54 \text{ in.}$$

If a 9/16 in. plate is used,* the necessary joint efficiency should be $0.82 \times 0.54/0.5625 = 78.8$ per cent. This efficiency will fall within the range attainable with double-riveted joints. On the other hand, if a 1/2 in. plate is selected, the efficiency will have to be $0.82 \times 0.54/0.50 = 88.5$ per cent. This would probably require a quadruple-riveted joint. To settle in a perfectly rational manner between the two possibilities thus presented, it would be necessary to compute the cost of the thicker shell with the cheaper joint as against the cost of the thinner shell with the more expensive joint. It is almost impossible to secure data on the cost of riveted joints in a form that would make computation a simple matter. In general the tendency in the United States is to increase the material cost in preference to the labor costs, if the difference is not too great. We shall therefore tentatively decide to use the thicker shell and the double-riveted joint.

2. From Table 15, a cover strap of thickness t_c is selected to go with the 9/16 in. plate. Since the narrow strap transmits less pull than the wide strap, theoretically it could be thinner. However, calking requires a firm edge, and, to avoid carrying a great number of plate thicknesses in stock, both straps are made of the same thickness. The strap thickness for a 9/16 in. plate would then be 7/16 in.

3. We will establish, tentatively, a rivet pitch at the calked edge of about eight times the thickness of the cover plate or strap as $8 \times 7/16 = 3\frac{1}{2}$ in. The pitch of the outer row in a double-riveted joint will be twice this inner pitch, or 7 in. The

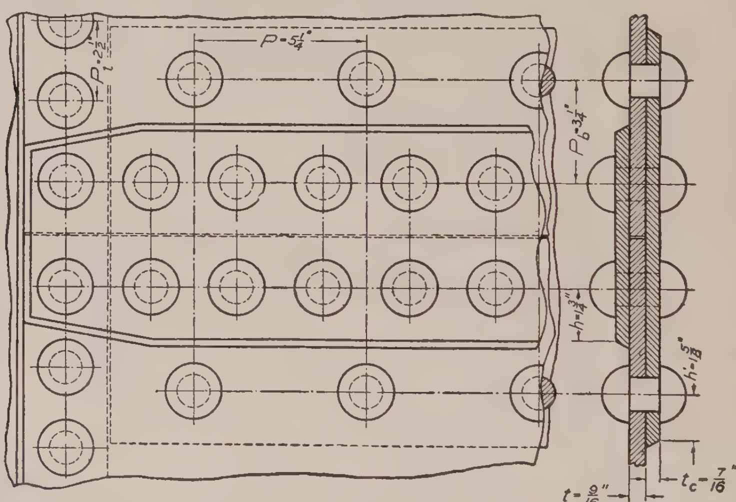


FIG. 73. DOUBLE-RIVETED BUTT JOINT JOINING SINGLE-RIVETED LAP JOINT.

rivet arrangement selected is illustrated in Fig. 73. The length of a unit strip of the joint is equal to 7 in., the pitch along the outer row of rivets.

4. The rivet diameter d may be determined by making the total shear strength of the rivets on one side of the joint equal to 1.2 to 1.3 times the strength of the solid

* The standards of the Hartford Steam Boiler Inspection and Insurance Company do not specify double-riveted butt joints in plates over 1/2 in. thick, claiming that the rivet size necessary is excessive. It will be instructive, however, to determine that by an actual trial design.

plate. While this proportion will give a theoretical shear strength in excess of the tearing strength at the joint, it is necessary because all the rivets are not stressed equally, the outer rivets being stressed excessively when failure is imminent.

The strength of the solid plate per unit length would be $7 \times 0.5625 \times 55{,}000 = 216{,}500$ lb. The total rivet strength in shear per unit length should be at least $1.2 \times 216{,}500 = 260{,}000$ lb. In a unit strip there are two rivets in double shear and one rivet in single shear, making a total of 5 effective shearing areas. Each area must be equal to $260{,}000/(5 \times 44{,}000) = 1.18$ sq. in., which corresponds to a rivet hole diameter of 1.23 in. The driven diameter of the rivet is equal to the diameter of the hole, and should be 1/16 in. greater than the diameter of the cold rivet. Standard rivet diameters are 5/8, 3/4, 7/8, 1, $1\frac{1}{8}$, $1\frac{1}{4}$, and $1\frac{3}{8}$ in. Acceptable hole diameters for different plate thicknesses are listed in Table 15.

In this problem the next larger standard rivet hole would be $1\frac{5}{16}$ in., corresponding to $1\frac{1}{4}$ in. rivets. Such rivets are not used in the standards of the Hartford Steam Boiler Inspection and Insurance Company for plates less than 25/32 in. thick. In plates 9/16 in. thick the rivet size is 1 in. and the hole diameter is $1\frac{1}{16}$ in. with a cross-sectional area of 0.8959 sq. in. If we wish to use this size and still keep the excess rivet strength, we must decrease the pitch from 7 in. to $7 \times 0.89/1.18 = 5\frac{1}{4}$ in. A still further reduction in rivet size may be accomplished by changing from a double-riveted to a triple-riveted joint, so as to have a greater number of rivets per unit length. We shall retain the double-riveted joint and proceed with the efficiency computation on the basis of the revised proportions.

5. The strength of the joint will now be determined. As a complicated joint may fail in a variety of ways, the A.S.M.E. Boiler Code lists the computations for the various modes of failure. A double-riveted butt joint may fail by any of the following methods, exclusive of A.

A = Strength of solid plate $= Pts_t = 5.25 \times 0.5625 \times 55{,}000 = 162{,}000$ lb.

B = Strength of plate between rivet holes in the outer row $\doteq (P - d)ts_t$, d being the diameter of the rivet hole. This value $= (5.25 - 1.0625) \times 0.5625 \times 55{,}000 = 130{,}000$ lb.

C = Strength of rivets in double shear, plus the strength of rivets in single shear $= NAs_s + nAs_s$, N being the number of rivets in double shear, n the number in single shear, A the cross-sectional area of the driven rivet, and s_s the shearing stress (Table 13). This shearing strength $= 5 \times 0.89 \times 44{,}000 = 196{,}000$ lbs.

D = Strength of the plate between rivets in the second row, plus the strength of the rivets in single shear in the outer row $= (P - 2d)ts_t + nAs_s = (5.25 - 2 \times 1.0625) \times 0.5625 \times 55{,}000 + 0.89 \times 44{,}000 = 136{,}000$ lb.

E = Strength of the plate between the rivets in the second row, plus the strength in crushing of the butt strap in front of the rivets in the outer row $= (P - 2d)ts_t + dt_cs_c = (5.25 - 2 \times 1.0625) \times 0.5625 \times 55{,}000 + 1.0625 \times 0.4375 \times 95{,}000 = 141{,}000$ lb. (From Table 13, $s_c = 95{,}000$ p.s.i.)

F = Strength of the plate in crushing in front of the inner rivets, plus the strength in crushing of the butt strap in front of the outer rivets $= Ndts_c + ndt_cs_c = 2 \times 1.0625 \times 0.5625 \times 95{,}000 + 1.0625 \times 0.4375 \times 95{,}000 = 157{,}500$ lb.

G = Strength of the plate in crushing in front of the inner rivets, plus the strength of the rivets in single shear $= Ndts_c + nAs_s = 2 \times 1.0625 \times 0.5625 \times 95{,}000 + 0.89 \times 44{,}000 = 153{,}000$ lb.

6. The efficiency of the joint is determined by dividing the least rupturing strength as computed above by the strength of the solid plate. The efficiency so found must be at least as great as the one previously assumed with the actual plate thickness. In this problem, the efficiency $e = 130{,}000/162{,}000 = 80$ per cent, which is satisfactory.

7. The other dimensions may now be determined to complete the joint. The *margin*, or the distance from the center line of the outer row of rivets to the nearest edge of the plate, is made from 1½ to 1¾ times the rivet hole diameter, or larger when recalking is to be expected. At every calking the edge must be planed or chipped off at least 1/8 in. In this case, the inner margin (h', in Fig. 73) could be made 1⅝ in., the outer margin (h in Fig. 73) 1¾ in.

The *back pitch*, or distance between the successive rows of rivets, should be at least 2 times the rivet hole diameter. Somewhat larger values are used if the ratio of the pitch to the rivet hole diameter exceeds 4.

The longitudinal pitch of the inner row is 2⅝ in. Since this value is less than 4×1.0625, the *minimum* back pitch would be $2 \times 1.0625 = 2\frac{1}{8}$ in. In this case, however, where two cover straps of unequal widths are used, it is necessary to insure ample space between the outer row of rivets and the edge of the narrow cover strap to permit heading and calking. A 1 in. rivet has a head diameter of $1.75 \times 1 = 1\frac{3}{4}$ in. (see Fig. 63). Allowing 5/8 in. from the outside of the head of the rivets in the outer row to the edge of the cover plate, the distance from the center of the rivets to the edge of this plate would be $1\frac{3}{4}/2 + 5/8 = 1\frac{1}{2}$ in. The *back pitch* would then be $1\frac{3}{4} + 1\frac{1}{2} = 3\frac{1}{4}$ in. This distance should be further checked to insure that the area in the main plate diagonally between the inner and outer rows of rivets is at least as great as the area of the plate between the rivets in the outer row. The head of the rivet in the outer row and the adjoining edge of the cover strap should be drawn to scale to make sure that ample space has been provided for the calking tool.

The proportions of the single-riveted lap girth joint must now be determined. The rivet pitch is obtained by equating the strength of the rivets in single shear to one-half the strength of the solid plate. As the strength of 5¼ in. of the plate is 162,000 lb. and the strength of one rivet in single shear is 39,000 lb., we require $0.5 \times 162{,}000/39{,}000 = 2.08$ rivets in this space. Thus the rivet pitch for the lap joint is $5.25/2.08 = 2.52$ in., say 2½ in., as this distance is sufficient for heading 1 in. rivets. The tearing strength of the plate between the rivets is manifestly greater than one-half of the strength of the solid plate, since $(2.5 - 1.0625)$ is more than $2.5/2$. The method of fitting the two joints together at their juncture is shown in Fig. 73.

78. Stays. The flat heads of boilers and pressure vessels are subjected to bending stresses and would have to be very thick unless they are supported. This support may be provided by means of *through stays*, in the form of long bolts extending through the vessel from end to end. *Corner stays*, as in Fig. 74, are probably more commonly used. *Gusset stays*, as shown in Fig. 75, are very strong but are also very stiff. For this reason they can be used only in unheated vessels. In heated vessels, the distortions due to temperature variations necessitate flexibility in the supports in order to prevent cracking.

Stays are computed on the assumption that they must withstand the total load on that portion of the head area which they are required

FIG. 74. CORNER STAY.

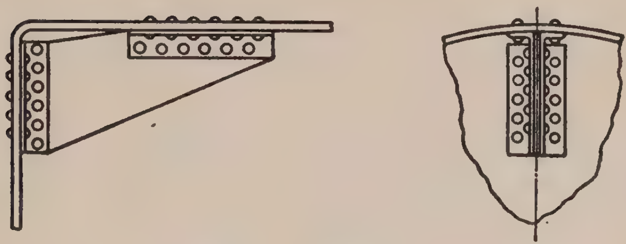


FIG. 75. GUSSET STAY.

to support. If, for instance, there are ten equally spaced stays in a head area of 100 sq. in., each stay should be computed to withstand a total load equal to 10 sq. in. times the pressure per sq. in. on the head. Corner stays which are inclined not less than 60 deg. to the stayed head are computed as through stays with an allowed tensile stress 10 per cent less than for a through stay.

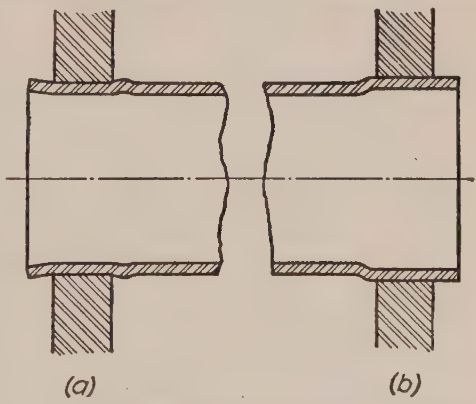


FIG. 76. FORMS OF EXPANDED BOILER TUBES.

79. Tubes. Boiler tubes expanded in the head and then beaded over have considerable holding capacity, and if properly spaced, no further staying is necessary in the tube zone.

Tubes are used in boilers, stills, condensers, heat exchangers, etc. The fixed ends of the tubes are usually expanded, as in Fig. 76a and b.

or expanded and beaded, as in Fig. 77a and b. At the fire box end of locomotive boilers (Fig. 78a), the tubes are beaded, expanded, and welded, and the tubes may be lodged in a ferrule to give added tightness and flexibility. Tubes in the water legs of water tube boilers may be flared only as shown in Fig. 76a. Detailed directions for computations of all types of stays as well as tubes are found in the A.S.M.E. Boiler Code.

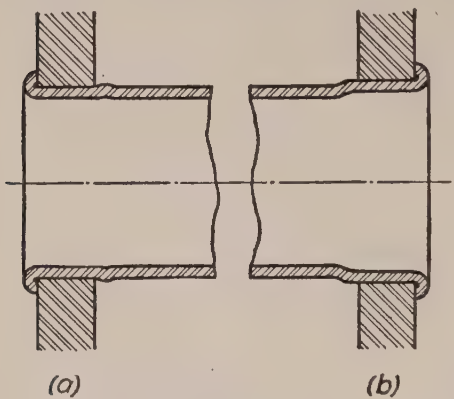


FIG. 77. FORMS OF EXPANDED AND BEADED TUBES.

80. Riveted Joints Carrying Eccentric Load. Structural joints may be subjected to a load which tends to twist the joint as well as to slide the connected members relatively to each other. This results from an eccentric shearing load and should be avoided whenever possible. An eccentrically loaded joint is shown in Fig. 79, where the load transmitted from the angle to the channel by the gusset plate intersects the center line of the channel at a point a inches above the center of gravity of the rivet group. In order that the gusset plate shall be in equilibrium, the total horizontal shearing stress in the rivets must equal the load F, and their combined resisting moment must equal the external turning moment. The direct load F in lb. presumably will be divided equally among the rivets; this produces a direct shear stress in each, parallel to F, and equal to $F/(nA)$, where n is the number of rivets and A the cross-sectional area in sq. in. of each rivet. The turning moment Fa about the center of gravity must be balanced by the summation of the shear forces in the individual rivets times their respective distances from the center of gravity. Thus

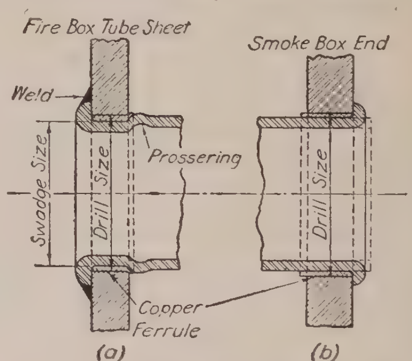


FIG. 78. LOCOMOTIVE BOILER TUBE.

$$Fa = F_1l_1 + F_2l_2 + F_3l_3 + \cdots + F_nl_n,$$

where $F_1, F_2, F_3, \cdots, F_n$ are the individual forces in lb. and $l_1, l_2, l_3, \cdots, l_n$ the individual distances in in. to the center of each rivet,

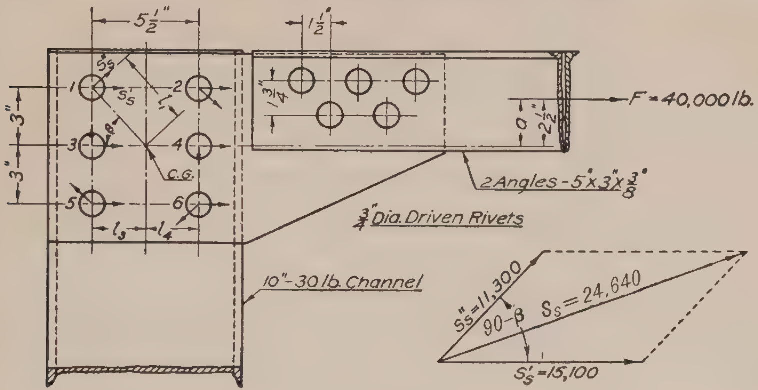


FIG. 79. RIVETED JOINT ECCENTRICALLY LOADED.

respectively. If the gusset plate is considered rigid, then its movement and hence the force * on each rivet are directly proportional to the distance from the center of rotation. Therefore,

$$\frac{F_1}{l_1} = \frac{F_2}{l_2} = \cdots = \frac{F_n}{l_n},$$

or

$$F_2 = F_1 \frac{l_2}{l_1}, \qquad \cdots, \qquad F_n = F_1 \frac{l_n}{l_1}.$$

Then

$$Fa = \frac{F_1}{l_1}[l_1^2 + l_2^2 + l_3^2 + \cdots + l_n^2].$$

If l_1 is the greatest distance, F_1 is the greatest force, and this force determines the shear stress due to the turning moment.

For the conditions shown in Fig. 79,

$$l_1 = l_2 = l_5 = l_6 = \sqrt{3^2 + 2.75^2} = 4.07 \text{ in.}$$
$$l_3 = l_4 = 2.75 \text{ in.}$$

A, area of 3/4 in. rivet (nominal) = 0.4418 sq. in.†

$$\cos(90° - \beta) = \frac{3}{4.07} = 0.737$$

* Since the intensity of the shear force is proportional to the distance from the center of rotation, the force at the inner edge of a rivet area is less than at the outer edge. Considering that the diameters of the rivet areas are small in comparison with the distances from the center of rotation, no serious error is involved in assuming that the force is concentrated at the center of the rivet.

† Since this problem is a *structural* joint, the nominal rivet area is used, in accord with structural steel practice. An allowable stress of 13,500 p.s.i. is commonly used.

Shear from direct load, $s_s' = \frac{F}{nA} = \frac{40{,}000}{6 \times 0.4418} = 15{,}100$ p.s.i.

$$Fa = 40{,}000 \times 2.5 \doteq \frac{F_1}{4.07}(4 \times 4.07^2 + 2 \times 2.75^2).$$

$$F_1 = \frac{100{,}000 \times 4.07}{81.385} = 5000 \text{ lb.}$$

Shear from moment, $s_s'' = \frac{5000}{0.4418} = 11{,}300$ p.s.i.

$$s_s^2 = s_s'^2 + s_s''^2 + 2s_s's_s'' \cos \beta.$$

$$s_s^2 = 1000^2(15.1^2 + 11.3^2 + 2 \times 15.1 \times 11.3 \times 0.737) = 10^8 \times 6.072$$

$$s_s = 24{,}640 \text{ p.s.i.}$$

This stress is too high and would require a redesign of the joint. The effect of the eccentricity has practically doubled the direct shearing stress, which proves that it is highly desirable to have the line of force pass through the center of gravity of the rivet group.

PROBLEMS

1. A locomotive boiler shell 6 ft. 3 in. in diameter and under an internal pressure of 200 p.s.i. is to be made of steel plate of uniform thickness having an ultimate tensile strength of 55,000 p.s.i. and a factor of safety of 5. The efficiency of the triple-riveted longitudinal butt joint is 85 per cent and of the double-riveted circumferential lap joint is 70 per cent. Determine to the nearest 16th higher, the thickness of the shell.

2. A drum 3 ft. in diameter and 8 ft. long is to carry fluid at a pressure of 300 p.s.i. The drum is made of boiler plate and is to have a double-riveted longitudinal butt joint. Select the thickness of plate required.

3. A double-riveted butt joint of 3/4 in. plate is to be designed for equal strength in tension, shear, and crushing. Determine the proportions if butt straps are of equal width. What is the efficiency of the joint as designed?

4. Design a single-riveted butt joint for a tank whose diameter is 40 in., to withstand 125 p.s.i.

5. Calculate the possible ways of failure of a double-riveted butt joint of prescribed proportions for 1/2 in. plate. What is the efficiency?

6. A double-riveted butt joint for a 1 in. plate is to have two 7/8 in. straps of equal width. Design by equating tensile and shear strengths and shear and crushing strengths. Determine the efficiency. Would this joint be fluid tight?

7. A triple-riveted butt joint of the pattern shown in Fig. 69 has the following dimensions: $t = 1\frac{1}{4}$ in., $t_c = 7/8$ in., $d = 1\frac{7}{16}$ in., $P = 12$ in. Calculate the strength of the joint in the possible methods of failure and the efficiency. How could the efficiency of this joint be increased?

8. A quadruple-riveted butt joint has three rows of rivets through both cover plates and one row through the wider cover plate. The pitch of the three inner rows is one-half the outer. The proportions are $t = 1\frac{1}{4}$ in., $t_c = 7/8$ in., $d = 1\frac{9}{16}$ in., $P = 22$ in. Calculate the strength of the joint in the possible methods of failure. What is the efficiency? How might this efficiency be improved?

9. A quadruple-riveted butt joint has two rows of rivets through both cover plates and two rows through the wider cover plate. The pitches of the next to outer row

and outer row of rivets are two and three times, respectively, that of the two inner rows. The joint proportions are: $t = 1/2$ in., $t_c = 7/16$ in., $d = 1\frac{1}{16}$ in., $P = 15$ in. Calculate the strength of the joint in the possible methods of failure. What is the efficiency?

10. Redesign the joint analyzed in the example of the text for triple-riveting.

11. Redesign the riveted bracket discussed on page 105, Fig. 79.

12. A vertical load of 6000 lb. is carried on a bracket 14 in. from the center line of a column. The bracket is fastened to the side of the column by four rivets placed symmetrically with the center line on a 7 in. diameter pitch circle. Determine the necessary diameter of the rivets if the stress is limited to 13,500 p.s.i. of nominal rivet area.

13. A channel is riveted to the side of a column with three rivets, all of the same diameter. The two rivets nearest the load are on the same vertical center line, each rivet 2 in. from the horizontal center line of the channel. The other rivet is on the horizontal center line of the channel, $4\frac{1}{2}$ in. to the left of the center line of the two rivets. A load of 3000 lb. is applied 15 in. to the right of the two rivets. Determine the necessary rivet diameter to limit the maximum induced shear stress to 10,000 p.s.i. of nominal rivet area.

CHAPTER 6

WELDING

81. Welding as a Production Method. Welding is now a production method of great importance, having advanced to that position after years of intensive research and its proven application on repair work and special fabrication. Welding is used in industry in a variety of forms and on products covering a considerable range of size. Boiler drums and storage tanks are generally of welded construction; automobile bodies, thin containers of various shapes, miscellaneous domestic and industrial appliances, machine parts, and frequently structures are of welded construction. The particular method of welding used depends upon the material and the proportions of the part, the service expected of the weld, the relative suitability of the process, and the relative cost.

82. Welding Processes. Welding is the process of joining metal under the influence of heat and may be accomplished with or without an accompanying pressure. Welding processes are divided by Llewellyn * into two general classifications: *plastic welding,* or union by pressure without a separate weld metal, and *fusion welding,* or union without pressure with a separate weld metal. Plastic processes are *forge welding,* and *electric resistance welding;* fusion processes are *gas* torch welding, electric *arc welding,* and *Thermit* chemical reaction welding.

83. Plastic Welding. *Forge* welding is accomplished by heating the two metal parts to a plastic state in a furnace and then uniting them by pressure. This pressure may be produced by means of a hand-wielded sledge, by a machine hammer, or by pressure rolls. The hand process is an old one, but it still has its uses. Pressure rolls are widely used in welding sheet into pipe. This form of welding is applicable not only to iron and steel, but also to nickel, gold, platinum, and a number of other metals.

Electric resistance welding utilizes the heat generated by the resistance of the welded metal to the passage of a large electric current going through the points to be welded. The parts are pressed together to effect the union after sufficient heat has been secured. In *butt* welding,

* *Outline of Welding,* by F. T. Llewellyn, Metal Progress, December, 1930 and January, 1931.

when parts of similar section are pressed together (Fig. 80), the current and pressure may be steady, as in *upset* welding, or the parts may be separated after the initial heating to cause sparking at the junction point before pressure is applied, as in *flash* welding. Sheets may be lap welded by the resistance process either along a seam or at certain spots. In *seam* welding (Fig. 81) the current and pressure are supplied by a wheel rolling along the seam, the work being supported on a base or on a horn. *Spot* welding (Fig. 82) joins the sheets at successive points instead

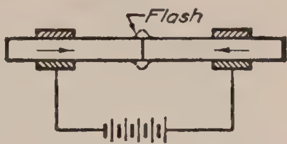


FIG. 80. BUTT WELD.

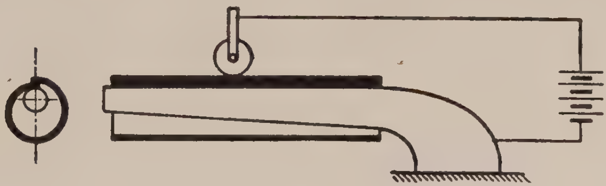


FIG. 81. SEAM WELD.

of along the whole joint, and is cheaper where strength and fluid tightness are not essential. Seam welding is limited economically to thin sheets, but spot welding may be applied to thicknesses of 1/2 in. or even more. The higher the electrical conductivity of the sheets, the more easily the current spreads, and consequently the more difficult they are to spot or seam weld. According to one prominent manufacturer of resistance welders, commercial welding is performed within the values of the following table, although special methods may extend the range.

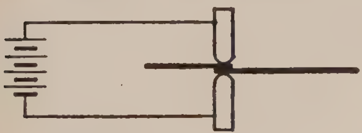


FIG. 82. SPOT WELD.

TABLE 17

THICKNESSES OF MATERIAL FOR SPOT AND SEAM WELDING

Material	Thickness of Sheet	
	Spot Weld	Seam Weld
Steel	0.01 to 0.50 in.	0.01 to 0.08 in.
Brass	0.01 to 0.13 in.	0.01 to 0.03 in.
Aluminum	to $\frac{3}{16}$ in.	none
Stainless steel	to $\frac{3}{16}$ in.	to 0.05 in.
Copper	Very little commercially	none

Projection welding (Fig. 83) is a resistance process where sheets are pressed together at mating projections, the welds being completed at different points simultaneously. In this process, the sheets must be thicker than 1/16 in.

Fig. 83. Projection Weld.

Shot welding is a very rapid resistance process used in welding stainless steel, and is similar to spot welding. Great rapidity is necessary to prevent metallurgical changes which would destroy the rust resisting properties of this steel.

84. Fusion Welding. The *fusion* process of welding melts the material at the points to be joined and fills the joint with metal from a filler rod which is generally of approximately the same composition as the base, or parent metal. *Gas* welding utilizes the heat of a gas torch which generally burns a controlled mixture of acetylene and oxygen. The gases are stored under pressure in drums, and are conducted through hose and a regulator to the torch tip, where they are adjusted to mix in the proper proportions. No power is required and the whole outfit is readily transported. The joined surfaces and the separate filler rod are made fluid and puddled together and on cooling form a homogeneous body. This process may be used for many constructions and is used extensively for pipe lines and airplane fuselages. Most fusion welding of thin sheet is done by this method.

The same basic equipment may be used in gas *cutting* of steel and iron. Automatic cutting attachments produce very smooth edges on metal several inches in thickness; sections 12 in. or more thick are readily cut. Reproducing mechanisms make it possible to cut complicated shapes, eliminating more expensive machine forming. Slight changes in successive pieces may be obtained inexpensively by changing the guiding cam. Special machine frames, bases, cams, etc., may be produced conveniently by the gas cutting method.

The *electric arc* process is another welding method of great adaptability in which portable equipment may also be used. A special electric motor-generator set, generally producing direct current, sends low-voltage, high-amperage current through a circuit of which the work and an electrode are a part. When a carbon electrode is used, the heat of the arc melts the surfaces to be united and also the filler rod, giving the same cast junction as is obtained with the gas weld. When the filler rod is used as the electrode, we call the method used the metallic arc. The carbon arc gives a hotter flame, is used for special materials, for automatic welding, and at times for filling in large amounts of welding material. The electromagnetic action of the

metallic arc carries particles of the electrode into the weld and thus makes overhead welding possible.

The electrode metal used depends upon the material to be welded and the property of the weld required. While the metal at the junction surfaces is in a fluid condition, it absorbs oxygen and nitrogen from the air and the resultant weld, although of ample strength for most uses, lacks ductility and resistance to repeated stresses. For this reason, the weld should be completed in an atmosphere protected from these active gases. One method of obtaining this protection is to surround the weld with gas produced by the burning of a special powdered material, which may be piled along the seam or carried as a coating on the electrode. The coating melts at the heat of the arc, liberating a gaseous shield for the weld and depositing a protective coating of slag over the weld, thus beneficially retarding cooling. The material used for this purpose should be one having a strong affinity for oxygen, for instance, ferromanganese or aluminum, and should be mixed with a slag-forming material and arc stabilizer, such as clay or lime.

According to the Lincoln Electric Company, welds made with coated electrodes have about 30 per cent more strength and greater ductility (over 20 per cent in 2 in.) than welds produced with bare rods. They claim also that the added cost of the coated rod is more than justified by the greater speed of welding. For this reason they do not recommend the use of a bare rod.

Arc welding, besides being a very versatile manufacturing process, is also adapted to more rapid production work when operated automatically. If the arc is to be shielded, the arc plows through powder piled along the seam. The arc-welding process is generally applicable to and is extensively used for machine parts, structures, tanks, drums, etc. Very thin parts are difficult to weld by this process.

In the *atomic hydrogen* process of welding, an arc established between two tungsten electrodes is surrounded by a stream of hydrogen directed upon it. The hydrogen protects the weld from the atmosphere and at the same time contributes to the heat of the weld. Under the influence of the arc the molecular hydrogen is dissociated into atomic form, which, on recombining at the weld, liberates additional heat. This process is especially adapted to the welding of thin plates. The finished weld is exceptionally clean and smooth.

Thermit welding utilizes the heat produced by the combustion of a mixture of finely divided aluminum and iron oxide from which a temperature of about 5000° F. is obtained. In this method the weld metal is all added at once instead of layer by layer as in the gas and electric-arc processes. The general procedure is to separate the

surfaces to be welded, shape a wax mold of the form desired in the final weld, and enclose this wax mold in a sand mold. The wax is then melted, leaving a hollow space for the weld metal. The aluminum-iron oxide mixture is ignited by a cartridge in a separate container and flows into the mold. The iron formed melts and superheats the parts while the slag rises to the top of the mold. When the weld cools, a homogeneous casting has been obtained. This method is used in bonding rails, uniting the ends of pipe under pressure, and for heavy repairs, particularly in railroad and marine equipment. Because of the disadvantage of requiring a mold, Thermit welding is largely confined to the repair of broken parts.

85. Welded Joints. Plates may be joined together with a lap weld by lapping one plate over the other and welding the edge of one to the surface of the other, as in Fig. 84; or with a butt weld by bringing the edges close together and filling the space between with weld metal, as in Fig. 85. Lap welds may have the welded bead placed along the edges parallel to the direction of the load, as in Fig. 86, or along the edge, or edges, perpendicular to the load (Fig. 87). The joint with the bead perpendicular to the line of action of the load is about 30 per cent

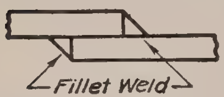


FIG. 84. LAP WELD.

FIG. 85. BUTT WELD.

stronger than the one with the bead parallel, other factors being equal. In either case, the plates of the lap joint and weld are subjected to a severe bending action due to the offset pull; for this reason butt joints are to be preferred (Fig. 88).

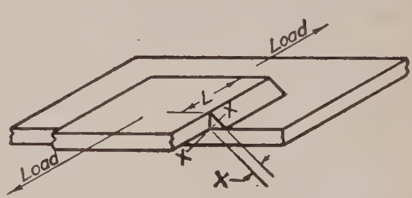


FIG. 86. WELD IN LONGITUDINAL SHEAR.

Thin plates may be butt-welded without special preparation of the edges, except cleaning off of all scale and foreign matter. For thicker plates, the edges of the plates are beveled for a single V weld, as in Fig. 89a, and for still thicker plates a double-V weld, as in Fig. 89b, may be used. While an included angle of 90 deg. is frequently recommended as best, an angle of 60 deg. will usually give ample penetration and requires less weld metal. A U-shaped weld as shown in Fig. 90 uses less of the expensive weld metal than the V shape and is used for very thick

plates. Fig. 91 shows how the successive layers of the bead are applied.

The designated size of a fillet weld is the length of the side of the triangle, which, in lap welds, is equal to the thickness of the edge welded. The fillet is generally flat, with a 45 deg. inclined side, although a 60 deg. fillet gives a better stress distribution and is coming into favor for important joints.

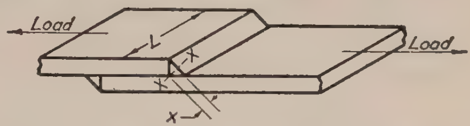


FIG. 87. WELD IN CROSS SHEAR, AND TENSION.

Fig. 92 shows approved fillet proportions. A large amount of reinforcement of the bead in the butt weld does more harm than good, because the outer portion of the bead carries no stress and the abrupt change in contour causes a sharp stress concentration at the edge of the bead. The bead should be gently rounded, as in Fig. 92-1.

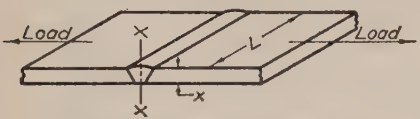
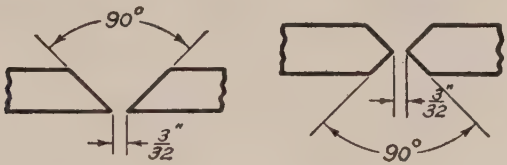
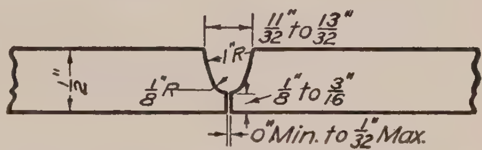


FIG. 88. WELD IN TENSION.



(a) (b)

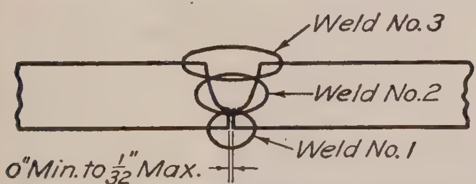
FIG. 89. V WELDS.



From General Electric Arc Welding Manual

FIG. 90. PREPARATION OF JOINT FOR U WELD IN 1/2 IN. PLATE.

The T-joint in Fig. 92-2 shows a joint which is satisfactory if the load is not high. The fillets are terminated gradually in arcs for improved stress distribution, but the discontinuity between the abutting plates produces high stress concentration. If the horizontal plate is prepared with a double-V, as in Fig. 92-3, the weld metal is continuous and the stress flow is regular. For the same reason Fig. 92-5 is preferred to Fig. 92-4 for heavy loads. The refinement of the double-V and the additional welding are econom-



From General Electric Arc Welding Manual

FIG. 91. METHOD OF APPLYING SUCCESSIVE LAYERS OF METAL TO U-EDGE WELD.

ically justifiable only when the loads are appreciable. When loads are small, parts are frequently tack welded, or joined at successive spots only. For strength greater than that of tack welds over a long seam, intermittent welds are used, beads several inches long being distributed along the seam.

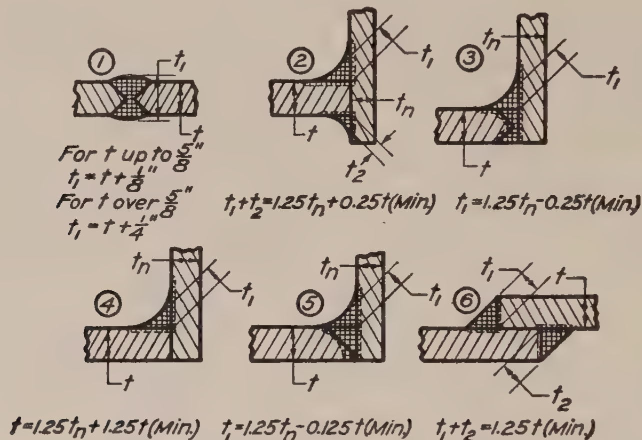


FIG. 92. FILLET PROPORTIONS OF WELDS.

86. Welded Pressure Vessels. Welding is rapidly supplanting riveting for pressure vessels, especially for higher pressures. This development has proceeded under rigid regulation by the A.S.M.E. and other boiler codes. Fired pressure vessels (boiler drums) must have all welds made on plate up to 4 in. thick, radiographed either by X-ray or gamma rays to insure the absence of poor weld penetration, gas holes, cooling cracks, and other defects. Two types of welded storage drum are illustrated in Fig. 93, A being a butt joint on a

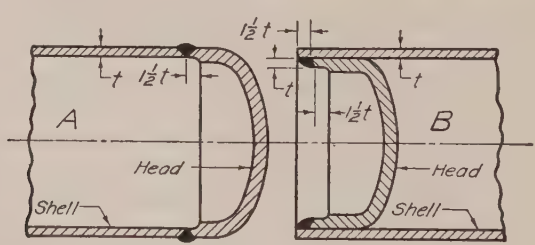


FIG. 93. WELDED DRUM HEADS.

"bumped" head and B being a lap joint on a "dished" head. Several different methods of welding nozzle outlets to drums, both with unreinforced and reinforced construction, are illustrated in Fig. 94.

87. Strength of Welds. Where excessive strength is not necessary, steel plate selected for welding generally has a carbon content of 0.3 per cent or less. Steels of higher carbon content and alloy steels may

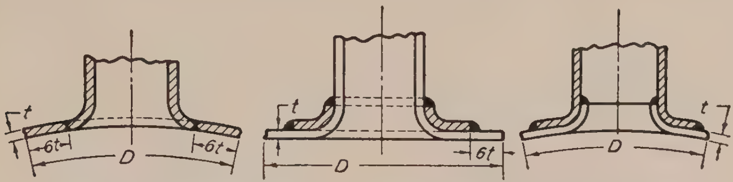


FIG. 94. UNREINFORCED AND REINFORCED NOZZLE OUTLETS.
Stress relieving is required over a minimum width of band denoted by *D*.

be welded, if a proper welding rod is used. Proper procedure and materials will enable the full strength of the parent metal to be developed. Bare welding rods are allowed an average tensile strength of 45,000 p.s.i. by the code for Fusion Welding (Structural). Welds produced in a shielded arc have a tensile strength of from 60,000 to 80,000 p.s.i. The shearing strength is less, and, as nearly all welds are subjected to some shear stress, fillet welds are calculated on the basis of 11,300 p.s.i. Welds made with a shielded arc may be calculated on a basis of 14,000–15,000 p.s.i.* Fillet welds in shear fail across the section *X-X*, Figs. 86 and 87, so that the effective area used in determining the average stress is *Lx*. On this basis, the following table, taken from the Lincoln Electric Company's *Handbook on Arc Welding*, shows the allowable shear load in pounds for one linear inch of fillet weld for steady loads.

TABLE 18

SAFE ALLOWABLE LOADS FOR FILLET WELDS IN LONGITUDINAL SHEAR

Size of Fillet Weld Leg Dimension	Pounds per Linear Inch	
	"Fusion Code" (Structural)	Shielded Arc Process
⅛ in.	1000	1250
3/16 in.	1500	1875
¼ in.	2000	2500
5/16 in.	2500	3125
⅜ in.	3000	3750
½ in.	4000	5000
⅝ in.	5000	6250
¾ in.	6000	7500

* *Welding design*, Charles H. Jennings, Transactions, A.S.M.E., MSP-58-1, gives values for working stresses in welds and methods of calculation for various types of connections.

Figure 95, after the Lincoln Electric Company, gives the rupturing force for various sizes and types of weld. Values obtained from this chart may be divided by the factor of safety required.

As a welded joint has been severely heated locally and cooled rapidly, considerable thermal stresses may remain in the joint and in the adjacent material. Sometimes these stresses must be relieved by

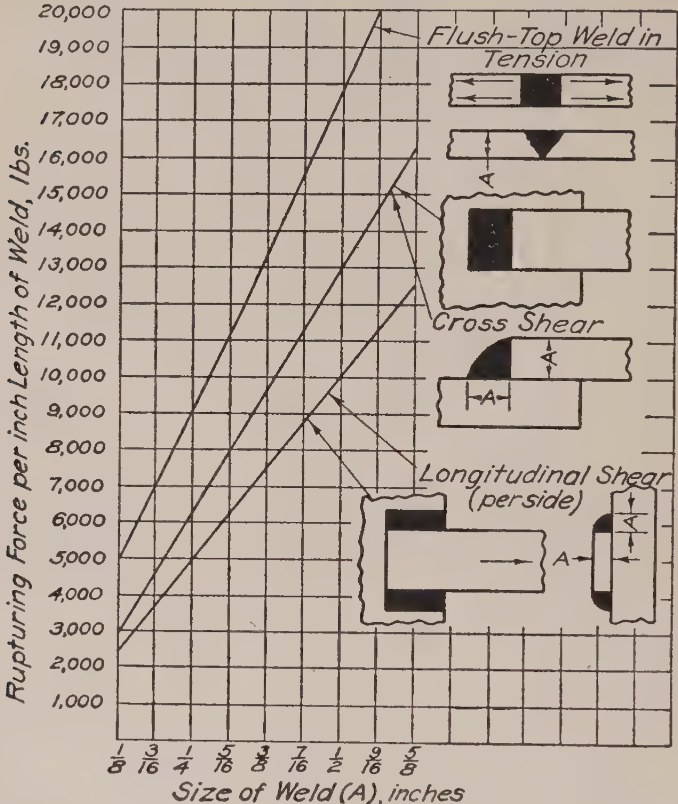


FIG. 95. RUPTURING FORCE FOR VARIOUS SIZES AND TYPES OF WELDS.

heating or cold-working (generally peening with a hammer). In Fig. 94, the distance D indicates the minimum band for reheating. When making long welds, especially on structural steel, great care must be exercised to prevent excessive warping of the parts. Long seams can be welded by the "step-back" method, in which the weld metal is applied in short sections laid down in a direction opposite to the direction of progress of the weld.

88. Welding Design. Welded machinery bases, gear blanks, special pipes and drums, and even entire machines of welded construction are in frequent demand. The design of such products demands ingenuity and a familiarity with the welding processes. The design should not be a copy of the casting or forging that might be used, but should be a new design of such form as to permit the member to function properly and to allow economical construction. Welding saves pattern cost and is particularly advantageous economically as compared with casting when the number of similar parts to be made is small. For this reason welding has been found an effective means of saving both time and expense in the manufacture of such devices as jigs and fixtures.

Plans drawn for welding should show in detail the welding requirements, and not merely state "weld" and leave the rest to the operator. The kind of welding rod to be used must be known, so that proper calculations may be made for strength. The preparation of the plate and the location, size, and length of the fillet must be specified. Either the engineering department, or the welding supervisor, should specify the size of welding rod, as well as the current and voltage, in case the electric arc is to be used. Successful welding is largely dependent upon supervision and efficient planning, trained operators, rigid inspection, and a constant check of the work in order to insure consistently reliable results.

PROBLEMS

1. A lap joint like Fig. 87 is welded with two 3/8 in. fillets, 4 in. long. What is the safe load for the weld? If the plate is 1/2 in. thick and 4 in. wide, what is the safe load for the plate? Give values for both uncoated and coated rod on boiler plate steel.

2. An angle clip is attached to a column by two 3/8 in. fillet welds loaded in transverse shear. What must be the length of each weld to support 8500 lb. if an uncoated rod is used?

3. A welded T-joint has a 1/2 in. double fillet weld. What length of weld is necessary to support a load of 25,000 lb. if a coated rod is used? If an uncoated rod is used? Joint is loaded in longitudinal shear.

4. A collar is welded to a shaft by a 1/4 in. fillet weld between the surface of the shaft and one face of the collar. What torque will the collar transmit to the 3 in. shaft? A coated rod is used.

5. Design a clevis made of sheet steel to transmit a pull of 18,000 lb.

CHAPTER 7

SCREW THREADS

89. General Remarks. Screw threads are used on bolts and screws mainly for the purpose of uniting parts. To a much lesser extent, screws threads are used for adjustments; as a means of making precision measurements, as in calipers and micrometers; and for the transmission of power or motion, as on lathe screws or jack screws. For these different applications, several different forms of threads are used.

90. Forms of Screw Threads. Threads of angular type, as represented by the *American* (*National*) *Standard* thread (Fig. 96a), the

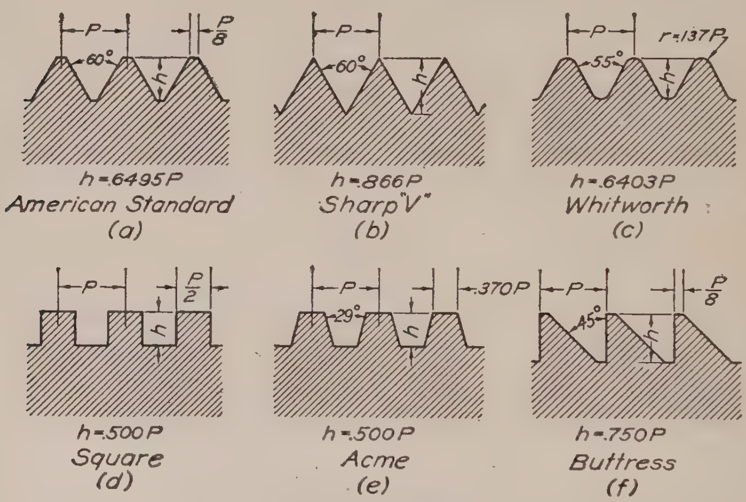


FIG. 96. FORMS OF SCREW THREADS.
P = pitch of thread.

sharp V (Fig. 96b), and the *Whitworth* thread (British Standard, Fig. 96c) are used for fastenings. The sloping sides of these threads produce a wedging action with the threads in the nut, thereby increasing the holding action. The threads are easily cut and are strong in bending and shear. The American Standard thread, as the name indicates, is the adopted standard in this country. The flat crest at the top protects the thread from being damaged, while the flat at the bottom gives added root strength to the bolt and facilitates cutting.

Angular threads are invariably used for precision measuring screws because it is usually necessary to employ a split nut as a means of eliminating lost motion between the threads.

For the transmission of power, the *square* thread (Fig. 96d) and the *Acme* thread (Fig. 96e) are the forms commonly used. The square thread is the most efficient form of power screw, since the normal thread force is exerted in a plane parallel to the axis of the screw. It is difficult to cut, however, is the weakest type at the base of the thread, and cannot be used with a split adjusting nut. The Acme or 29-deg. thread is a modification of the square thread and is more extensively used as a power screw than the square thread. Its efficiency is high, although not so high as the square thread; it is more easily cut; it is stronger at the base, and can be used with a split nut if necessary.

The *buttress* thread (Fig. 96f) is used for transmitting or resisting forces in one direction only, as in screw jacks or the ram adjusting screw of a punch press. It has the efficiency of the square thread in one direction, yet it possesses the strength of a V-type thread at the base.

91. Thread Action. A screw thread is essentially an inclined plane wound upon the surface of cylinder or cone, externally for a screw (Fig. 97) and internally for a nut (Fig. 98). In overcoming a resistance, as in holding two or more parts together with a screw fastening, or applying a force as with a power screw, the engaging threads slide on each other under the application of a torque. The space curve, that is, the projection of the thread on the cylindrical or conical surface, is a "helix" (see Fig. 99).

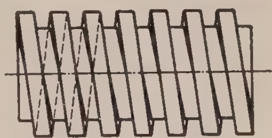


FIG. 97. EXTERNAL THREAD.

Threads may be either right-handed or left-handed. A *right-handed* thread (Fig. 97), if turned in a clockwise direction, advances into a threaded hole, whereas a *left-handed* thread, in order so to advance, must be turned in a counter-clockwise direction.

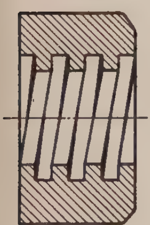


FIG. 98. INTERNAL THREAD.

92. Pitch, Lead, and Multiple Threads. The *pitch* of a screw thread is the distance between corresponding points on adjacent threads, measured parallel to the axis. The *lead* is the distance a nut advances in one revolution. Threads may be cut single or multiple. A *single* thread, as the name implies, is one continuous uninterrupted thread. In this case, the pitch and lead are equal. In multiple threads, two, three, four, or more threads are cut parallel to each other

on the same shaft (Fig. 99). The lead of a double thread is then twice that of a single thread, etc. Multiple threads are used when a rapid advance is required without using a coarse thread, as in the threaded

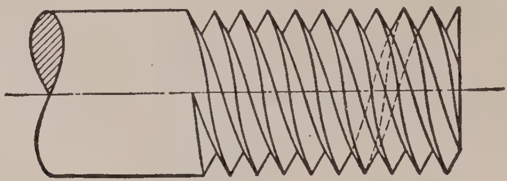


FIG. 99. TRIPLE V THREAD (RIGHT-HAND).

connection between a fountain pen cap and barrel; or to increase the efficiency of power screws, as will be explained later. A screw thread is always understood to be single right-hand unless otherwise specified.

93. American Standard Screw Threads. One of the most important standards and one of far-reaching significance involves the standardization of screw threads. The American Standards Association* recently approved and adopted the American (National) form of thread (Fig. 96a) which was previously known as the Sellers or United States Standard. A total of five series of screw threads was approved, all of the same form, but varying the pitch for a given diameter. Two of the series of most importance are the *coarse* series, based upon the old U. S. Standard, and the *fine* series, based upon the S.A.E. (Society of Automotive Engineers) standard (see Table 19 for dimensions).

The American Standards Association also recommends three series of finer threads for somewhat special applications, as follows:

8-*Pitch Series* has 8 threads per in. for all diameters ranging from 1 in. to 6 in. It is used where initial tightness is necessary, as in high-pressure cylinder head bolts, pipe flanges, etc.

12-*Pitch Series* has 12 threads per in. for all diameters ranging from 1/2 in. to 6 in. It is used in boiler practice, for thin nuts on sleeves, and for general machine construction.

16-*Pitch Series* has 16 threads per in. for all diameters ranging from 3/4 in. to 4 in. It is used primarily for threading adjusting collars and bearing retaining nuts.

94. S.A.E. Extra Fine Thread Series. In addition to the American Standard thread series, the S.A.E. has standardized an *extra fine* thread series. This series, which is given in Table 19, is suitable for threads on thin tubular sections, on parts where fine adjustments are required, and where tightness against vibration is a very important factor.

* American Standards Association, B1, 1–1935.

TABLE 19

AMERICAN STANDARD SCREW THREADS. COARSE AND FINE THREAD SERIES. S.A.E. EXTRA FINE THREAD SERIES

Size	Basic Major[a] Diameter (In.)	Coarse Series (NC)			Fine Series (NF)			Extra Fine Series (EF)		
		Threads per In.	Minor[a] Diameter	Root Area	Threads per In.	Minor[a] Diameter	Root Area	Threads per In.	Minor[a] Diameter	Root Area
0	0.0600	——	——	——	80	0.0438	0.0015	——	——	——
1	0.0730	64	0.0527	0.0022	72	0.0550	0.0024	——	——	——
2	0.0860	56	0.0628	0.0031	64	0.0657	0.0034	——	——	——
3	0.0990	48	0.0719	0.0041	56	0.0758	0.0045	——	——	——
4	0.1120	40	0.0795	0.0050	48	0.0849	0.0057	——	——	——
5	0.1250	40	0.0925	0.0067	44	0.0955	0.0072	——	——	——
6	0.1380	32	0.0974	0.0075	40	0.1055	0.0087	——	——	——
8	0.1640	32	0.1234	0.0120	36	0.1279	0.0128	——	——	——
10	0.1900	24	0.1359	0.0145	32	0.1494	0.0175	——	——	——
12	0.2160	24	0.1619	0.0206	28	0.1696	0.0226	——	——	——
$\frac{1}{4}$	0.2500	20	0.1850	0.0269	28	0.2036	0.0326	32	0.2117	0.0352
$\frac{5}{16}$	0.3125	18	0.2403	0.0454	24	0.2584	0.0524	32	0.2742	0.0591
$\frac{3}{8}$	0.3750	16	0.2938	0.0678	24	0.3209	0.0809	32	0.3367	0.0890
$\frac{7}{16}$	0.4375	14	0.3447	0.0933	20	0.3725	0.1090	28	0.3937	0.1211
$\frac{1}{2}$	0.5000	13	0.4001	0.1257	20	0.4350	0.1486	28	0.4562	0.1635
$\frac{9}{16}$	0.5625	12	0.4542	0.1620	18	0.4903	0.1888	24	0.5114	0.2054
$\frac{5}{8}$	0.6250	11	0.5069	0.2018	18	0.5528	0.2400	24	0.5739	0.2586
$\frac{3}{4}$	0.7500	10	0.6201	0.3020	16	0.6688	0.3513	20	0.6887	0.3725
$\frac{7}{8}$	0.8750	9	0.7307	0.4193	14	0.7822	0.4805	20	0.8131	0.5192
1	1.0000	8	0.8376	0.5510	14	0.9072	0.6464	20	0.9387	0.6922
$1\frac{1}{8}$	1.1250	7	0.9394	0.6931	12	1.0167	0.8118	18	1.0568	0.8972
$1\frac{1}{4}$	1.2500	7	1.0644	0.8898	12	1.1417	1.0238	18	1.1818	1.0975
$1\frac{1}{2}$	1.5000	6	1.2835	1.2938	12	1.3917	1.5212	18	1.4418	1.7120
$1\frac{3}{4}$	1.7500	5	1.4902	1.7441	——	——	——	——	——	——
2	2.0000	$4\frac{1}{2}$	1.7113	2.3001	——	——	——	——	——	——
$2\frac{1}{4}$	2.2500	$4\frac{1}{2}$	1.9613	3.0212	——	——	——	——	——	——
$2\frac{1}{2}$	2.5000	4	2.1752	3.7161	——	——	——	——	——	——
$2\frac{3}{4}$	2.7500	4	2.4252	4.6194	——	——	——	——	——	——
3	3.0000	4	2.6752	5.6209	——	——	——	——	——	——

[a] Major diameter is the nominal diameter of the shaft on which the thread is cut. Minor diameter is the diameter at the root of the thread.

95. Screw Thread Fits. The American Standards Association has also established four classes of fits for threaded parts as follows:

Class 1 *Fit.*—"Recommended only for threaded parts where clearance between mating parts is essential for rapid assembly and where shake is not objectionable."

Class 2 *Fit.*—"Represents a high quality of commercial thread product and is recommended for the great bulk of interchangeable screw thread work."

Class 3 *Fit.*—"Represents an exceptionally high quality of commercially threaded product and is recommended only in cases where the high cost of precision tools and continual checking is warranted."

Class 4 *Fit.*—"Intended to meet very unusual requirements. It is a selective fit if initial assembly by hand is required. Not yet adaptable to quantity production."

Tables are provided in the Standard (ASA B 1. 1—1935), also standard handbooks, giving detailed dimensions and tolerances for meeting these specifications. Symbols are used to designate the thread series and fit as, 1″—8—NC—2, meaning 1 in. diameter, 8 threads per in., National Coarse series, class 2 fit.

96. American Standard Bolts and Nuts. The American Standards Association has standardized wrench-head bolts and nuts, and wrench openings for both square and hexagon bolts, as given in Table 20.

TABLE 20

AMERICAN STANDARD SQUARE AND HEXAGON BOLT HEADS, NUTS, AND JAM NUTS. REGULAR SERIES

Diameter of Bolt	Width Across Flats		Height of Bolt Heads			Thickness of Nuts			
						Regular		*Jam*	
	Unfinished and Semi-finished Square and Hexagon Bolt Heads	Finished Hexagon Bolt Heads, Regular and Jam Nuts	Unfinished Square and Hexagon	Semi-finished Square and Hexagon	Finished Hexagon	Unfinished Square and Hexagon and Finished Hexagon	Semi-finished Square and Hexagon	Unfinished Square and Hexagon and Finished Hexagon	Semi-finished Square and Hexagon
1/4	3/8	7/16	11/64	5/32	3/16	7/32	13/64	5/32	9/64
5/16	1/2	9/16	13/64	3/16	15/64	17/64	1/4	3/16	11/64
3/8	9/16	5/8	1/4	15/64	9/32	21/64	5/16	7/32	13/64
7/16	5/8	3/4	19/64	9/32	21/64	3/8	23/64	1/4	15/64
1/2	3/4	13/16	21/64	19/64	3/8	7/16	27/64	5/16	19/64
9/16	7/8	7/8	3/8	11/32	27/64	1/2	15/32	11/32	21/64
5/8	15/16	1	27/64	25/64	15/32	35/64	17/32	3/8	23/64
3/4	1 1/8	1 1/8	1/2	15/32	9/16	21/32	41/64	7/16	27/64
7/8	1 5/16	1 5/16	19/32	9/16	21/32	49/64	3/4	1/2	31/64
1	1 1/2	1 1/2	21/32	19/32	3/4	7/8	55/64	9/16	35/64
1 1/8	1 11/16	1 11/16	3/4	11/16	27/32	1	31/32	5/8	39/64
1 1/4	1 7/8	1 7/8	27/32	25/32	15/16	1 3/32	1 1/32	3/4	11/16
1 3/8	2 1/16	2 1/16	29/32	27/32	1 1/32	1 13/64	1 9/64	13/16	3/4
1 1/2	2 1/4	2 1/4	1	15/16	1 1/8	1 5/16	1 1/4	7/8	13/16
1 5/8	2 7/16	2 7/16	1 3/32	1 1/32	1 7/32	1 27/64	1 23/64	15/16	7/8
1 3/4	2 5/8	2 5/8	1 5/32	1 3/32	1 5/16	1 17/32	1 15/32	1	15/16
1 7/8	2 13/16	2 13/16	1 1/4	1 3/16	1 13/32	1 41/64	1 37/64	1 1/16	1
2	3	3	1 11/32	1 7/32	1 1/2	1 3/4	1 11/16	1 1/8	1 1/16
2 1/4	3 3/8	3 3/8	1 1/2	1 3/8	1 11/16	1 31/32	1 29/32	1 1/4	1 3/16
2 1/2	3 3/4	3 3/4	1 21/32	1 17/32	1 7/8	2 3/16	2 3/32	1 1/2	1 13/32
2 3/4	4 1/8	4 1/8	1 53/64	1 11/16	2 1/16	2 13/32	2 5/16	1 5/8	1 17/32
3	4 1/2	4 1/2	2	1 7/8	2 1/4	2 5/8	2 17/32	1 3/4	1 21/32

Unfinished bolt heads are not machined on any surface.
Unfinished nuts are threaded but not machined on any other surface.
Semi-finished bolt heads are machined under head only, either plain or washer-faced.
Semi-finished nuts are threaded and machined on bearing surface, either plain or washer-faced.
Finished bolt heads are machined on all surfaces with washer face on bearing surface.
Finished nuts are threaded, machined on all surfaces, and washer-faced.

97. Types of Screw Fastenings and Their Uses. *A through* bolt and nut (Fig. 100) should always be used as a fastening when the hole can be drilled through both the parts to be held together. Sufficient clearance should be provided, ordinarily 1/32 in. for smaller sizes and 1/16 in. for larger sizes, to permit free assembly and to allow for inaccuracies in alignment, etc.

98. Studs. A stud is a fastening with threads at each end (Fig. 101). It is used when a through bolt and nut is not suitable, that is, when the hole cannot be drilled through both parts. By screwing

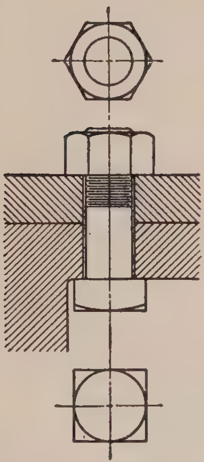


Fig. 100. Square Head Machine Bolt.

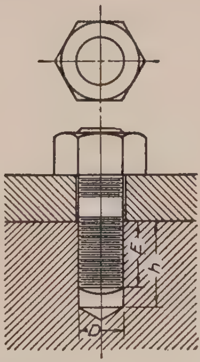


Fig. 101. Stud.

$E = 1.5D$ for cast iron.
$h = 2D$ for cast iron.
$E = D$ for steel.
$h = 1.5\,D$ for steel.

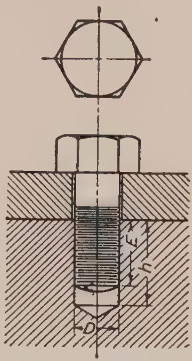


Fig. 102. Tap Bolt.

$E = 1.5D$ for cast iron.
$h = 2D$ for cast iron.
$E = D$ for steel.
$h = 1.5D$ for steel.

one end tightly in the tapped hole, the nut only is removed when parts are disassembled, thus preserving the threads in the machine member. The depth of thread engagement is specified in Fig. 101.

99. Tap Bolts. A tap bolt (Fig. 102) may be used in place of a stud in cases where a bolt and nut cannot be used and when the parts that are held together are removed only occasionally. The name tap bolt is given to regular bolts (Table 20) when used without a nut.

100. Cap Screws. *Cap* screws are used for the same purpose as tap bolts. In addition to the hexagon head, several types of slotted heads are also provided, as shown in Fig. 103 and Table 21. It should be noted that the proportions of the hexagon head cap screw differ from those of the hexagon head bolt.

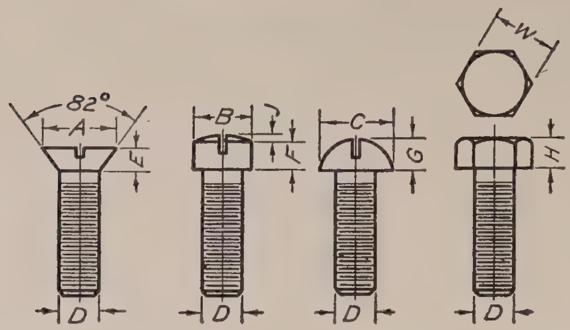


Fig. 103. Cap Screws.

TABLE 21

Approved American Standard Cap Screws. Flat Head, Fillister Head, Button Head, Hexagon Head, Coarse Threads

Diameter D	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1	1 1/8	1 1/4
A	1/2	5/8	3/4	13/16	7/8	1	1 1/8	1 3/8				
B	3/8	7/16	9/16	5/8	3/4	13/16	7/8	1	1 1/8	1 5/16		
C	7/16	9/16	5/8	3/4	13/16	15/16	1	1 1/4				
E	.146	.183	.220	.220	.220	.256	.293	.366				
F	11/64	13/64	1/4	19/64	21/64	3/8	27/64	1/2	19/32	21/32		
G	.191	.246	.273	.328	.355	.410	.438	.547				
H	3/16	15/64	9/32	21/64	3/8	27/64	15/32	9/16	21/32	3/4	27/32	15/16
J	.044	.050	.064	.071	.084	.091	.099	.112	.126	.146		
W	7/16	1/2	9/16	5/8	3/4	13/16	7/8	1	1 1/8	1 5/16	1 1/2	1 11/16

101. Machine Screws. In this classification are included small screws mostly designated by numbers except in the larger sizes (see Fig. 104 and Table 22). Machine screws as approved by the American Standards Association are provided with four types of slotted heads and a coarse and fine series of threads.

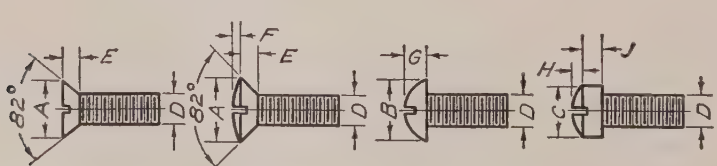


Fig. 104. Machine Screws.

TABLE 22

APPROVED AMERICAN STANDARD MACHINE SCREWS

Nominal Size	2	3	4	5	6	8	10	12	1/4	5/16	3/8
Diameter—D	.086	.099	.112	.125	.138	.164	.190	.216	.250	.312	.375
Thds. per in. (Coarse)	56	48	40	40	32	32	24	24	20	18	16
Thds. per in. (Fine)	64	56	48	44	40	36	32	28	28	24	24
A (max.)	.172	.199	.225	.252	.279	.332	.385	.438	.507	.636	.762
B "	.162	.187	.211	.236	.260	.309	.359	.408	.472	.591	.708
C "	.140	.161	.183	.205	.226	.270	.313	.357	.414	.519	.622
E "	.051	.059	.067	.075	.083	.100	.116	.132	.153	.192	.230
F "	.029	.033	.037	.041	.045	.053	.061	.069	.079	.098	.117
G "	.070	.078	.086	.095	.103	.119	.136	.152	.174	.214	.254
H "	.028	.032	.035	.039	.043	.050	.057	.064	.074	.092	.109
J "	.055	.063	.072	.081	.089	.106	.123	.141	.163	.205	.246

102. Set Screws. Set screws are used to prevent relative motion between two parts by the pressure exerted by the point of the screw (Fig. 105). Some are provided with square heads (Fig. 105a), and other forms are headless (b, c, and d). The two latter types require special wrenches for tightening. A number of different points are available, as shown, although the cup point is the one usually carried in stock. Set screws are made of hardened steel to prevent "mushrooming" the points and to enable the cup or round point to indent the surface of the part to be held. The headless types are used for revolving parts, as a safety measure, although square heads may be used if they are properly guarded.

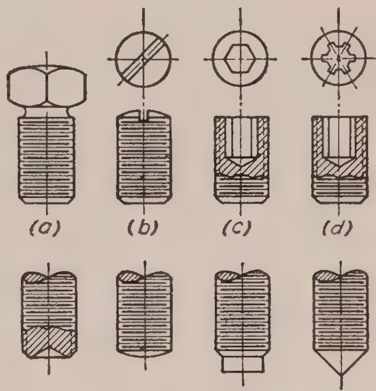


FIG. 105. SET SCREWS.

103. Pipe Threads. The American Standard pipe thread (formerly the Briggs Standard) is a 60 deg. thread cut on a taper of 3/4 in. per ft. The crest and the bottom of the thread are slightly flatted. Pitches are as follows: 1/8 in. pipe—27; 1/4 and 3/8—18; 1/2 and 3/4—14; 1, 1¼, 1½, and 2—11½; and from 2½ to 10 in. inclusive—8 threads per in.

104. Other Types of Bolts and Screws. Many other types of screw fastenings have been developed for various specialized uses, such as lag screws, expansion bolts, stove bolts, carriage bolts, patch bolts, wood

screws, and many others. In Figs. 106 and 107 are shown two forms of T-head bolts which are used when it is desirable to avoid the use of

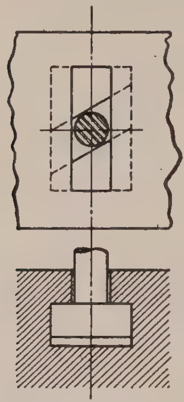


FIG. 106. SPECIAL T-HEAD BOLT.

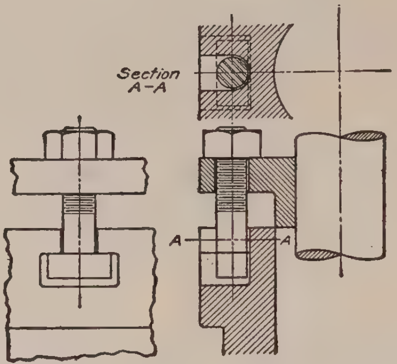


FIG. 107. T-HEAD BOLT.

tapped holes. In these applications, slots are formed in the casting to accommodate the heads and to prevent them from turning.

105. Nut Locks. Various forms of locking devices are used to prevent nuts from working loose. Representative types found in common use are shown in Fig. 108.

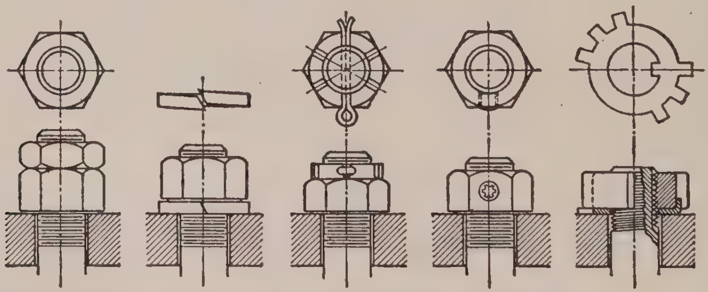


FIG. 108. NUT LOCKING DEVICES.

106. Theory of Screw Thread Action. In tightening up a screw fastening or applying a force by means of a power screw, an external torque or couple of sufficient magnitude must be applied on the screw to overcome the following forces:

(a) The external load along the axis of the screw.
(b) Frictional resistance between the threads in the nut and the screw.

(c) Frictional resistance between the collar, or nut, and the surface against which it bears (Fig. 109), or between the end of a screw and the surface supporting the load (Fig. 110).

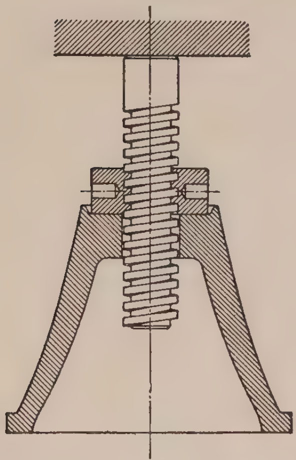


FIG. 109. SCREW JACK.

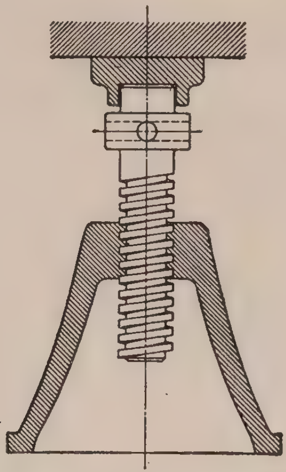


FIG. 110. SCREW JACK.

The screw torque proper, that is, the torque necessary to overcome (a) and (b), is equal to $F_m r_m$, where F_m is the force applied in a direction perpendicular to the axis of the screw at the mean radius r_m. To develop the mathematical expression for this torque, we may proceed by observing that the resultant force between two plane surfaces sliding on one another is inclined to the normal by the amount of the friction angle.

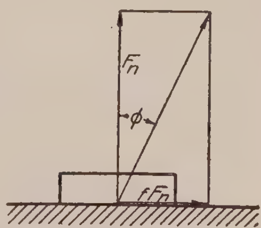


FIG. 111. FORCES ACTING ON SLIDING BODY.

The fundamental action of applied and friction forces between solid surfaces should be clear at this time. Review of these fundamentals will simplify the explanation of screw thread action.

If two bodies act on one another without friction, there can be no force along the contact surface, and the resultant force between them is normal to this surface. If there is friction and the coefficient of friction is f, then the friction force along the contact surface is fF_n, where F_n is the normal force (Fig. 111). The resultant of F_n and fF_n is inclined to F_n by an angle ϕ, the tangent of which is $fF_n/F_n = f$. The angle ϕ is called the friction angle.

Consider now the conditions at the mean thread radius of a square threaded screw (Fig. 112) for raising a load W. The normal force exerted by a supporting thread of the screw on the engaging thread of a nut is F_n. This normal force F_n is inclined to the screw axis and the direction of the load to be lifted by the mean helix angle α. The resultant force is therefore inclined to the normal by the friction angle ϕ and to the load direction by the angle $(\alpha + \phi)$. The magnitude of the resultant force, OC, manifestly must be such that its component in the load direction balances the load W. This means that the component OD in a plane at right angles to the screw axis will be $W \tan (\alpha + \phi)$. It is this force that the turning force F_m at the mean thread radius must overcome. We have

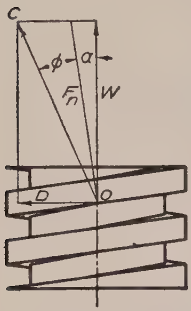


FIG. 112. FORCES ACTING ON SQUARE THREADED SCREW.

$$F_m = W \tan (\alpha + \phi) = W \frac{\tan \alpha + f}{1 - f \tan \alpha}, \tag{1}$$

where $\tan \alpha$ = lead of thread/mean circumference.

When the thread elements are not perpendicular to the axis of the screw, but inclined to that axis at an angle θ, as in the V and similar threads, sufficient accuracy is obtained if the normal to the thread F_n (Fig. 113) is taken equal to F_n for a square thread divided by $\cos \theta$. The friction force then is $fF_n/\cos \theta$. The effect, then, of using an angular thread instead of a square thread is to increase the coefficient of friction from f to $f/\cos \theta$. We can therefore take for the angular threads the expression

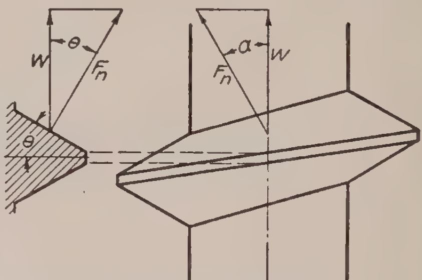


FIG. 113. FORCES ACTING ON ANGULAR THREADED SCREW.

$$F_m = W \frac{\tan \alpha + f/\cos \theta}{1 - f \tan \alpha/\cos \theta}. \tag{2}$$

(Note that formula (2) may be used for square threads, since $f/\cos \theta = f$.)

The total torque required to advance the nut is the sum of the screw torque (a) and (b) and the collar friction torque (c). The friction on the collar, or nut, may be considered as concentrated at the radius r_c, and the coefficient of collar friction as f_c. The resultant

collar friction torque is then $f_c W r_c$. The total torque becomes

$$(3) \qquad F_m r_m + f_c W r_c = W \left[r_m \left(\frac{\tan \alpha + f/\cos \theta}{1 - f \tan \alpha/\cos \theta} \right) + f_c r_c \right].$$

107. Efficiency of Screw Threads. The efficiency of a screw is of importance in connection with power screws. It is simply the ratio of the work done without friction to the work done with friction. For example, for a square screw thread without regard to the friction of nut or end bearing surface, it is

$$(4) \qquad \text{Eff.} = \frac{2\pi r_m W \tan \alpha}{2\pi r_m W \tan (\alpha + \phi)} = \frac{\tan \alpha}{\tan (\alpha + \phi)}.$$

For an angular type thread with half apex angle θ and an allowance for nut or end friction on a radius r_c, the efficiency formula is

$$(5) \qquad \text{Eff.} = \frac{r_m \tan \alpha}{r_m \dfrac{\tan \alpha + f/\cos \theta}{1 - f \tan \alpha/\cos \theta} + r_c f_c}.$$

108. Overhauling Screws. For most purposes it is desired that a screw be self locking, that is, it should not descend or overhaul under load. In some cases, as in steering gears, a reversible screw is wanted. The helix angle of the thread causes an "unwinding" force at the mean thread radius equal to $W \tan \alpha$ (see Fig. 114). If overhauling is not to take place, it is evident from Fig. 114 that the friction angle ϕ for a square thread must be large enough to cancel this force, that is, $W \tan (\phi - \alpha)$ must be $\geqq 0$. Neglecting collar friction, if the friction angle is greater than the helix angle, it will take force to unwind the screw; if it is less, the screw is reversible or overhauling. Also, if the efficiency is less than 50 per cent, the screw is non-reversible.

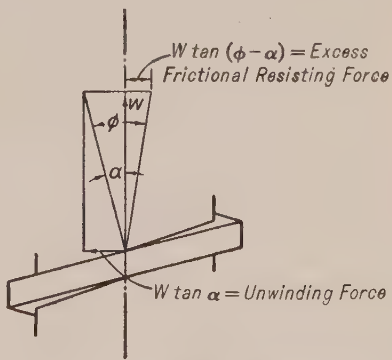


FIG. 114. OVERHAULING SCREW.

109. Holding Action and Stresses in Fastening Screws. A bolt which is to hold together two machine parts without the possibility of a gap forming between them must be tightened with a tensile force greater than the force tending to separate the parts. If it is not tightened with this excess tension, it will stretch when the external

load exceeds the tension in the bolt, and a gap will form. In computing bolts which are to hold machine parts firmly together, an allowance must be made for the excess tension required. A common allowance is to multiply the separating force by 1.25 in computing the stress on the root area of the bolt.

It is impossible, however, to regulate the pull exerted by a person in tightening a bolt. Experiments have shown that a half-inch standard coarse-thread bolt may be easily broken by the pull that can be applied at the end of a standard wrench.

As an example of the use of the formulas given, we may investigate the stress situation in a 1/2 in. bolt tightened by a pull of 100 lb. at the end of a 6 in. wrench.

Such a screw has 13 threads per in., the root diameter is 0.4001 in., and the mean diameter 0.45 in.

$$\tan \alpha = \frac{1}{13 \times \pi \times 0.45} = 0.054.$$

With a half thread apex angle of 30 deg. and a coefficient of friction of 0.15, which would apply to somewhat greasy threads and surfaces, we have $f/\cos\theta = 0.15/0.866 = 0.173$. We have further

$$\frac{\tan \alpha + f/\cos\theta}{1 - f \tan \alpha/\cos\theta} = \frac{0.054 + 0.173}{1 - 0.173 \times 0.054} = 0.229.$$

If we assume that the coefficient of friction between the nut and the surface on which it rests is 0.15, and that this friction force is concentrated on a radius of 0.35 in., then with a mean thread radius of 0.225 in., we have, with the load W on the screw, $6 \times 100 = W \times 0.229 \times 0.225 + W \times 0.15 \times 0.35$, or $W = 5770$ lb.

The root area of the bolt is 0.126 sq. in. and the direct tensile stress on this is

$$\frac{5770}{0.126} = 45{,}800 \text{ p.s.i.}$$

The screw torque is $5770 \times 0.229 \times 0.225 = 297$ in. lb.

The resultant torsional shear stress is $297 \times 16/(\pi \times 0.4^3) = 23{,}600$ p.s.i.

The resultant maximum shear stress is

$$\sqrt{\frac{45{,}800^2}{4} + 23{,}600^2} = 32{,}900 \text{ p.s.i.,}$$

which is equivalent to a tensile stress of 65,800 p.s.i.

Such a stress will readily break the bolt. Wherever rough and injudicious tightening is to be feared, it is, therefore, well not to use bolts as small as 1/2 in.

Certain tests at Cornell University gave the conclusion that experienced machinists tighten nuts with a pull roughly proportional to the bolt diameter. Cardullo therefore proposed a bolt formula

$$W = s_t(0.55d^2 - 0.25d), \tag{6}$$

where W is the permissible load in lb., s_t the working stress in tension, and d the nominal outside diameter in in. It will be seen readily that this formula gives little or no permissible load on bolts of 1/2 in.

diameter and less. It is rather advisable to use this formula where perfect tightness is desired and where there is no particular need for light construction.

110. Bolts or Tie Rods Subjected to Shock Loads. Bolts or tie rods subjected to shock loads should be designed not only for strength but also for their ability to absorb impact energy. Consider, for example, the bolt shown in Fig. 115, which is subjected to a shock load due to the weight W falling through a free distance H. Upon striking the bolt head, the shank yields and elongates a distance h in absorbing the impact energy and bringing the weight to rest. This assumption is based upon an unyielding support, bolt head, and weight, and that all of the live-load energy is absorbed by the bolt shank. These conditions, of course, are not theoretically true. However, the bolt is the "yielding" member, relatively, and by disregarding the deformation of the other parts, our assumption is on the safe side.

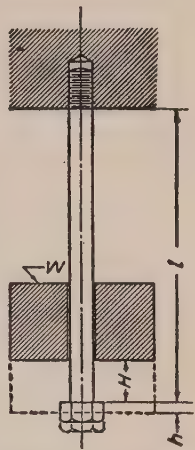


FIG. 115. BOLT SUBJECTED TO SHOCK LOAD.

The total energy to be absorbed by the bolt within the elastic limit is therefore $W(H + h)$. In absorbing this energy, the bolt offers a resistance which is zero at the beginning of impact and is a maximum when the bolt is elongated to h. If the maximum resisting force is F, the average resisting force is $F/2$ and the total internal work done in the bolt shank is $hF/2$. Equating these energies

$$W(H + h) = \frac{hF}{2} = \frac{hs_tA}{2}$$

or

$$s_t = \frac{2W}{A}\left(1 + \frac{H}{h}\right), \tag{7}$$

where s_t is the tensile stress in the bolt shank and A is the shank area.

It is to be observed from the above formula that with all other factors the same, the stress s_t becomes smaller the higher the elongation h becomes. A high elongation can be obtained by either increasing the length of the bolt or by reducing the area of the bolt shank. Since the tensile strength of a bolt is based on the area at the root of the thread, by reducing the shank area to agree with the root area, the bolt will have a uniform stress throughout and the maximum elongation for a given stress and length.

Referring to Fig. 115, let us take as an example a 3/4 in. coarse thread bolt with the distance $l = 15$ in. If the load W acts through a free distance of 0.1 in. before striking the bolt head, compare the allowable load W for a bolt of standard proportions with one having the shank area reduced. We will assume a working stress of 10,000 p.s.i. and $E = 30{,}000{,}000$.

The shank area of a 3/4 in. standard bolt = 0.442 sq. in. The root area = 0.302 sq. in. The stress in the shank is then $10{,}000 \times 0.302/0.442 = 6830$ p.s.i. The elongation $h = 15 \times 6830/30{,}000{,}000 = 0.00341$ in. Then

$$W(0.1 + 0.00341) = 0.00341 \times 6830 \times 0.442/2,$$
$$W = 50 \text{ lb.}$$

If the shank area is reduced, the elongation is

$$h = 15 \times 10{,}000/30{,}000{,}000 = 0.005 \text{ in.},$$

and

$$W(0.1 + 0.005) = 0.005 \times 10{,}000 \times 0.302/2,$$
$$W = 72 \text{ lb.}$$

While the values of W as computed are both apparently low, it is due to the fact that the support, weight, and bolt head were considered as absolutely rigid. However, the *relative* values are important. An increase in load capacity of approximately 50 per cent occurs when the shank area is reduced. The loss of motion in a connecting rod bearing, for instance, may cause impact loads which can be sustained only by bolts designed with reduced shanks (see Fig. 157). The shank area may be reduced by turning the shank diameter to the same size as the root diameter of the threaded portion, or by drilling a hole axially through the unthreaded portion of the bolt shank.

111. Power Screws. A screw jack is the simplest form of a power screw. In the form shown in Fig. 109, the screw bears directly on the body to be lifted and does not rotate. The nut is turned and lifts the load. The bearing surface between nut and pedestal can be made sufficiently large to confine the bearing pressure to a moderate value. On the other hand the radius on which the friction force may be regarded as concentrated is fairly large, and the friction moment will therefore be large unless there is adequate lubrication.

In the arrangement shown in Fig. 110, the screw is rotated while the nut is stationary. In this case, the end of the screw again supports the entire load. The bearing pressure between screw and socket is high, on account of the small area, but the friction radius is small; this tends to make the friction moment small, unless the coefficient of friction is very high. In both of these arrangements, the screw must be proportioned to withstand the screw torque as well as the vertical load.

Jack screws may require a considerable amount of friction to prevent descent under load. Friction in screw presses and many other power screw applications means loss of efficiency.

According to tests by Kingsbury (Transactions A.S.M.E., vol. XVII, p. 96), coefficients of friction for square threads with bearing pressures varying between 3000 and 10,000 p.s.i. were found to lie within the approximate limits listed in Table 23.

TABLE 23

COEFFICIENTS OF FRICTION FOR SQUARE THREADS UNDER BEARING PRESSURES OF 3000 TO 10,000 P.S.I.

Metal Combination	Heavy Machinery Oil Lubricant	Heavy Machinery Oil and Graphite
Steel on steel	0.14 to 0.15	0.11
Steel on cast iron	0.13 to 0.15	0.07 to 0.11
Steel on bronze or brass	0.12 to 0.14	0.04 to 0.07

It will be observed that very substantial reductions in friction may be attained by a steel and bronze, or steel and brass, combination, and by the use of a lubricant containing graphite. Brass or bronze possesses a smoother surface than steel, and graphite aids in filling depressions. In addition, graphite has a strong capillary attraction for oil, causing it to enter and to be retained between the metallic surfaces. Certain oils of the nature of organic acids, particularly *castor oil,* have a similar capillary attraction to metals; for this reason, they reduce screw friction very considerably, even without graphite.

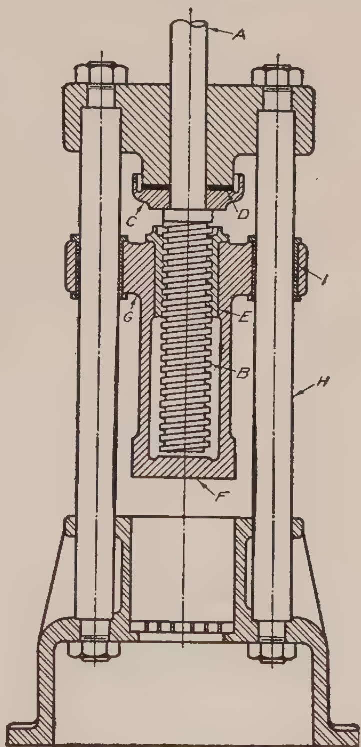


FIG. 116. MACARONI SCREW PRESS.

112. Design of a Power Screw. To illustrate the computations involved in the design of a power screw, we shall consider as an example the screw mechanism of a macaroni press (Fig. 116). Power is transmitted through the neck or shaft A to the screw B. The thrust is taken on the collar C, with a fiber washer D interposed between the collar and the frame. The screw, which may be taken to be of 0.4 per cent carbon steel, runs in a bronze nut E, which is anchored to the piston F. The piston is provided by cross arms G, which are guided through bushings I on stanchions H, so as to prevent rotation relative to the frame. For ease of assembly, the cross arms in commercial designs are often devel-

oped into a split cross head, which is clamped to the piston. For simplicity of design, we will assume them to be cast in with the piston.

We shall assume a cylinder bore of 13.5 in. and a pressure of 2000 p.s.i., producing a force on the piston of $\pi \times 13.5^2 \times 2000/4 = 286{,}000$ lb. Since the load on the screw varies between zero and a maximum in one direction and comes on gradually, a factor of safety of 4 is ample, giving, with steel of 80,000 p.s.i. strength, a working stress in tension or compression of 20,000 p.s.i.

The screw is subject to compression and to torsional shear. To arrive at a root diameter, we shall assume a compressive stress of 12,000 p.s.i., thus leaving a considerable margin for torsion. The root area of the screw is then $286{,}000/12{,}000 = 23.8$ sq. in. and the corresponding root diameter $\sqrt{23.8 \times 4/\pi} = 5.5$ in.

Assuming a ratio of outside diameter to root diameter of approximately 10 to 8, the outside diameter will be 5.5/0.8, say $6\frac{3}{4}$ in. The mean thread diameter is $(5.5 + 6.75)/2 = 6\frac{1}{8}$ in. With square threads the pitch is 1.25 in. The tangent of the mean helix angle is then $1.25/(\pi \times 6.125) = 0.065$. With a coefficient of friction of 0.05, the tangential force at the mean thread diameter is

$$F_m = 286{,}000 \left(\frac{0.065 + 0.05}{1 - 0.05 \times 0.065} \right) = 33{,}000 \text{ lb.}$$

In consequence, the screw torque is $33{,}000 \times 6.125/2 = 101{,}000$ in. lb. The torsional shear $= (101{,}000 \times 16)/(\pi \times 5.5^3) = 3100$ p.s.i. The maximum shear stress is

$$\sqrt{12{,}000^2/4 + 3100^2} = 6{,}750 \text{ p.s.i.,}$$

which is equivalent to a tensile stress of 13,500 p.s.i.

This is a comparatively low stress, and as far as the stress on the root area of the screw is concerned, the screw diameter could be reduced. It happens, however, that the bearing stress on the screw thread would require too high a nut, if the thread diameter were reduced. For this reason, it is better to retain the dimensions just computed.

Bach regards a bearing pressure of about 2200 p.s.i. (see *Maschinenelemente*, 12th ed., p. 151) as the maximum permissible for steel power screws on hard bronze nuts, yet most other authorities recommend pressures less than half this value. In our case, however, even 2000 p.s.i. will lead to undesirable dimensions. To avoid this difficulty, it has been found that manufacturers of presses of this type use castor oil lubrication and carry the pressure as high as 3000 p.s.i. The total bearing area required in our case is then $286{,}000/3000 = 95.3$ sq. in. The area of one thread is $\pi \times 6.125 \times 0.625 = 12.0$ sq. in. The number of threads required to carry the load is then $95.3/12.0 = 7.9$. The nut height would be $7.9 \times 1.25 = 9.9$ in. A nut height of 10 in. would be very nearly 1.5 times the screw diameter of $6\frac{3}{4}$ in.

Bach (*Maschinenelemente*, 12th ed., p. 150) shows with an illustration how the load with very long nuts may be concentrated only on the extreme threads. This condition is particularly true if the nut is in tension and the screw in compression, as in our problem, or vice versa. While nuts two diameters high may occur in practice, it certainly is better, particularly where the stresses are of opposite kinds in the screw and the nut, to limit the height to $1\frac{1}{2}$ diameters.

The threads must now be checked for shear and bending. Very likely an Acme type of thread would actually be used for strength, but for simplicity we shall assume that the thread is square. If the square thread is strong enough, the Acme thread will be even stronger. As far as bearing pressure is concerned, there is very little

difference between the two. In consequence no re-figuring for the Acme thread is necessary, in case it should be used.

According to formula (13), page 16, the shear stress at the neutral axis of the section at the root of the screw thread is $3 \times 286{,}000/2 \times \pi \times 5.5 \times (10/2) = 4960$ p.s.i.

The shear stress in the nut is lower than this value, since the root diameter of the thread is larger. Such a shear stress is permissible, even for bronze.

The thread should also be checked for bending when it is considered as a cantilever beam having a distributed load concentrated along the pitch line. The beam width is the total thread length at the root, that is, $7.9 \times (10/9.9) \times \pi \times 5.5 = 138.0$ in. The bending moment is $286{,}000 \times 0.625/2 = 89{,}400$ in. lb. The stress is $(89{,}400 \times 6)/(138.0 \times 0.625^2) = 9950$ p.s.i.

This stress is a more serious one than the transverse shear stress. It is perfectly safe for steel, but for bending in bronze it might be better not to exceed 8000 p.s.i. Since the root diameter of the nut thread stands to the root diameter of the screw thread very nearly in the ratio of 10 to 8, it would seem that the bending stress on the bronze would be about $9950 \times 0.8 =$ approx. 8000 p.s.i.

The shoulder that carries the collar may be computed for a compressive stress of 20,000 p.s.i. The required shoulder area $\pi(6.75^2 - d_n^2)/4$, where d_n is the neck diameter, is equal to $286{,}000/20{,}000 = 14.3$ sq. in. We find $d_n = 5.23$, say $5\frac{1}{4}$ in.

With a bearing pressure on the collar of 3000 p.s.i. the bearing area required is $286{,}000/3000 = 95.3$ sq. in. If d_c is the outside diameter of the collar bearing area, we have $d_c = \sqrt{(95.3 \times 4/\pi) + 5.25^2} = 12.2$, say $12\frac{1}{4}$ in.

The assumption is often made that, if wear sets in, the bearing load is concentrated toward the center, where the rubbing velocity is least. The greatest friction torque is obtained, however, on the assumption of a uniformly distributed bearing pressure. This torque is

$$2\pi f p \int_{r_n}^{r_c} r^2 dr = \frac{2\pi f p (r_c^3 - r_n^3)}{3}\,.$$

With a coefficient of friction $f = 0.05$, with $p = 3000$ p.s.i., $r_c = 6.125$ in., and $r_n = 2.625$ in., we have the friction torque equal to 66,700 in. lb. The total torque on the neck of the screw shaft is equal to the collar friction torque plus the screw torque, that is, $101{,}000 + 66{,}700 = 167{,}700$ in. lb. The torsional shear stress in the neck is then equal to $167{,}700 \times 16/\pi \times 5.25^3 = 5900$ p.s.i. This stress is equivalent to 11,800 p.s.i. in tension and is reasonable even with some allowance for key seats.

It is impossible to compute with scientific exactness the strength of a piece having the section of the thrust collar C. Perhaps the best approximate analysis that can be made is to assume that the added strength derived from the oil cup rim more than compensates for the lacking lower corner of the section. On this assumption the collar may be computed as one of rectangular section with an outside diameter equal to the outside diameter of the bearing surface, that is, $12\frac{1}{4}$ in. Such a collar may be considered to break across a diametral section. To compute the bending moment with respect to this section, the bending moment of the shoulder reaction is most conveniently obtained by regarding the reaction as concentrated at the center of gravity of a semi-annular ring having a mean diameter equal to the mean diameter of the shoulder. The mean diameter is $(6\frac{3}{4} + 5\frac{1}{4})/2 = 6$ in. The distance of the center of gravity of the semi-annular ring from the mean diameter is 0.6366 times the radius, that is, 1.91 in. The moment is then $286{,}000 \times 1.91/2 = 274{,}000$ in. lb.

The moment of the load on the semi-annular bearing area is

$$
\begin{aligned}
0.6366\pi p \int_{r_n}^{r_c} r^2 dr &= 0.2122\pi p(r_c^3 - r_n^3) \\
&= 0.2122\pi \times 3000(6.125^3 - 2.625^3) \\
&= 425{,}000 \text{ in. lb.}
\end{aligned}
$$

The net moment is 425,000 − 274,000 = 151,000 in. lb.

With a stress of 20,000 p.s.i. for steel the thickness of the collar is then

$$= \sqrt{(151{,}000 \times 6)/(12.25 - 5.25) \times 20{,}000} = 2.54, \text{ say } 2\tfrac{1}{2} \text{ in.}$$

113. Stanchions. The stanchions guide the piston and balance the screw torque. They also take up the vertical reaction from the force on the piston and tie the two ends of the press together. To compute the bending force on the stanchions due to the screw torque, it is necessary to estimate the moment arm, that is, the distance from the axis of the screw to the center line of the stanchions. Assuming the stanchion diameter to be 4 in., the thickness of the cylinder wall 1 in., and the clearance between stanchions and wall likewise 1 in., the distance would be $2 + 1 + 1 + 6\frac{3}{4} = 10\frac{3}{4}$ in. The cross bending force is then equal to half the screw torque divided by this radius, that is, $101{,}000/(2 \times 10.75) = 4700$ lb.

The stanchions are subjected to a peculiar type of bending. The top part of the press is joined to the base only through the stanchions. If the stanchions deflect, the top will pivot relatively to the base. However, the ends of the stanchions will remain parallel to one another and to their original direction. Moreover, since the drive comes from a mechanism rigidly connected with the top and not with the bottom part, the top should be regarded as the point of anchorage, at which the transverse reaction is taken up. At the lower end there is merely a bending moment which keeps the ends lined up with the original direction. There may be some torsion in the stanchions, but this may be neglected. Under these conditions, it will be found that if a beam of length l is bent by a cross bending force F, the maximum bending moment is $Fl/2$ and will occur when F is applied at the distance l from the anchorage. In our case, this condition occurs when the piston is about in its lowest position. By estimating the various distances and dimensions which determine the length of the stanchion, we find this length to be 44 in. The maximum moment is then equal to $4700 \times 44/2 = 103{,}400$ in. lb.

The straight pull on each stanchion is 143,000 lb. Allowing 25 per cent additional to keep nuts tight, the root area of each stanchion at the thread is

$$1.25 \times 143{,}000/20{,}000 = 8.95 \text{ sq. in.,}$$

with a permissible stress of 20,000 p.s.i. With standard national coarse threads this value requires a $3\frac{3}{4}$ in. screw. With a shoulder pressure of 20,000 p.s.i. and a stanchion diameter d_s, we have

$$\pi(d_s^2 - 3.75^2)20{,}000/4 = 1.25 \times 143{,}000,$$

or

$$d_s = \sqrt{(4 \times 1.25 \times 143{,}000)/(\pi \times 20{,}000) + 3.75^2} = 5.05,$$

say 5 in.

The straight tensile stress in the stanchion will be

$$(143{,}000 \times 4)/(\pi \times 5^2) = 7280 \text{ p.s.i.,}$$

and the bending stress $(103{,}400 \times 32)/(\pi \times 5^3) = 8420$ p.s.i. The total stress in the extreme fiber is then $8420 + 7280 = 15\,700$ p.s.i.

The 5 in. diameter as computed gives a longer moment arm and a smaller cross bending force than was originally assumed on the basis of stanchions whose diameter is 4 in. Also the cross bending force is somewhat distributed in the bushing and will not produce a moment quite as high as assumed. The 5 in. diameter, however, was determined on the basis of a high root stress in the thread and a fairly high compressive stress on the shoulder. The design may therefore be considered satisfactory.

PROBLEMS

1. A square-threaded screw, 2 in. outside diameter, with a single thread of 2¼ turns per in., is used to raise a load W. If a turning force of 80 lb. is applied at the end of a 3 ft. wrench used on the nut, determine the value of W and the efficiency of the screw. The mean radius of the collar may be taken as 1.6 in. and the coefficient of friction as 0.2. Determine the direct stress in the screw. [Consider the depth of the thread equal to 7/16 of the pitch.]

2. A short jack is used to raise a load of 10,000 lb. The screw has an Acme single thread of two turns per inch and an outside diameter of 2½ in. For a coefficient of friction of 0.2, determine (a) the torque necessary to raise the load, (b) the efficiency of the screw, and (c) the maximum resultant stress in the screw if the torque is transmitted over that section that carries the load. [Consider the depth of thread as approximately one-half the pitch and the collar friction as negligible.]

3. An automatic machine has a right and left hand power screw used to separate two non-rotating nuts against a force of 2000 lb. The power screw has 2 Acme threads per inch, is double threaded, and has an outside diameter of 2 in. (Depth of thread is 0.26 in.) Assume a coefficient of friction of 0.12 and negligible collar friction, as a ball bearing is used. (a) What horsepower is necessary to separate the nuts at 1 ft. per sec.? (b) What is the efficiency? (c) What is the necessary height of each nut to limit bearing pressure to 300 p.s.i.?

4. A screw press carries a load of 4500 lb. The screw is made of S.A.E. 1035 steel, and has an extension below the bushing of 20 in. Assuming the end conditions as fixed at one end and guided at the other and a coefficient of friction of 0.12, (a) determine the necessary root diameter of the screw, (b) select a suitable pitch for square thread and determine the torque necessary to raise the load, (c) determine the efficiency of the thread. No collar friction is to be considered.

5. The screw in a C-clamp is 3/8 in. NC. For an assumed coefficient of friction of 0.25, determine the force between screw and anvil produced by a force of 20 lb. applied at a radius of 6 in. What is the torsional stress induced in the body of the screw?

6. A compression testing machine has a capacity of 100,000 lb. The table is to be actuated by three square-threaded screws. Design the screws and nuts.

7. A steam cylinder 14 in. in diameter has a head held in place by 12 studs. What diameter of stud is required for fluid tightness against 250 p.s.i.?

8. A circular flange, 10 in. outside diameter, is bolted to a column by four 1 in. NC bolts on a 7 in. circle. A load of 20,000 lb. is applied 3 in. from the face of the column, parallel to it. What is the maximum resultant stress induced in the bolts if the bolts are on horizontal and vertical center lines? On 45 deg. center lines?

9. A wall hanger supports a bearing carrying a load of 5000 lb., 16 in. from the wall. The hanger is attached to the wall by three bolts, one 3 in. from the lower edge of the hanger and two 21 in. from the lower edge of the hanger. What is the proper size for the bolts?

CHAPTER 8

CYLINDERS, PISTONS, AND STUFFING BOXES

114. Cylinders. A simple hydraulic cylinder is shown in Fig. 117. This particular cylinder is cast with one closed end and has incorporated at the other end a stuffing box to allow for the passage of a plunger. Many cylinders are simply containers for fluids under pressure and for such purposes they are extensively used in the chemical

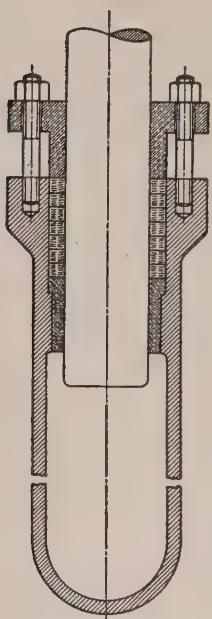


FIG. 117. HYDRAULIC CYLINDER.

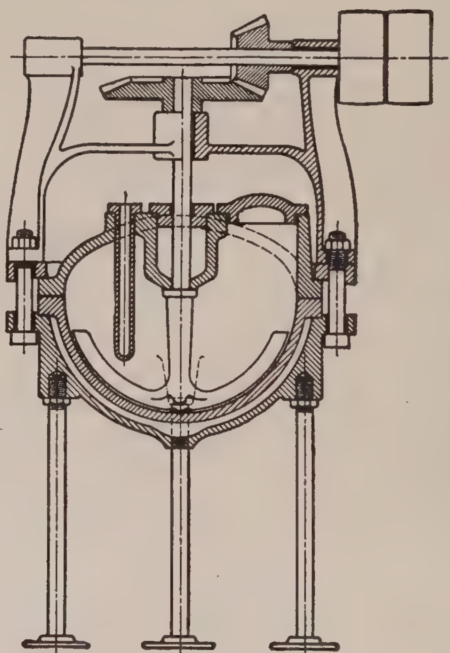


FIG. 118. AUTOCLAVE.

industry. Vessels of this type may contain mixing paddles or agitators, as illustrated by the autoclave in Fig. 118. Frequently they contain pipes or tubes, as in condensers and heat exchangers. Such tubes may be screwed or welded in at one end, but if they carry fluids of a temperature different from that which surrounds them, they must be free to

slide at the other end. At the sliding end there should then be some simple form of stuffing box, such as that shown in Fig. 119.

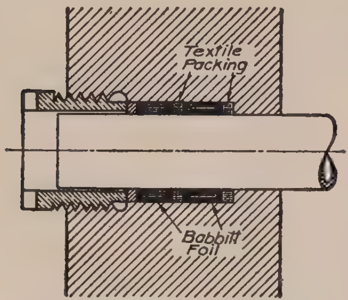


FIG. 119. STUFFING BOX FOR THE SLIDING END OF CONDENSER TUBE.

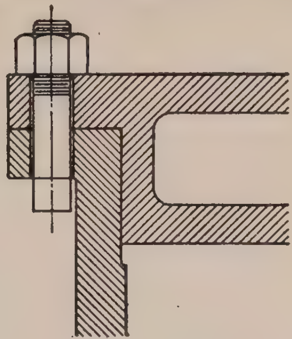


FIG. 120. APPLICATION OF T-HEAD BOLTS FOR FASTENING HEAD TO CYLINDER.

Heads are generally attached to cylinders by means of bolts or studs. The use of through-bolts and nuts is preferred, since plain holes are cheaper to make than threaded holes, and the inconvenience and expense of thread failure of tapped holes in a casting are eliminated.

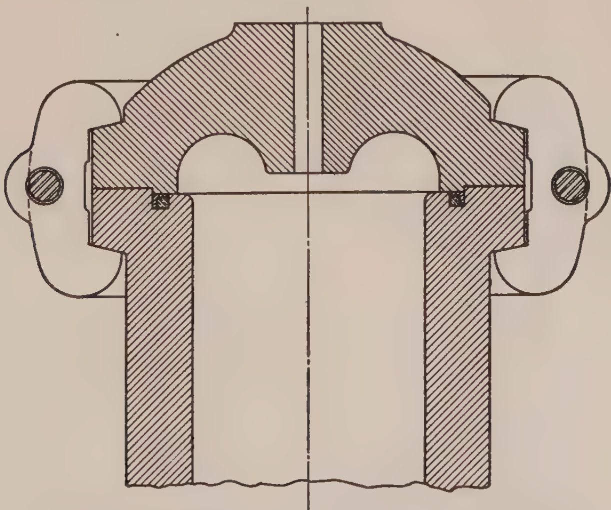


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FIG. 121. APPLICATION OF CLAMPING RINGS FOR FASTENING HEAD TO CYLINDER.

When bolts are used as fastenings, the end of the cylinder terminates in a flange of sufficient radius to accommodate the head of the

bolts. If the pressure within the cylinder is high, the bending stress in this flange may be considerable. By reducing the overhang of the flange and thus the moment arm of the bolts, the bending stress can be reduced. This construction may be accomplished by using T-head bolts (Fig. 120), or square head bolts with one side cut off flush with the bolt body. Another method is to eliminate the flange entirely and use T-bolts inserted in slots, as shown in Fig. 107 (page 126). If very high pressure is involved, it may be necessary to avoid the use of bolts and use clamping rings instead (Fig. 121). The methods of computing the strength of flanges and cylinder walls for both thick and thin cylinders have already been given in Chapter 2.

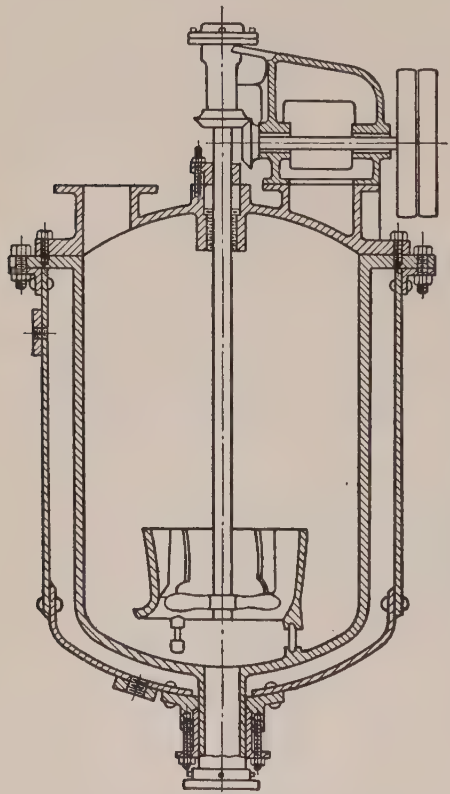


FIG. 122. STEAM-JACKETED AUTOCLAVE.

Cylinders are frequently encased in a jacket as a means of retaining or transferring heat. Figure 122 shows an autoclave with a jacket for steam-heating the contents. To remove the heat developed in compressors and internal combustion engines, water is circulated through the head and around the cylinders (Fig. 123). Small cylinders usually have the jacket cast integral with the cylinders, as in automobile engines. For larger cylinders and comparatively high temperatures, it is better to design the cylinder with a separate inner sleeve which is free to slide at one end and thus adjust itself to different rates of expansion (Fig. 124). To secure water tightness with this construction, a rubber ring in a groove may be used, as shown.

Cast iron is commonly used for cylinders, although cast steel is used where high strength is essential. If the cylinder supports a sliding piston, cast iron is much preferred, as cast iron rubbing on cast iron gives satisfactory wearing action. Where steel rods or steel plungers,

or even cast iron plungers, enter steel cylinders, bronze bushings should be provided as the bearing element. In certain applications, resistance

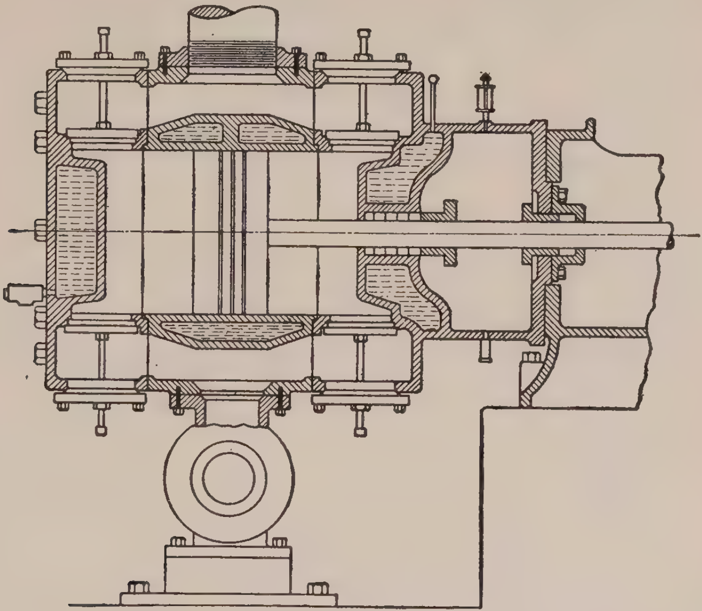


FIG. 123. WATER-JACKETED COMPRESSOR.

to corrosion is important, and for this purpose bronze, Monel metal, and nickel-chromium steel are used. For special applications and resistance against corrosion, steel tanks or shells are often lined with stainless steel, rubber, glass, or enamel.

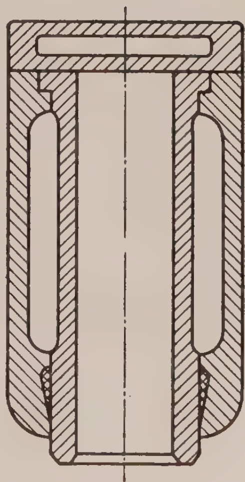


FIG. 124. CYLINDER SLEEVE.

115. Pistons. The simplest type of double-acting piston is the so-called Swedish piston shown in Fig. 31 (page 39, Chapter 2). With a conically dished web to obtain lightness and a replaceable rim, this type of piston is regularly used in locomotives and in reciprocating marine steam engines (Fig. 125). In stationary engines, compressors, etc., the somewhat sturdier box piston shown in Figs. 123 and 126 is in common use.

Double-acting pistons are forced against a shoulder on the piston rod and are held by a nut, as shown in Fig. 125. In some cases, the

piston is forced on the rod with a taper fit, but this construction, while considerably more expensive and difficult to produce accurately, is not necessarily any better than a good tight cylindrical fit. To reduce clearance to a minimum with flat cylinder heads, the nut that holds the

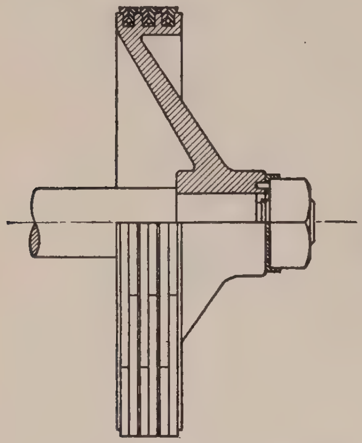


Fig. 125. Piston with Dished Web.

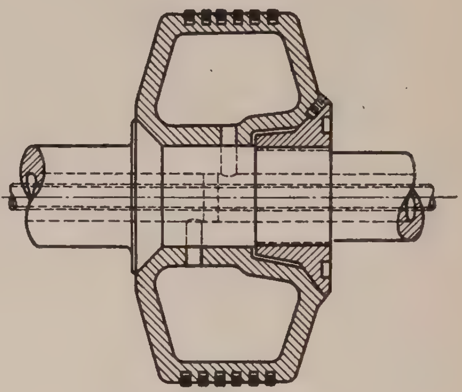


Fig. 126. Box Piston.

piston on the rod may be recessed in the head. The nut should be locked securely in place by some positive type of nut locking device. Two different methods are shown in Figs. 125 and 126. Formulas for the computation of the wall thickness of pistons, both Swedish and box types, were given on pages 39 and 40, Chapter 2.

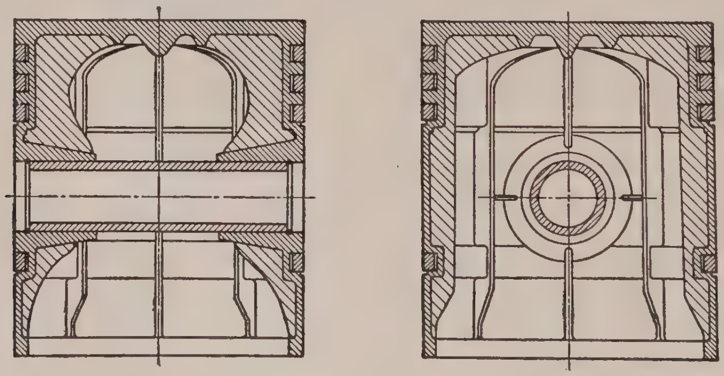


Fig. 127. Trunk Piston.

Trunk type pistons, as shown in Fig. 127, are used in single action steam engines, compressors, and internal combustion engines. The angularity of the connecting rod causes a side thrust against the

cylinder wall; this bearing pressure should be limited to 20 or 30 p.s.i. to avoid excessive wear.

In horizontal engines, the weight of the piston must be added to the thrust force. Haeder suggests a wall pressure of only 6 p.s.i. for double-acting pistons bearing on the cylinder wall of double-acting horizontal engines.

Aluminum-alloy pistons are extensively used in high-speed internal-combustion engines as a means of reducing inertia forces and vibration. Furthermore, aluminum is a better conductor of heat than cast iron; hence higher compression ratios are possible. Since aluminum has a different coefficient of expansion than cast iron, the skirts of automotive pistons are usually slit at a slight angle to maintain a snug fit and prevent piston "slap." Spring expanders are frequently inserted in the skirt to expand the piston when it becomes worn.

116. Piston Rings. Piston rings are universally used as the means of sealing the piston in the cylinder against the pressure of fluids. There are many types of patented ring designs, especially for automotive applications, and these vary in complexity and merit. The common type of ring is of one piece construction, cut through on one side to permit assembly and to exert a spring pressure against the cylinder wall. The joint may be formed with an overlap, as in Fig. 128, but it is more common practice to cut the ring at right angles or on a slant, usually at 45 deg.

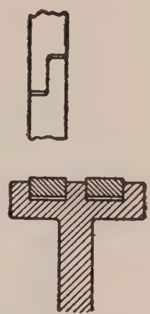


FIG. 128. PISTON RING WITH OVERLAPPED JOINT.

Piston rings in the past have generally been made of high grade cast iron, hard enough to possess good wearing qualities, yet not so hard as to cause excessive cylinder wear. A ring must have high elasticity, freedom from permanent distortion, and sufficient strength to enable it to be sprung over the piston crown.

To prevent the restrained fluid from flowing through, there must be at least two rings with the joints staggered. In horizontal engines, the rings should be pinned to the piston so as to prevent alignment of the joints. For heavy stationary or marine gas engines, six or more rings are used, but in light high-speed engines not more than three are employed.

If the rings are to be sprung over the piston head, the thickness should not exceed 1/30 of the diameter when the rings are of uniform thickness, and 1/25 of the diameter if the rings taper toward the joint.* If rings are thicker than this proportion they must be subdivided and

* For rational formulas for ring stress, see Norman, *Machine Design*, p. 209.

pressed outward by underlying springs. The recommended groove depth * for plain automotive piston rings is $(0.01D^2/8)^{1/2} + 0.005$, where D is the diameter of the piston in in. The width of the ring may be taken 1.5 times its thickness, if the joint is straight and twice the thickness if it is overlapped.

117. Stuffing Boxes and Packings. A cross-section of a single-acting pump is shown in Fig. 129. With a non-compressible fluid, the

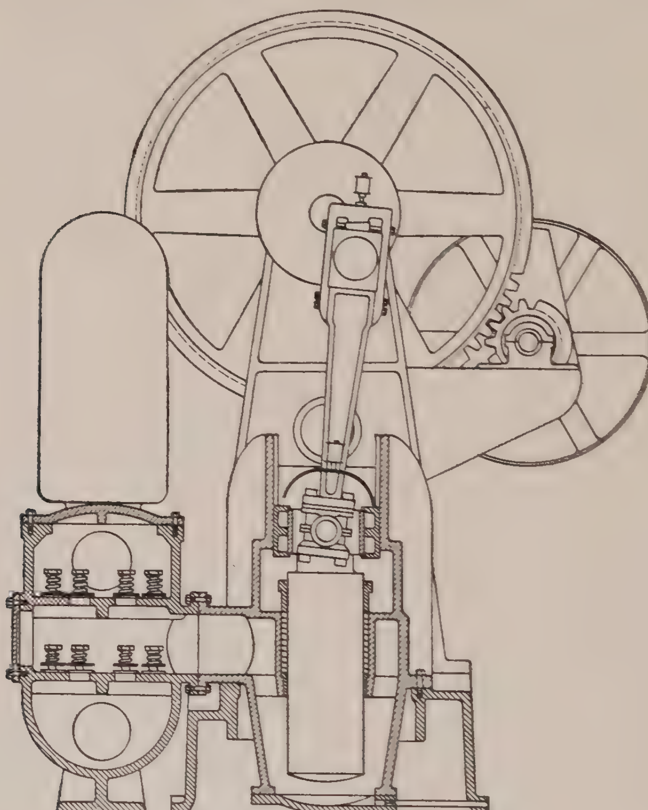


FIG. 129. SINGLE-ACTING PUMP.

plunger need not be tight against the walls of the cylinders, since the entering volume of the plunger causes the positive displacement of an equal volume of liquid. The stuffing box is then the only packing required.

* See Favary, *Motor Vehicle Engineering*, McGraw Hill, 1920, p. 111.

With a double-acting pump, such as that shown in Fig. 130, a packing on the piston is necessary to prevent the passage of fluid from one side of the piston to the other. This packing is arranged quite similarly to the stuffing box packing, that is, it consists of a number of plastic rings compressed into a recess and expanded sideways by a gland actuated by two or more studs. For cold fluids the rings

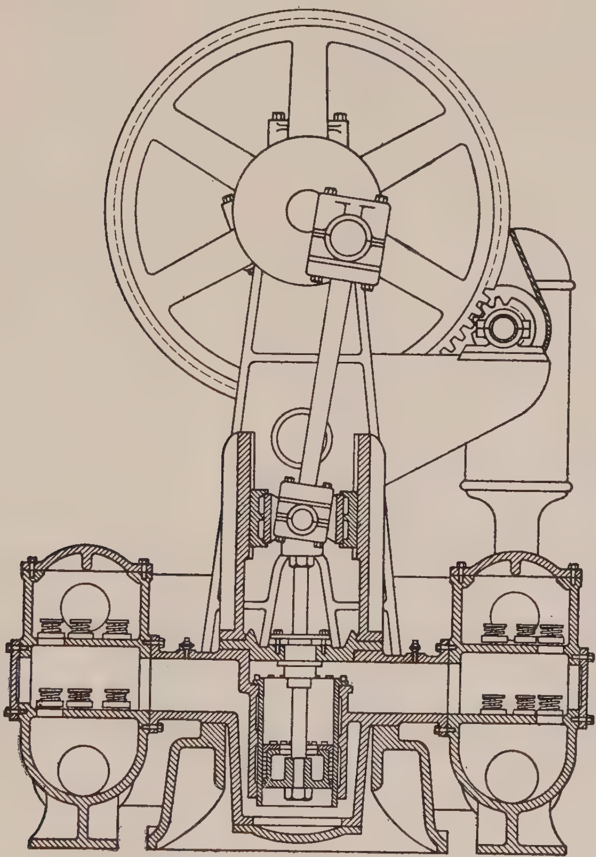
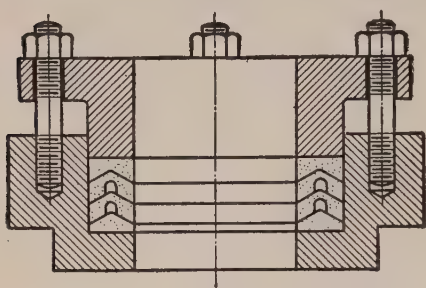


Fig. 130. Double-Acting Pump.

are made of cotton, hemp, or flax, whereas for hot fluids, asbestos material is used.

For hydraulic pumps and cylinders operating under moderate hydraulic pressure, packing rings of solid square section may be used, but for pressures in excess of 2000 or 3000 p.s.i. or more, rings expanded against the walls by the fluid pressure are necessary. Some rubberized

textile rings of this character are shown in Fig. 131. In the past U-leather rings (Fig. 132) were regularly used. Even a simple U-



The Garlock Packing Co.

FIG. 131. HYDRAULIC PACKING.

FIG. 132. U-LEATHER PACKING.

packing may be scarfed so that it can be slipped into its recess without removing the plunger. Less exacting workmanship is needed and more dependability is obtained if several rings are used in series with the joints staggered, and preferably a filler inserted at the bottom of the U to prevent collapsing (see Fig. 133).

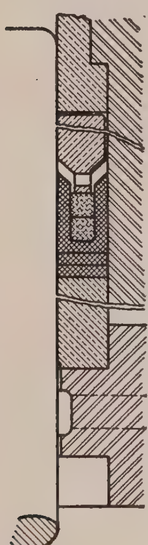


FIG. 133. U-PACKING.

The detail arrangement of a U-leather packing to give satisfactory service requires experience and expert technical judgment. An excellent treatment of this matter may be found in the book *Hydraulic Engineering* published by the research staff of E. F. Houghton & Co., Philadelphia.

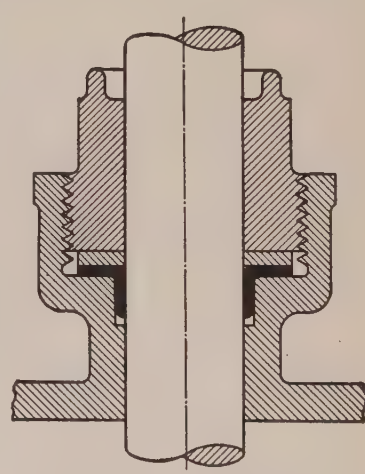


FIG. 134. HAT-LEATHER PACKING.

For comparatively low pressures and for smaller parts, U-packings may be replaced by cup packings or flanged packings ("hat leathers"). In Fig. 134 a valve stem is shown packed by means of a hat leather; in Fig. 135 a double-acting piston by means of cup leathers. U-leathers could be used for both applications.

118. Stuffing Box Proportions. The dimensions and general arrangement of an ordinary stuffing box for moderate pressures and temperatures are given in Fig. 136. The size of the gland bolts or studs may be determined from the total force on the ring area $\pi(D_1^2 - D^2)/4$. If there is no other force than that resulting from the action of a cylinder pressure p, the force would of course be given by the formula $\pi(D_1^2 - D^2)p/4$; but it is customary to multiply the pressure with a factor which allows for the friction of the packing against the rod or plunger, which tends to force the packing out of the recess when the rod or plunger is moving out. The minimum factor to use for this allowance is 1.25, although Bach recommends a factor of 3.

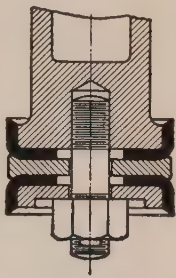


FIG. 135. DOUBLE CUP PACKING.

In general, in view of the fact that stuffing box bolts can be tightened only to a point where they are affected by the fluctuations in the load, it is advisable to select as large a factor and hence as low a stress on the root area of the bolts as can be done without reaching obviously impossible dimensions. A stress of 10,000 p.s.i. may be permissible with soft steel bolts, if a substantial allowance for friction is made. Without such an allowance, the stress should not exceed 5000 p.s.i. However, in hydraulic stuffing boxes for very high pressures, these rules lead to bolt sizes that can hardly be accommodated. In this class of machinery, one finds bolts that are designed without any allowance for friction and with root stresses as high as 15,000 p.s.i. The acceptance of bolt dimensions resulting from such a computation may be tolerated, because hydraulic plungers and pistons are usually slow-moving and the number of pressure changes in a given time is comparatively small.

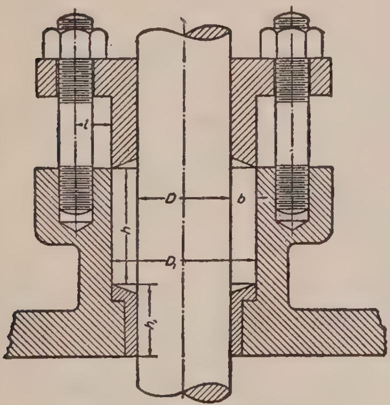


FIG. 136. STUFFING BOX.

$b = 0.4\sqrt{D}$ to $0.5\sqrt{D}$.
$D_1 = D + 2b$ or $D_1 = 1.22D + 0.6$ in.*
$h = D_1$ or $h_{max} = 6b$ to $8b$ (may be reduced somewhat for water or increased for steam).
$h_1 = D$ for horizontal rods or $h_{1max} = 3b$ to $4b$ for vertical rods.

* See Mark's *Handbook*.

Empirical proportions are often given for the thickness of the flange of the gland; for the stuffing box illustrated in Fig. 137, a ratio of

1.75 times the nominal bolt diameter may be used. It is better, however, to compute the thickness based on the assumption that the flange is a cantilever built in at the skirt with a load equal to the pull on the bolts. This method of computation is an approximation which disregards entirely the possibility of a serious distorting stress that may occur in the skirt at the root of the flange, particularly if the skirt is thin. Again referring to Fig. 136, if there are n bolts of root diameter d_r in. subjected to a tensile stress s_t, and a flange of thickness t in., having a distance l in. from the bolt circle to the root of the flange, then, if the bending stress in the flange is s_b, we have

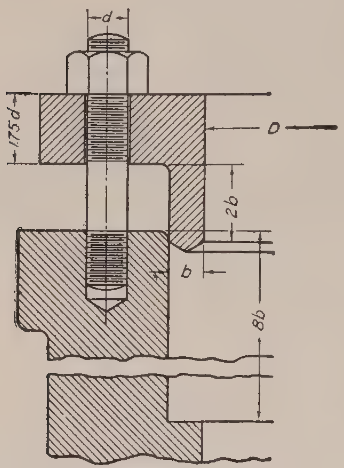


FIG. 137. HYDRAULIC STUFFING BOX.
D = Ram diameter in in.
$b = 0.25\sqrt{D}$.

$$\frac{\pi d_r^2 n l s_t}{4} = \frac{\pi D_1 t^2 s_b}{6},$$

$$(1) \qquad t = d_r \sqrt{\frac{3nls_t}{2D_1 s_b}}.$$

For the root diameter of the bolts we have

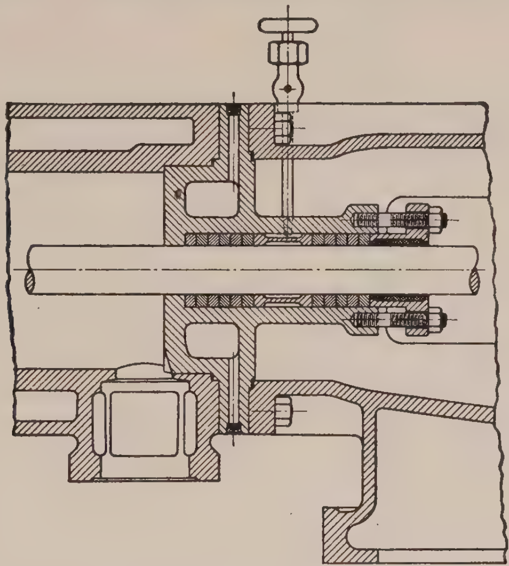
$$(2) \qquad \frac{\pi d_r^2 n s_t}{4} = \frac{\pi (D_1^2 - D^2)kp}{4}, \qquad \text{or} \qquad d_r = \sqrt{\frac{(D_1^2 - D^2)kp}{ns_t}},$$

where k is the allowance factor for friction.

119. Special Stuffing-Box Packings. If it is essential that no fluid leaking through the stuffing box should penetrate into the open, the designer may divide the packing into two parts, as shown in Fig. 138, placing a "lantern" between the two parts and draining the leakage from the lantern by means of a pipe. The arrangement in Fig. 138 is taken from an ammonia compressor, in which case the leakage is carried to the suction side of the compressor. An exactly similar arrangement may be used to prevent air infiltration into the suction chamber of a centrifugal pump, but here water from the high-pressure side is carried to the lantern chamber. Similarly, high-pressure steam may be carried to the stuffing boxes on the vacuum side of steam turbines so as to prevent drawing air into the condenser.

For high temperatures, such as those that occur in engines running on highly superheated steam, or in gas engines, metallic or semimetallic packings are used. In the full metallic packing (Fig. 139), the packing elements are subdivided cast iron rings held together by springs

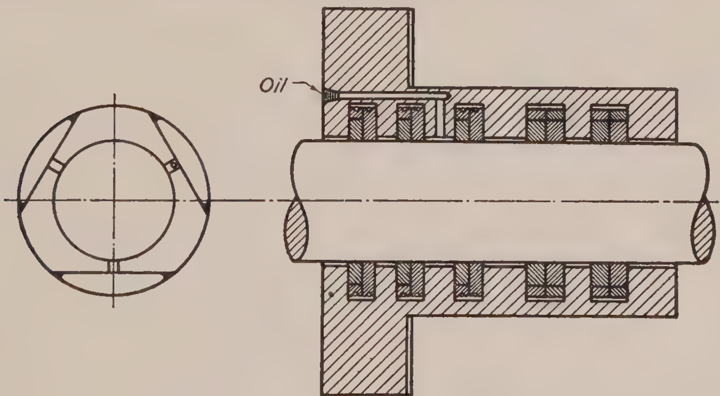
and located in chambers with a radial clearance at the outer circumference. In case of slight misalignment or whipping of the rod,



The Arctic Ice Machine Co., Canton, Ohio

FIG. 138. STUFFING BOX WITH LANTERN FOR AMMONIA COMPRESSOR.

the rings can yield. Semi-plastic packing consists of wear rings of soft metal, in contact with the rod and with the intervening filler rings



The Garlock Packing Co., Catalogue C, 1935

FIG. 139. FLEXIBLE METAL PACKING.

of a plastic material. The filler rings may be of babbitt or iron if textile rings are used at the ends to give the required elasticity.

120. Labyrinth Packings. It has been mentioned that the packing boxes on the shafts for centrifugal pumps may be of the type described in § 119 for reciprocating rods. In such applications, the cooling effect of the liquid against which the packing tightens may be depended upon to prevent overheating from friction. In steam turbines or rotary compressors for gases, no such cooling effect is available, and, to prevent overheating from friction, labyrinth packings were introduced, first by Sir Charles Parsons, the inventor of the reaction steam turbine. In Fig. 140 a labyrinth packing is shown in its original application, namely, to the so-called "dummy piston" which balances the end thrust on the rotor in a reaction turbine. There is no actual solid contact to prevent leakage. The steam is forced to travel through a series of constricted passages with intervening chambers into which it expands, with a reduction of pressure. There is also a repeated change of direction of the flow, which consumes pressure and causes an additional reduction of pressure due to energy loss.

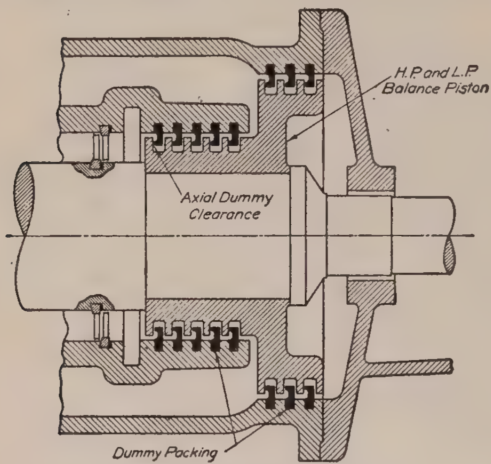


Fig. 140. Labyrinth Packing.

With expansive fluids, that is, gases and vapors, the constricted passages with intervening expansion chambers are sufficient to reduce the pressure to the point where very little pressure remains to cause leakage when the fluid arrives at the throat communicating with the atmosphere. In consequence, the labyrinth can take the form of a simple series of serrations or recesses in the sleeve surrounding the shaft. Such a "pseudo-labyrinth" is shown in Fig. 141.

For non-expansive fluids, that is, liquids, constricted passages and changes in the direction of flow are the principal means for reducing pressure. The arrangement in Fig. 142 may serve for liquids, especially if the expansion chambers are contracted to change the flow more abruptly. Such chambers are of value only as they may serve as vortex chambers in absorbing pressure by turbulence. Pseudo-labyrinths are therefore not effective for liquids. Annular recesses around valve stems, as shown in Fig. 143, have been commonly used.

They may be of some use for stopping leakage of gases, but not of liquids. Sometimes they may be of value as distributing points for lubricants.

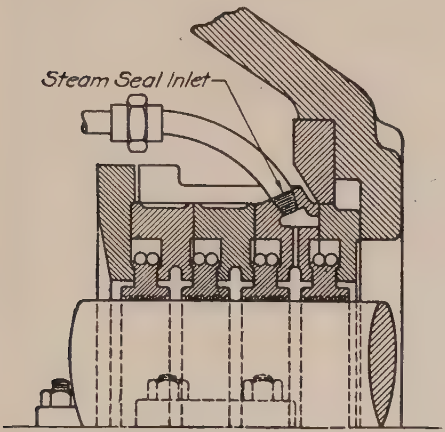


Fig. 141. Pseudo-Labyrinth Packing.

Fig. 142. Labyrinth Packing with Axial Restriction.

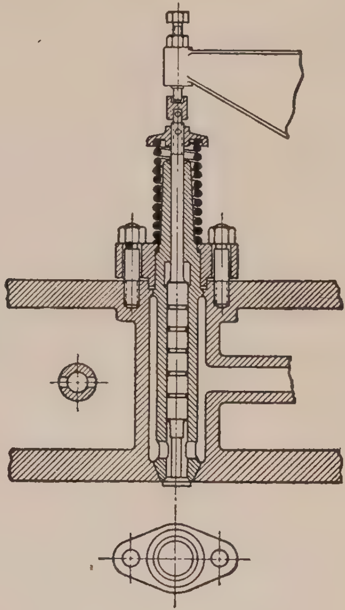


Fig. 143. Valve-Stem Packing.

PROBLEMS

1. A single-acting pump with a plunger $4\frac{1}{2}$ in. in diameter supplies a boiler with feedwater at 200 p.s.i. pressure. Determine the size of three stuffing-box studs if tightened with a 50 per cent overload. Determine the thickness of the flange if proportioned according to § 118. The center line of each stud is at a distance from the edge of the packing chamber equal to the stud diameter.

2. A 2 in. by 6 in. high-pressure pump delivers water against 1000 p.s.i. pressure. What type of packing should be used? Select packing-gland dimensions and bolts.

3. A cylinder head that supports a pressure of 500 p.s.i. is fastened to a 12 in. cylinder with 12 studs. (a) Determine the diameter of the studs to insure tightness. (b) Considering the head a flat plate, simply supported, determine the thickness of the cast-iron head.

CHAPTER 9

LINKAGES

121. General Principles. The term link, as used in this chapter, shall simply mean a bar hinged or pivoted at the end. If the bar is pivoted at both ends and the pivots are considered to be frictionless, the bar can manifestly receive and transmit force only along its own axis. On the other hand, if one end is pivoted and the other is attached to a shaft, the bar becomes a *crank* arm. It can then transmit a useful force only at right angles to its axis. While links can transmit force only as stated, the actual motions produced at ends free to move in various directions will depend upon the interconnections with other links.

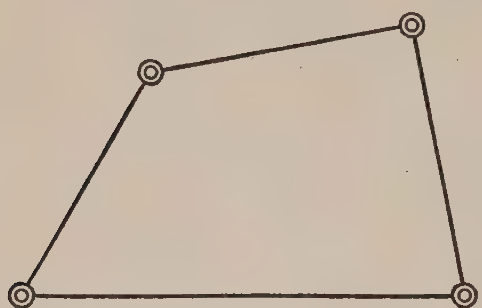


Fig. 144. Four-Bar Linkage.

A combination of several links is known as a *linkage*. Undoubtedly the most important linkage is the well known *four-bar* linkage (Fig. 144). In fact, it may be shown that any freely moving linkage consisting of more than four bars can be reduced to a combination of four-bar linkages.

A great variety of mechanisms in use are developed by anchoring one of the four bars and using one or more of the others as cranks. Drag links, a number of quick-return motions, and other mechanisms are examples of such linkage variations. The most important type of linkage, however, is the *slider-crank* chain, which is used to transmit motion from a piston to a crankshaft, or vice versa, in all reciprocating engines, pumps, and compressors. In this case, one of the links is theoretically of infinite length, so that the swinging end is made to travel in a straight line, either in the working cylinder itself, as in single-acting engines, or between special guides, as in double-acting engines.

The composition and velocity relations of the linkages themselves form the subject of a special study, namely, *mechanism* or *kinematics*. Here we will concern ourselves only with the forces in the links and the detailed design of their various parts.

122. Design of a Simple Link. A simple link consists of a rod with an eye (Fig. 145), or a clevis (Fig. 146) at the ends. If the length

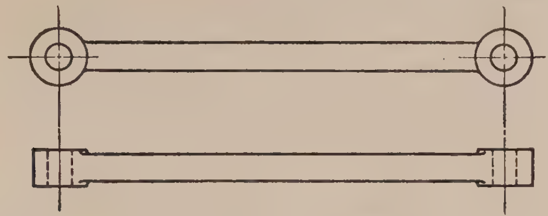


FIG. 145. LINK WITH EYE CONNECTION AT EACH END.

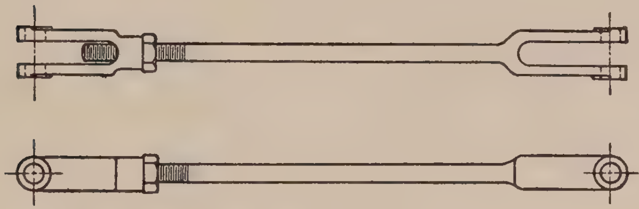


FIG. 146. LINK WITH CLEVIS AT EACH END.

is to be adjustable, the simplest construction is to screw the rod into the clevis and secure it by means of a lock nut (Fig. 146). Long tie rods with considerable adjustment may be provided with turnbuckles (Fig. 147). The clevis may be simply a bent piece of strap steel, a casting, or a forging. If it is a forging, it can be made by upsetting and forming to shape the ends of the rod itself, or it can be made as a separate piece and welded or screwed to the rod.

FIG. 147. TURNBUCKLE.

The rod is computed for tension or buckling, depending on whether it transmits a tensile or compressive force. The easiest way to compute the rod for buckling is first to obtain the necessary moment of inertia of the section by means of Euler's formula (38), page 25, with a very ample factor of safety, perhaps 15 or more, and then to derive the actual stress by means of Rankine's formula (39), page 26.

Consider, for instance, a round rod 20 in. long with both ends pivoted and required to withstand a buckling load of 100 lb. Assuming a factor of safety of 15, we have, from Euler's formula, $I = 15Fl^2/\pi^2E = 15 \times 100 \times 20^2/\pi^2 \times 30{,}000{,}000 = 0.002$ in.^{4}. The diameter that gives this moment of inertia is 0.45 in. (Note: $\pi D^4/64 = I$.) The radius of gyration is $D/4 = 0.112$ in. Substituting in Rankine's formula, $s_c = F(1 + q \times l^2/k^2)/A = 4 \times 100(1 + 0.0006 \times 20^2/0.112^2)/(\pi \times 0.45^2) = 12{,}700$ p.s.i. This stress is comparatively high, not permissible in case of alter-

nating loads. In the use of Euler's formula, it would have been better to have assumed a factor of safety of 20 or 25.

123. Design of Eye and Clevis. To obtain a first estimate of the section required for the eye, note that stress concentration at the inside of the eye may cause stresses about three times as high as the average tensile stress (see page 44). After selecting a tentative section, we may check the bending moments in the eye by the formulas given in § 22, page 36, and derive the bending stresses.

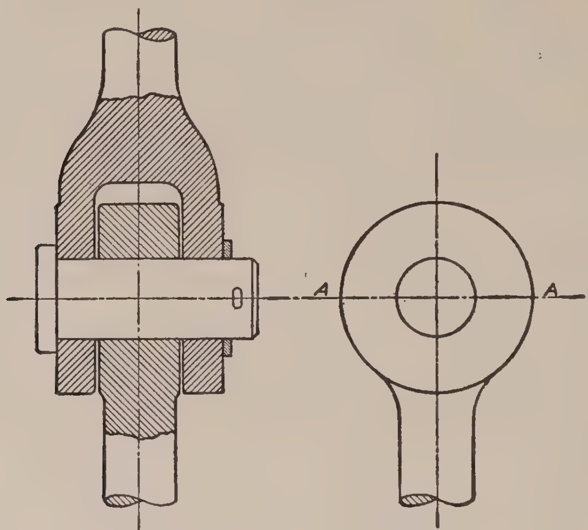


FIG. 148. EYE AND CLEVIS.

As an example, assume a rod of 1 in. diameter (Fig. 148) subjected to a stress of 15,000 p.s.i. The total force in the rod is then about 11,800 lb. Assume this rod terminating in an eye of $1\frac{1}{8}$ in. inside diameter and $1\frac{1}{8}$ in. width. To allow for stress concentration at the inside, the average direct tensile stress on the section A-A of Fig. 148 should not exceed 5000 p.s.i. The section required is then 11,800/5000 = 2.36 sq. in. In other words, the eye should have $2.36/(2 \times 1.125) = 1.05$, say $1\frac{1}{16}$ in. radial thickness at each side. The mean radius is then $0.562 + 0.531 = 1.093$ in. To determine the bending stress in the eye according to the moment equation, $M = 0.182Fr$ (page 36), the bending moment at the section A-A is equal to $0.182 \times 11{,}800 \times 1.093 = 2350$ in. lb. The bending stress is $(2350 \times 6)/(1.125 \times 1.062^2) = 11{,}100$ p.s.i. Adding the direct tensile stress of 5000 p.s.i., we find a total tensile stress at the inside of the eye of 16,100 p.s.i., or slightly higher than the working stress regarded as permissible.

If we assume the outer end of the eye (90 deg. from section A-A) to have a concentrated load applied at the middle, then according to the formula, $M = 0.318Fr$ (page 36), the bending moment amounts to $0.318 \times 11{,}800 \times 1.093 = 4100$ in. lb. The stress caused by this moment is 19,400 p.s.i.; in other words, it is greater than the stress on the transverse section.

In actual design in the past, the bending moments and stress concentrations have usually been neglected and the eye computed only for tension on the section A-A. This method leads to a much thinner eye; if failure does not occur in such designs, the reason is that the load is not quite concentrated at the point of application and that the most highly stressed fibers are backed up by much less seriously stressed material. If the radial-eye thickness is made 3/4 in., as drawn in Fig. 148, the maximum bending stress computed at the point of load application equals 33,000 p.s.i., which still is within the elastic limit and is no doubt considerably higher than the actual stress that occurs.

In the clevis, each side is designed for only half the load, the shanks being in straight tension or compression, and the eyes loaded in the same manner as the eyes in the rod. The yoke or cross piece joining the sides to the rod may be figured in bending, the most conservative assumption being that it is a beam freely supported at the ends with a concentrated load in the middle.

The inside faces of the clevis, as well as the faces of the eye, are usually finished by milling, although other machining methods are possible. These contact faces may be bossed to avoid undercutting in finishing.

124. Design of Hinge Pin. The dimensioning of the hinge pin that holds the eye and the clevis together depends upon whether the hinge is to constitute merely a flexible connection or is to be subjected to continuous rotation or oscillation. In the latter case the load-carrying surfaces in the eye or in the clevis, or in both, must be developed as bearings with permissible bearing pressures, which ordinarily should not exceed 1500 to 2500 p.s.i. on the projected bearing area, and often are very much less. We shall deal more fully with such bearings in discussing connecting rods. If the surfaces merely carry the load, but are not subjected to continuous rotation or oscillation, pressures of 4000 to 8000 p.s.i. on the projected area may be permissible, depending upon the closeness of the fit. Much higher stresses are used on rivets or pins in structural work.

After the proper bearing area has been ascertained, the strength of the pin in bending and in shear should be checked. The bending moment is usually computed on the assumption that a concentrated load occurs at the middle of the pin, which is considered freely supported at the middle of the clevis arms.

In the example already used (Fig. 148), the projected bearing area would be $1.125 \times 1.125 = 1.26$ sq. in., and the pressure on this area would be 9350 p.s.i., which, while high, may be considered permissible if no rotation is involved. Assuming each clevis eye to be 9/16 in. wide, the moment would be $11{,}800(0.5625 + 0.2812)/2 = 4970$

in. lb. The stress caused by this moment is $(4970 \times 32)/(\pi \times 1.125^3) = 35{,}500$ p.s.i. This stress is of course excessive for ordinary low carbon steel and the pin diameter would have to be increased. The shear stress computed as for a rivet in double shear would be only $11{,}800/(2 \times 0.995) = 5930$ p.s.i., which would be quite reasonable.

Experiments of Bach * with cast-iron pins showed that breakage actually occurred in the middle of the eye, which indicates that the method for figuring bending stress is not unreasonable. It will be seen that as a rule the computation of the shear stress may be omitted, since the bending stress is much more serious.

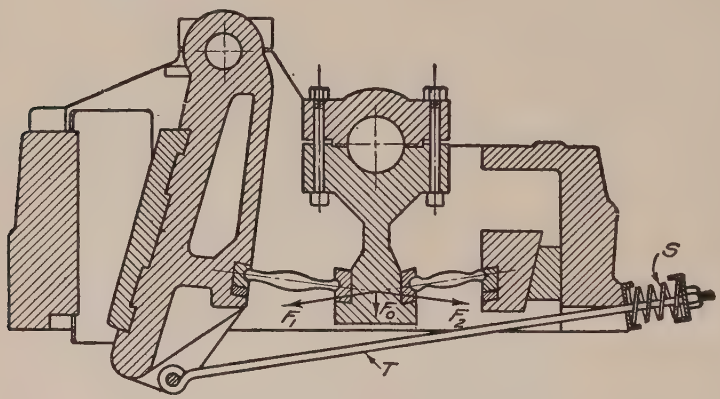


Fig. 149. Jaw-Type Rock Crusher.

Sometimes the bending action on the pin would lead to very unwieldy dimensions. This might be true, for instance, in the case of the toggle links in the jaw rock crusher shown in Fig. 149. In a toggle joint, a comparatively large movement in one direction is transformed into a small movement in a direction practically at right angles. With reference to the diagram in Fig. 150, it is clear that the forces F_1 and F_2 in the toggle links have only very small components in the direction F_0 resisting the motion of the lower end of the piston. It then takes very large forces in the links to resist the force in the pitman, or inversely, a very small force in the pitman produces very large forces in the links. To transmit these large forces, the ends of the links are made to bear directly on recessed pressure plates at the fulcrum point, the frame

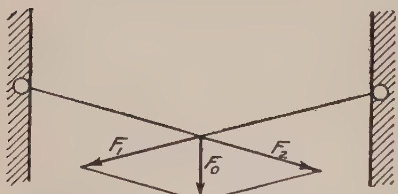
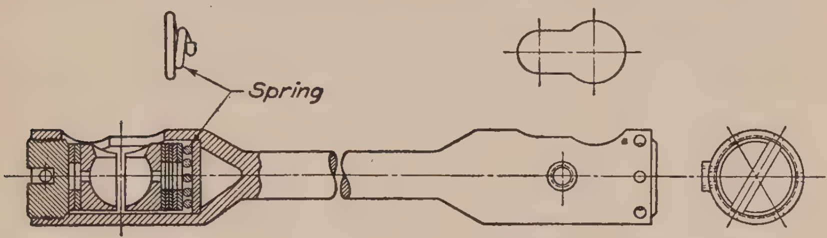


Fig. 150. Force Diagram of Toggle Links.

* *Elastizität und Festigkeit*, 8th ed., p. 415, plate XXII, Fig. 2.

support, and the oscillating jaw. The links are prevented from slipping out of the recesses by means of the tie rod T and the spring S.

There are times when the motion of a linkage is not in one plane. Such is the case, for instance, with the steering linkage in an ordinary automobile. If the motions do not deviate much from the same plane, the deviations *may* at times be taken up simply by some play in the hinges or by a small amount of distortion of the rods. Such a distortion occurs, for instance, in the rear springs of automobiles using transverse springs, if the drive is transmitted from the rear axle to the car frame through a rod of fixed length. In the steering linkage, however, the motion must be transmitted through ball joints. An automotive rod with sockets for such joints is shown in Fig. 151.



The Columbus Auto Parts Co., Columbus, Ohio

FIG. 151. AUTOMOTIVE TIE ROD.

125. Connecting Rods, Cross Heads, and Piston Rods. The connecting rod and cross head represent two of the links in a four-bar chain. In single-acting engines with trunk pistons, the piston itself functions as a cross head in guiding one end of the connecting rod.

The relation of forces acting between these elements is shown by the diagram in Fig. 152. The force F on the piston is directed along the

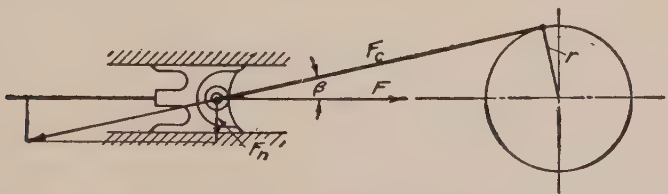


FIG. 152. FORCE DIAGRAM OF CROSS HEAD AND CONNECTING ROD.

axis of the machine, while the force in the connecting rod, except at dead centers, is at an angle β to this axis. Consequently, as in the toggle joint just considered, only a component of the force F_c in the connecting rod is available to balance the piston force. Another component F_n is at right angles to the force F and is transmitted to

cylinder walls by trunk pistons, or to the guides by the cross heads. From the parallelogram of forces, we have $F_c = F/\cos\beta$, and $F_n = F\tan\beta = F_c \sin\beta$.

The maximum value of β occurs when the crank arm r stands at right angles to the direction of the stroke. When the line of stroke passes through the crank center, if the length of the connecting rod is l, we have $\sin\beta = r/l$. In this position we have $F_n = F_c r/l = Fr/l\cos\beta$, and this is the maximum value of F_n, at least if F has its maximum value in this position.

A common value for r/l in stationary machinery is 1/5, in which case $\cos\beta = 0.98$ for maximum β. Even in very compact automotive or marine machinery, r/l is rarely more than 1/3.5, in which case $\cos\beta$ for maximum β reaches the value 0.962. In general, then, it involves an error of only 3 to 4 per cent to make $F_n = Fr/l$.

Regularly in combustion engines and often in steam engines and compressors, F does not have its maximum value when β attains its maximum value. It is then necessary to ascertain from the pressure curve the angle at which F_n is maximum.

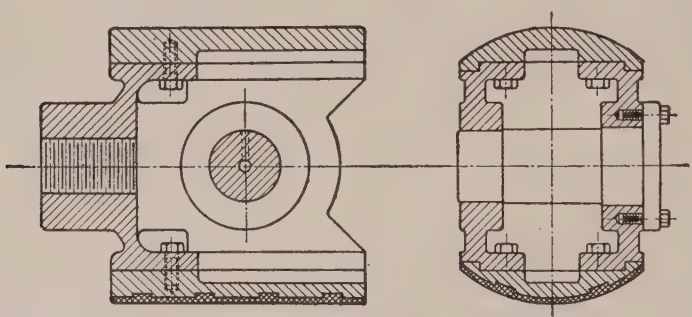


Fig. 153. Cross Head with Shoes Adjustable by Means of Shims.

126. Cross Heads. Designs of cross heads are shown in Figs. 153 and 154. The bearing pressure produced by the side thrust on the cross head should not exceed 30 to 60 p.s.i. The shorter the time during which the maximum pressure occurs, the higher the bearing pressure may be. Thus combustion engines or steam engines with a short cut-off may have higher pressures than steam engines with a long cut-off, or pumps which are subjected to the maximum pressure during practically the whole delivery stroke.

Shoes are adjustable for wear either by means of shims, as in Fig. 153, or by means of wedges, as in Fig. 154. The bearing surface of the shoe may be babbitted. If the engine does not reverse, there is pressure on only one of the shoes, except possibly for pressure arising from tipping

actions. Under such conditions, it may be satisfactory to babbitt and to make adjustable only one of the shoes. It is better, however, to make both shoes adjustable in order to center the rod ends.

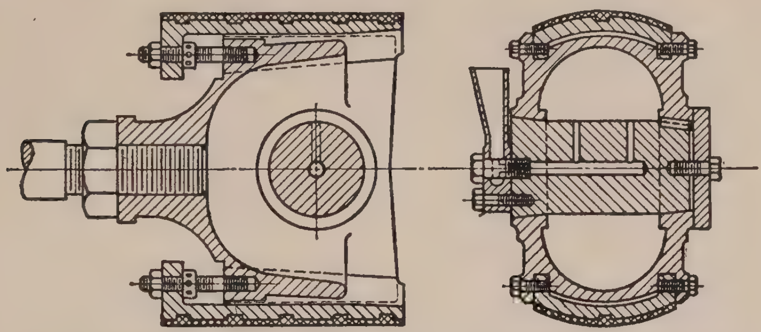


FIG. 154. CROSS HEAD WITH SHOES ADJUSTABLE BY MEANS OF WEDGES.

127. Cross-Head and Piston Pins. Two different methods of fastening the cross-head pin in place are illustrated in Figs. 153 and 154. The dimensions of the pin are obtained by computing it first for bearing pressure and then for bending, as already explained for the clevis on page 155. The bearing pressures permitted on cross-head pins vary from 1100 to 1400 p.s.i. for steam engines and compressors. On piston pins for stationary and marine engines, the pressures may be as high as 1800 to 1900 p.s.i. For automotive engines, according to Heldt, 2500 p.s.i. is permissible at the maximum explosion pressure, and it is possible that even higher pressures may be attained in extreme cases, if the best of modern pin and bushing materials are used. Copper-lead bearing alloys are being adopted for severe service.

Note that in Fig. 153 the cross-head pin is stepped, while in Fig. 154 the ends are tapered. By this means, scoring of the bearing surfaces is avoided when the pin is inserted. Cylindrical fits are easier to attain than taper fits with modern quantity production methods.

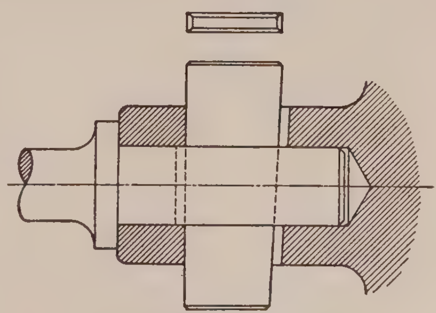


FIG. 155. COTTERED JOINT.

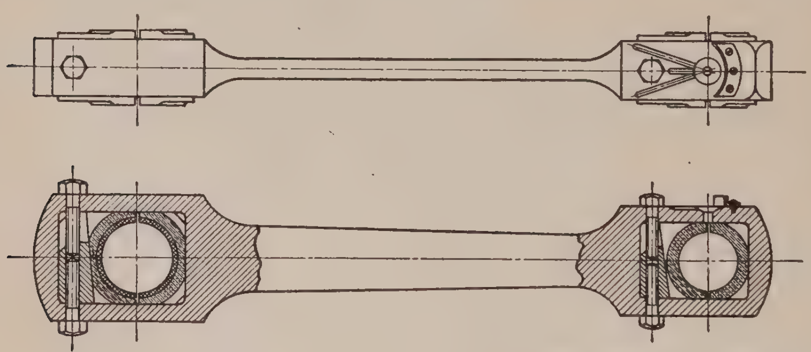
128. Piston Rods. The simplest method of fastening the piston rod in the cross head is to have it threaded on the end and secured by a lock nut in its proper position as shown in Fig. 154. The use of cotters, as in Fig. 155, has been common, but this is expensive

and inconvenient from the manufacturing point of view; and furthermore it does not admit of the same convenient lengthwise adjustment.

The most conservative method of computing the piston rod, as has already been explained for link rods in general on page 153, is to compute it for buckling by means of Euler's formula, with a factor of safety 15 to 20, and then apply Rankine's formula, with the assumption that it is entirely free to buckle on an effective length equal to the entire distance between the piston and the cross head. A less conservative method is to assume the rod guided in the stuffing box, and thus compute it on the basis of the shorter length between the stuffing box and the cross head, or between the stuffing box and the piston.

The piston rod was formerly computed as a cantilever beam rigid enough to carry the piston in horizontal machines without excessive deflection. In modern design, however, it is regarded as satisfactory to reduce the bearing pressure between the piston and the cylinder wall to values of 6 p.s.i. and less, and with improved methods of cylinder lubrication, the piston can be allowed to carry itself with these pressures without support from the rod.

129. Connecting Rods. While there are many different designs of connecting rods, we shall consider here only two fundamental types. One is the closed-end type shown in Fig. 156, which obviously must be



The Arctic Ice Machine Co., Canton, Ohio

Fig. 156. Closed-End Type Adjustable Connecting Rod.

assembled with the pin from the side. The other is the marine or split-end type shown in Fig. 157. This construction is suitable for center cranks or pins over which the rod must be clamped. There are also combination types, for instance, those in which the ends are designed very much like the closed-end type, but with detachable end straps.

The difference in the design regularly occurs in the large end. The small end, which fits on the cross-head or piston pin, may be designed in the same manner as the large end (Fig. 156); very frequently it is simply a bushed hole or an eye, as in Fig. 157.

The closed ends in Fig. 156 are provided with split bronze bearing blocks inserted from the side. Adjustment for wear is effected by means of the wedge block, which is locked in position by two cap screws, one pulling against the other. Obviously, the bearing blocks must not be clamped against the pin, but must bear on one another so as to leave the proper clearance around the pin. Either removable shims are inserted between these contact surfaces or the ends must be filed or ground off when wear has taken place.

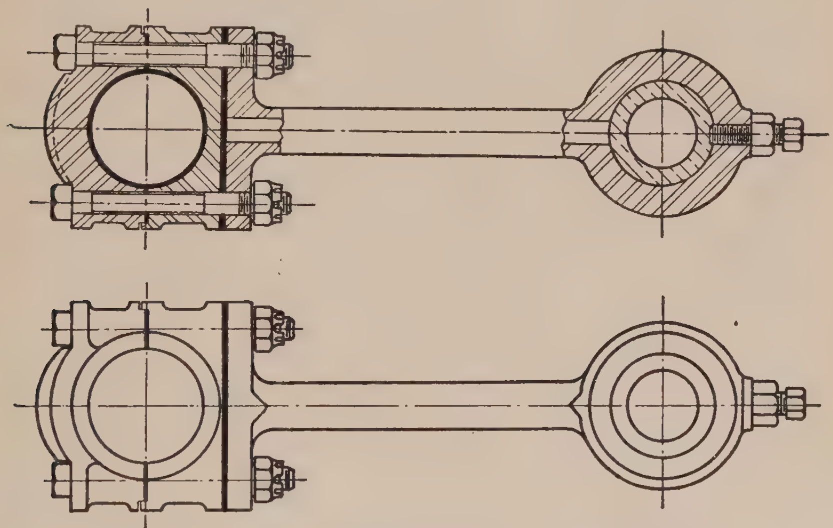


FIG. 157. MARINE OR SPLIT-END TYPE CONNECTING ROD.

In computing the dimensions for the large end of the rod, it is advisable to compute the end piece as a beam freely supported at the ends with the total force in the rod considered concentrated at the center (that is, if the width of the opening is b and the force is F, the bending moment will be $Fb/4$). The beam section should not be tapered very much toward the sides, since any distortion that may occur in it will produce rather complicated stresses at the corners and in the sides. With the end piece very rigid, the sides may be computed for straight tension with a factor of safety corresponding to a stress variation between zero and a maximum, and with some allowance for impact, which will occur if there is any looseness in the bearing. Where

extreme lightness is not necessary, that is, for slow and medium-speed machines, the factor of safety should be not less than 5 or 6. This same factor of safety may be used for the end piece. In view of the possibility of impact, the screws holding the adjustment wedge should be liberally dimensioned.

The wedge adjustment on the small end should be toward the rod for accessibility, as shown in Fig. 156, and the adjustment on the large end away from the rod. An adjustment at both ends will then tend to leave the distance between bearing centers comparatively unchanged.

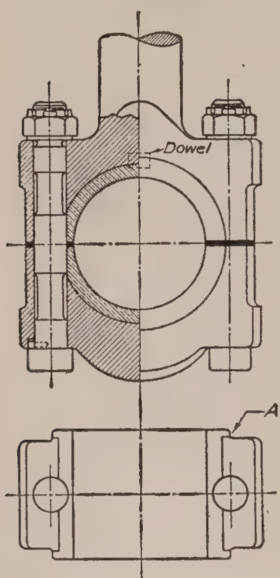


Fig. 158. Large End of Marine Type Connecting Rod.

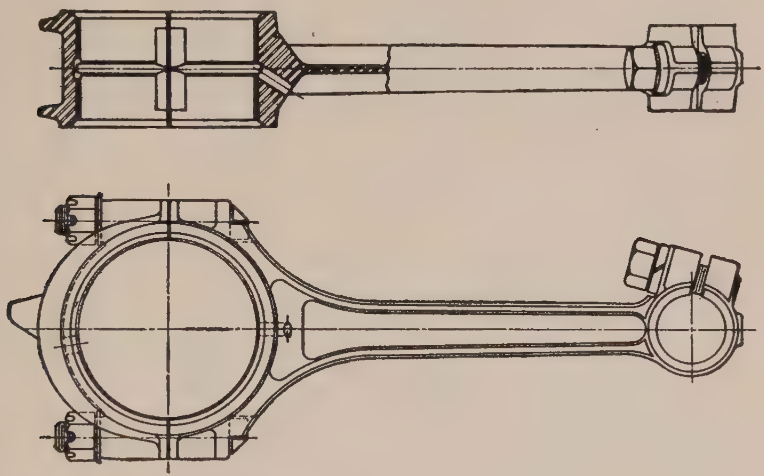
The large end of a marine rod should be designed as in Fig. 157, when weight and expense are not too serious considerations. In this construction, there are projections or offsets for taking up transverse bearing forces and shims for length adjustment. The large end may be babbitted with high quality tin-base babbitt, if the bearing pressure does not exceed 1000 p.s.i. For pressures approaching 1500 p.s.i., even if they are momentary, some manufacturers have found it advisable to introduce copper-lead bearing shells (A in Fig. 158), particularly if the speed is high. The babbitting may be done directly even on a steel rod if the steel surfaces are carefully cleaned with suitable solutions before babbitting and the babbitt is applied before any oxidation occurs.

The copper-lead bearings are applied as loose shells, as shown in the figure, with a dowel to prevent rotation, or with the bolts slightly recessed into the shells for the same purpose. In any case, the bolts should be located as close as possible to the bearing, so as to reduce weight and decrease the bending moment on the cap.

In double-acting machines of not too high speed (less than 350 r.p.m., let us say), the cap and the bolts are computed to stand the maximum rod force resulting from the pressure on the piston. For high-speed engines, the inertia forces may be more serious than the pressure forces. The effect of these forces will be discussed presently.

When lightness and price are important considerations, the large end is simplified. The offsets for the transverse forces are omitted, the

head is split straight across, and there are no shims for length adjustment. This type of design is shown in Fig. 159. Lightness, however, usually goes with high speed, and particular care is then necessary to make the cap and the bolts strong enough to withstand the inertia forces. Ribbing is resorted to in such designs, and in no case should nuts or bolt heads be recessed into the cap or base.



Chevrolet Motor Co., Detroit, Mich.

FIG. 159. AUTOMOTIVE CONNECTING ROD.

130. Inertia Forces in the Slider-Crank Chain. The somewhat approximate formula generally used for the inertia forces of the reciprocating parts of the slider-crank chain is as follows:

$$\text{(1)} \qquad \text{Inertia Force} = M\omega^2 r\left(\cos\alpha + \frac{r}{l}\cos 2\alpha\right).$$

In this equation, M is the mass of the reciprocating parts, which usually is taken to include one-third to one-half of the mass of the connecting rod, ω is the angular velocity of the crank arm in radians per sec., r the crank arm length in in., l the length of the connecting rod in in., and α the angle of displacement of the crank arm from the inner dead center position, O-A in Fig. 160. The expression consists of two terms, $M\omega^2 r \cos\alpha$, which is called the *primary* inertia force, and $(M\omega^2 r^2/l)\cos 2\alpha$, which is called the *secondary* inertia force. The secondary force is caused by the oscillation of the connecting rod around the central axial position, and goes through two complete cycles during one revolution. It vanishes if the connecting rod is infinitely

long. Its maximum value, however, is only the fraction r/l of the primary force.

The total inertia force reaches its greatest value at inner dead center, where it is $M\omega^2r(1 + r/l)$, and has at outer dead center (B in Fig. 160) the value $M\omega^2r(1 - r/l)$. To demonstrate how serious these maxima are at high speed, let us assume an automotive engine with reciprocating parts weighing only 1.5 lb. per cylinder, running at a speed of 3600 r.p.m. Let us assume $r = 2$ in. The angular velocity is then $60 \times 2\pi = 377$ radians per sec. The primary force is $(1.5 \times 377^2 \times 2)/(32.2 \times 12) = 1100$ lb. Assuming that the cylinders have a bore of 2.5 in., this force corresponds to a pressure of 225 p.s.i. of piston area.

In a four-cycle combustion engine, this force occurs at the beginning of the suction stroke. To get the force on the connecting-rod cap

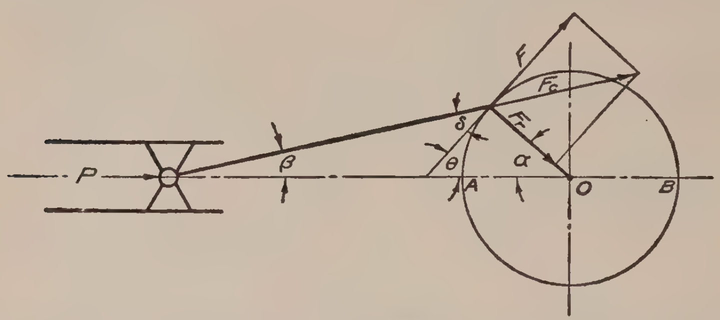


FIG. 160. FORCE DIAGRAM OF CONNECTING ROD AND CRANK ARM.

and bolts, we must add to this force the centrifugal force of a certain part of the connecting rod, which may be considered as rotating with the crank pin. Including the cap and the bolts, this weight is usually taken as two-thirds of the weight of the connecting rod; in this case, with aluminum pistons, it would be about 1 lb.* Without the cap, it may be about 0.75 lb. The centrifugal force is equal to $M_1\omega^2r$, where M_1 is the rotating mass. In this case, it would be 550 lb., and would be equivalent to 112.5 p.s.i. of piston area.

It will be seen that, even with these light parts, the stress from inertia force and centrifugal force on the cap and bolts of the connecting rod at fairly high speed is fully equal to the stress that would be produced by the maximum gas pressure, if it acted in the same direction. According to the investigation by Heldt, mentioned above, the crank-pin bearing pressures in the case of modern automotive engines should

* See article by P. M. Heldt in Automotive Industries, June 8, 1935, p. 774.

be computed regularly from the centrifugal and inertia forces at the beginning of the suction stroke and not from the maximum gas pressures in the cylinder.

In the preceding investigations, we have neglected the secondary inertia force, since the question at issue was only the magnitude of the inertia forces. The secondary force, with an r/l value of 1/4, adds, however, 25 per cent to the value of the inertia force at inner dead center. That is, in our case, it would add over 50 p.s.i. of piston area.

131. Computation of Rod Parts. After the determination of the force on the cap and the bolts, the cap is computed for bending and the bolts for tension, with a factor of safety which in general should not be less than 5 to 6, and in cases where extreme lightness is essential, certainly not less than 4.

Consider a connecting-rod cap for a 2 in. diameter pin that has a width of 2 in. and is subjected to a force of 1500 lb. The rod bearing is to be provided with copper-lead shells 1/8 in. thick, with the distance between bolt centers estimated at 2¾ in. (Fig. 159). The cap is to be a steel drop forging having a tensile strength of 80,000 p.s.i. We will assume as the working stress 16,000 p.s.i. If the cap at the center line is simply a rectangular section 2 in. wide, we have for the thickness

$$t = \sqrt{(1500 \times 2.75 \times 6)/(4 \times 16{,}000 \times 2)} = 0.44 \text{ in.}$$

If this thickness is considered to be too heavy, the cap must be ribbed and computed as a T- or U-section. The ribbed section is beneficial also because the ribs promote the radiation of heat. If the bolts are likewise computed for a stress of 16,000 p.s.i., the necessary root area with two bolts will be $1500/(2 \times 16{,}000) = 0.047$ sq. in. With national fine threads, bolts of 5/16 in. diameter would have the required root area; but, in view of impact loads and possible over-stressing in tightening, bolts of at least 3/8 in. diameter would be advisable.

If these bolts are to be used to locate the cap sidewise, or to resist transverse forces, or to prevent the shells from rotating, they should be finished and closely fitted in the holes. Some provision should be made to keep the bolts from rotating when the nuts are being tightened. Usually square or flat-sided round heads fitting against a recessed shoulder have been used for this purpose. Increasing emphasis is now being placed on the weakening effect of such recesses and on the seriousness of inertia loads. Heads may be held between ribs, but sharp re-entrant corners should be avoided.

As for the nuts, the castellated type locked by means of cotter pins are regularly used in aircraft and automotive work. Check or jam nuts should be avoided, since the two halves of the connecting-rod bearing seat on each other at the joint, and a check nut under these conditions gives no more safety than an equally well-tightened main nut.

The rod is computed for buckling and, to obtain first the necessary moment of inertia by means of Euler's formula, a factor of safety of 20 to 25 should be assumed. As a rule, it will be found that such a factor will result in stresses none too low when checked by Rankine's formula.

It is usually assumed that the tendency of the rod to buckle in the plane perpendicular to the plane of motion is somewhat relieved by the stiffening action of the bearings; consequently the moment of inertia in this direction could be much smaller. In view of the researches of Kirsch,* this assumption is very doubtful for the comparatively low l/k ratios usually occurring in rods; there is little doubt but that a rod with equal stiffness in all directions is the most satisfactory. To obtain maximum lightness and safety, a round hollow rod, finished all over, is unquestionably the best; rods of this type are found in many high-grade designs. Such rods have the added advantage that the tubular construction can be used to conduct oil from the crank pin to the cross-head or piston pin.

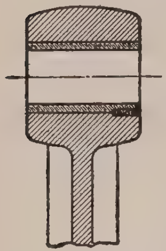


Fig. 161. Bushed Eye at the Small End of a Connecting Rod.

At the present time, however, the I-section is in more general use (see Fig. 159), and the stiffness sidewise is made considerably less than the stiffness in the direction of rotation.

As far as strength is concerned, the small end of the rod, if developed as a simple eye, is computed as already set forth on page 154. The eye should be lined with a high-grade bronze bushing, pressed in place. If oil holes are to be kept in proper alignment, the bushing may be kept from turning by means of a small headless set screw, threaded half in the bushing and half in the eye (Fig. 161).

The bearing pressures at the large end of the rod will be dealt with in the following articles on cranks.

132. Cranks and Crank Arms. Cranks and crank arms are sometimes dealt with as a part of shafting; but, since they almost always form part of a complete or modified four-bar chain, they will be dealt with here. There are two kinds of cranks, *side cranks* (Fig. 162) and *center cranks* (Fig. 166). The center cranks may be developed into multi-throw crank shafts.

133. Computation of Side Crank. The *crank pin* (Fig. 163) is computed as follows.

* Zeitschrift des Vereines Deutscher Ingenieure, 1906, p. 907.

First, determine the maximum force F_c in the connecting rod. (See Fig. 160.) We will assume for example that this force is 9820 lb. in a certain steam engine running at 150 r.p.m. A suitable bearing pressure on the pin is then selected by reference to Table 30 on page 235. In this case we will assume 1000 p.s.i. The next step is to assume a certain ratio of pin length to pin diameter. Usually this ratio is unity or a little more, in our case, say 1.1. If the diameter of the pin is d in., we have

$$1.1d \times d \times 1000 = 9820; \qquad d = 3 \text{ in.}$$

The length l is then $1.1 \times 3 = 3.3$, say $3\frac{5}{16}$ in. We should now compute the pin for bending, using a moment equal to $F_c l/2$ on the reduced diameter where the pin is inserted in the arm.

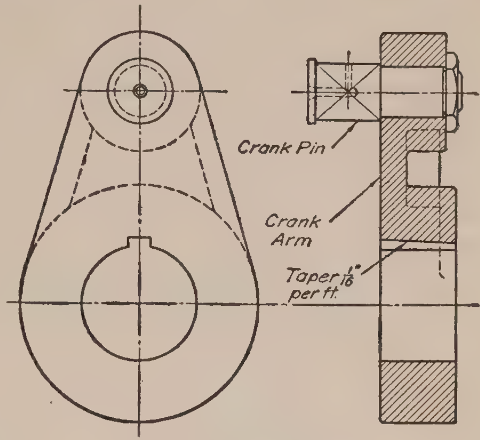


Fig. 162. Side Crank.

The section modulus of this section is $\pi d_r^3/32$, where d_r is the reduced diameter at the root. We may take $d_r = 2.75$ in. The bending stress is then $(9820 \times 3.313 \times 32)/(2 \times \pi \times 2.75^3) = 7950$ p.s.i. At inner dead center, the moment bends the pin toward the center of rotation, while near outer dead center, it bends it in the opposite direction. With some allowance for impact or suddenness of load, the factor of safety should be at least 8 or 10. With 80,000 p.s.i. steel the working stress would be 8000 to 10,000 p.s.i. The pin as proportioned is then safe in bending.

In addition to being computed for bending and for bearing pressure, the pin must also be computed for heating. In our case the rubbing speed is $\pi \times 3 \times 150/12 = 118$ ft. per min. This speed multiplied by the average bearing pressure gives a pV-value, which may be compared with accepted empirical values listed in Table 31, page 241. If we assume that in emergencies the engine under consideration may be run at nearly 100 per cent cut-off, the pV-value would be $1000 \times 118 = 118{,}000$. According to the table, this value is permissible.

The pin may be held in the arm by means of a nut, as shown in Fig. 162, but it is also common practice to hold it simply by means of a force fit (Fig. 163). A set screw, tapped half in the pin and half in the arm, may be used as an added locking device.

134. The Crank Arm. The crank arm is subject to a multiplicity of moments and forces, varying in the course of a revolution. In an arbitrary position, as shown in Fig. 160, the force F_c in the connecting rod resolves itself into a tangential turning force F and a radial force F_r along the arm. The angle θ between F and the axis is equal to $90° - \alpha$, and is also equal to $\beta + \delta$. We consequently have $\delta = 90° - \alpha - \beta$,

$$F = F_c \sin(\alpha + \beta), \qquad F_r = F_c \cos(\alpha + \beta).$$

For α and β each equal to zero, that is, in the dead-center position, F

is zero and F_r is a maximum and equal to F_c; for $(\alpha + \beta) = 90$ deg., that is, when crank arm and connecting rod are at right angles, F_r is zero and F is a maximum and equal to F_c.

The forces on the crank pin are very nearly the same in both positions, if F_c does not change very much, but the forces on the arm are very different. For instance, on the section Z-Z (Fig. 163), we have, at dead center with a rectangular arm section of thickness t and width b, a compressive stress $F_c/(tb)$ and a bending moment F_cx. In the 90 deg. position, again, there is on the section a bending moment F_cz, a twisting moment F_cx, and a shear stress $3F_c/(2tb)$. (See page 16.) The torsional shear stress caused by F_cx has a maximum at the middle of the long side and a somewhat smaller maximum at the middle of the short side of the rectangular section. The bending stress has a maxi-

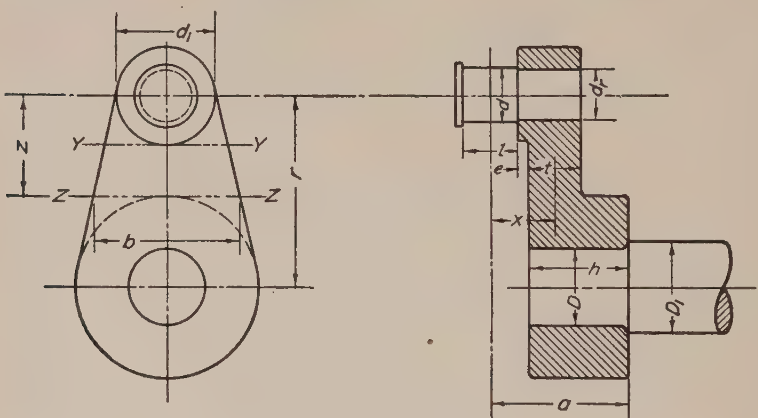


Fig. 163. Side Crank.

mum value of uniform amount along the whole short side. Evidently this maximum bending stress combines itself with the shear stress at the middle of the short side to form a maximum shear, or a maximum deformation stress, which must be computed to ascertain whether the arm is safe. The transverse shear stress $3F_c/(2tb)$ has a maximum at the middle of the long side, and this shear stress is added directly to the maximum torsional shear occurring at the same point.

If, as in gas engines, the cylinder pressure drops rapidly from the dead-center position, it is necessary to compute the stresses at a series of positions and to see where the resultant stresses are most serious. As combustion engine diagrams do not vary a great deal, the location of these important positions can be established quite definitely once for all. According to Güldner, the maximum F value is about $0.4P$ for explosion

engines and $0.6P$ for Diesel engines, where P is the total maximum force on the piston. The maximum turning moment occurs at about 33 or 34 deg. from dead center. This position is also the angle at which the maximum bending moment occurs on the arm in the plane of motion.

Often, particularly when the stroke is short, the crank arm is developed to a complete disc, as shown in Fig. 164. The disc is

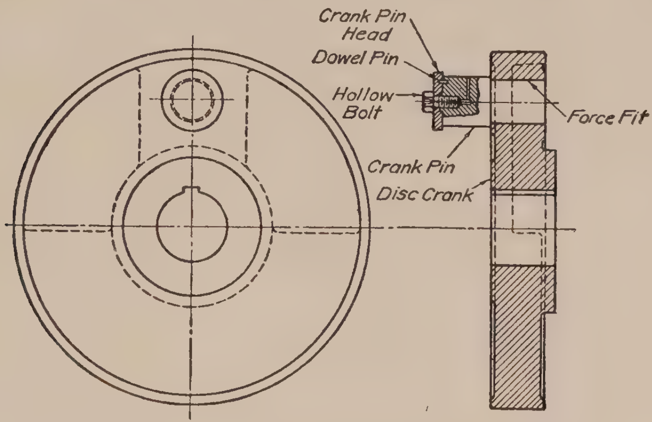


FIG. 164. CRANK ARM DEVELOPED TO A DISC.

recessed around the pin and is left solid at the other end in order to balance the centrifugal force. If the arm is very short and the forces high, it may happen that the pin and the crank shaft so nearly touch each other that it is necessary to make the pin integral with the disc, as shown in Fig. 165. Such a disc is usually a steel casting. If the arm is still shorter, it is necessary to enclose both the pin and the crank shaft in the large end of the connecting rod. The arm then becomes an *eccentric,* a brief discussion of which will follow later. As an example, we may now compute an ordinary crank arm complete.

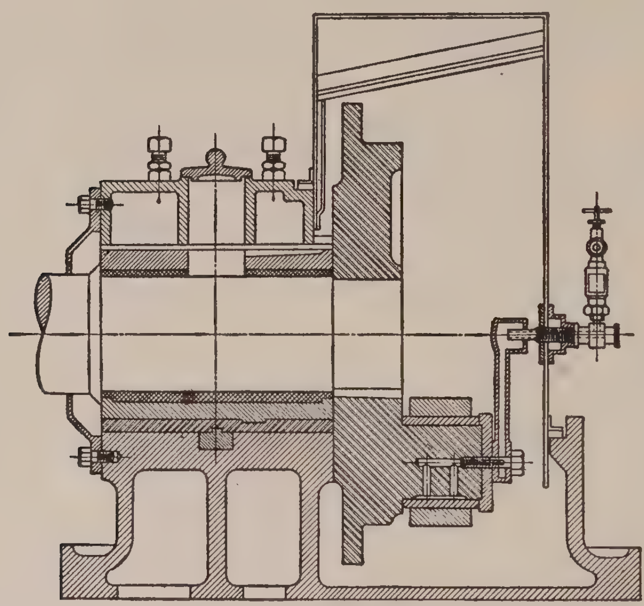
EXAMPLE. Let us assume that the pin computed previously is used in connection with a crank arm 9 in. long, and that the piston force is again 9820 lb. We will assume that the ratio of connecting-rod length to crank arm length is 5. To determine the hub length h, we must first establish the crank-shaft diameter D. With reference to Fig. 163, the diameter will be computed for a combined stress due to the bending moment $F_c a$ and the torque Fr, where r is the length of the crank arm. We will assume that the maximum steam pressure still occurs in the position where the connecting rod and the crank arm are at right angles. In this position, we have $\tan \beta = 1/5$, $\beta = 11° 20'$, and with a piston force of 9820 lb., the force $F_c = F/\cos \beta = 9820/\cos 11° 20' = 10{,}010$ lb. The maximum torque on the shaft is then $M_t = 10{,}010 \times 9 = 90{,}090$ in. lb.

To obtain an idea of the crank-shaft diameter, we may assume a shear stress of only 3000 p.s.i., equivalent to 6000 p.s.i. in tension, so as to leave a margin for bending stress. Based on the torsional strength only, the crank-shaft diameter D would then be

$$= \sqrt[3]{(90{,}090 \times 16)/(\pi \times 3000)} = 5.35,$$

say $5\frac{1}{2}$ in. The wall thickness at the hub can be taken 0.4 × bore + 3/8 in., say $2\frac{5}{8}$ in., so that the hub diameter would be $10\frac{3}{4}$ in.

If the length of the shaft stump entering the arm is taken equal to the diameter and there is a boss e 1/4 in. high at the crank pin, the moment arm a to the bearing will be $3.313/2 + 5\frac{1}{2} + 1/4 = 7.4$ in. The bending moment will be $10{,}010 \times 7.4 = 74{,}070$ in. lb., and the bending stress $(74{,}070 \times 32)/(\pi \times 5.5^3) = 4500$ p.s.i. Combined in



The Arctic Ice Machine Co., Canton, Ohio

Fig. 165. Integral Design of Crank Arm and Crank Pin.

maximum shear according to formula (3), page 12, we have an equivalent tensile stress equal to $\sqrt{4500^2 + 4 \times 3000^2} = 7500$ p.s.i. The bending stress changes sign as the crank moves from inner to outer dead center, and the torque stress changes sign when the flywheel carries the reciprocating parts over dead center. For this reason, the stress may be regarded as of reasonable amount. If it is desired to have the shaft of the same diameter through the bearing and the arm, then the moment arm of the bending moment should be taken to the center of the bearing. The diameter d_1 of the boss at the pin may be taken equal to twice the pin diameter d_r for forged cranks or steel castings, and somewhat more than this for cast iron.

After the hub diameters at the shaft and at the pin have been determined, the sides of the crank arm can be drawn and the widths b at the sections Z-Z and Y-Y (Fig. 163) scaled. Suppose the width at Z-Z to be 8 in. and at Y-Y 7.5 in. Tenta-

tively we may assume the thickness t of the arm to be 1.5 in. The moment arm x is then $1.66 + 0.25 + 0.75 = 2.66$ in.

At dead center, the maximum bending stress will occur in the section Y-Y, and this stress, s_{b_1}, is $(9820 \times 2.66 \times 6)/(7.5 \times 1.5^2) = 9300$ p.s.i. In addition there is a direct stress $9820/(7.5 \times 1.5) = 875$ p.s.i. These stresses are combined directly and their sum is 10,175 p.s.i. This stress might be regarded as high for conservatively designed steam engines, since the stresses change sign between the two dead centers and there may be some impact. For Diesel engines, such stresses would be considered permissible simply because, with lower stresses, the necessary dimensions would be difficult to accommodate.

In the position where the connecting rod and the crank arm form an angle of 90 deg. with one another, we have the following situation. On the section Z-Z, there is a bending moment $10{,}010(9 - 5.375) = 36{,}500$ in. lb., which results in a bending stress $(36{,}500 \times 6)/(1.5 \times 8^2) = 2270$ p.s.i. There is also a torque $10{,}010 \times 2.66 = 26{,}700$ in. lb. To determine the resultant stress from this torque, we may use formula (27), page 23. We have the ratio of the long to the short side equal to $8/1.5 = 5.33$, and the constant α from Table 5 $= 0.293$. The torsional shear stress is then $26{,}700/(0.293 \times 8 \times 1.5^2) = 5100$ p.s.i. This stress occurs in the middle of the long side and does not combine with the maximum bending stress which occurs along the short edge. In the middle of the short edge, there is, however, a smaller torsional stress, which with the constants in Table 5 amounts to $0.745 \times 5100 = 3800$ p.s.i. This stress, combined in maximum shear with the bending stress, gives an equivalent tensile stress of $\sqrt{2270^2 + 4 \times 3800^2} = 7900$ p.s.i. This stress is less serious than the equivalent tensile stress at the middle of the long side, which is $2 \times 5100 = 10{,}200$ p.s.i. In case it is considered that these stresses are too high, it should be pointed out that the equivalent deformation stress is only $\sqrt{3 \times 5100^2} = 8800$ p.s.i., which is not unreasonable (see formula (6b), page 13). There is, however, also a transverse shear stress which at the middle of the long side has a maximum value according to formula (13), page 16, of

$$(10{,}010 \times 3/2) \div (1.5 \times 8) = 1250 \text{ p.s.i.}$$

This stress should be added to the torsional shear stress, so that the total shear stress is 6350 p.s.i., which is equivalent to a tensile stress of 12,700 p.s.i. under the maximum shear theory, or to 11,000 p.s.i. under the deformation theory.

Both of these stresses are excessive, and the thickness might better be increased to 2 in. It should be remarked that strengthening by means of U-sections or webs is not of as much advantage in torsion as it is in bending, the percentage increase in torsional strength being roughly equal to the percentage increase in perimeter (see page 24). It is therefore better to thicken the web somewhat.

Since the torsional stress is greater but the bending stress is less on section Y-Y than on the section Z-Z, the arm should be checked also at the Y-Y section.

135. Center Cranks. The most common application of center cranks today is undoubtedly in automotive and other multi-cylinder combustion engines. The exact computation of the stresses in multi-throw crank shafts is exceedingly complicated, and may be illusory if the stiffness of the frame is not sufficient to eliminate appreciable distortions. As we shall see later, there are also stresses caused by vibrations that may be more serious than the stresses caused by direct

loads. For these reasons, an approximate method of computation of these stresses is sufficient.

Güldner * computed the stresses on the assumption that the multi-throw shaft is made up of stiff parts, hinged at the bearings; he found that the stresses so computed as a rule appear to be greater than stresses computed more exactly, for instance, by means of deformation-work formulas derived by Max Ensslin.† Güldner's method of computation is therefore safe, and means that no bending moments are assumed to be transmitted through the bearings, except possibly the bending moment due to an overhung flywheel. We can therefore illustrate the method of computing center cranks by means of a single throw with an overhung flywheel. The engine may be assumed to be vertical, so that the bending moments from the piston forces and from the flywheel weight are nearly in the same plane. To simplify matters, we shall assume that they are actually in the same plane, and that the crank arms are at right angles to the shaft axis. This latter condition would be true of heavy stationary or marine engine crank shafts, finished all over, but it would not be true of automotive crank shafts.

Example. Assume a steam engine running at 150 r.p.m. with a stroke of 14 in. and hence a crank arm of 7 in. The overhung flywheel (Fig. 166) weighs 5000 lb. and the rod force F is 10,000 lb. at dead center.

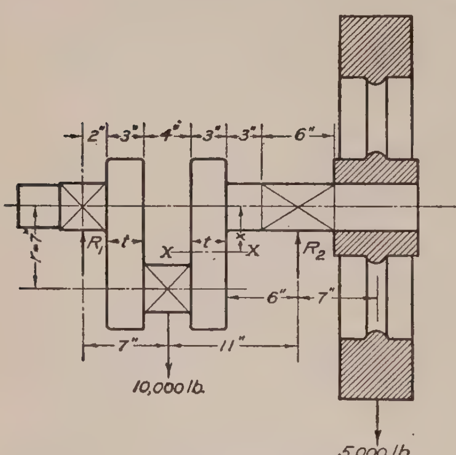


Fig. 166. Center Crank.

To proceed with this problem some tentative dimensions will first be assumed in order that the bearing reactions may be calculated. We will assume, for instance, that the piston force is divided equally on both bearings, 5000 lb. on each, and that the whole flywheel weight is on the right bearing. The total load on this bearing is then 10,000 lb.

If we assume that the left bearing has $l/D = 1$, with a bearing pressure of 300 p.s.i. (see Table 30, page 235) the diameter will be $\sqrt{5000/300} = 4.08$, say 4 in. With a bearing pressure of 1000 p.s.i. (see Table 30) on the crank pin and a pin with $l/D = 1$, the diameter would be $\sqrt{10{,}000/1000} = 3.16$ in. With 400 p.s.i. on the right bearing and a ratio of length to diameter of 1.5, the diameter of the right bearing would be $\sqrt{10{,}000/(1.5 \times 400)} = 4.08$ in. Usually all bearings are given the same diameter, which in this case would not be less than 4 in. The crank arms

* *Verbrennungskraftmaschinen*, 3rd German ed., p. 204.
† *Mehrmals Gelagerte Kurbelwellen*, Stuttgart, 1902.

should have a thickness t of 0.6 to 0.75 times the crank-pin diameter, let us say in this case 3 in. Assuming 3 in. between the right bearing and the crank arm for eccentrics or gears, we get the tentative dimensions shown in Fig. 166.

If the engine is double-acting, the piston force will be directed upward as well as downward, and the resulting bearing reactions and bending moments must be computed for both cases.

With the piston force down, we have

$$R_1 = \frac{10{,}000 \times 11 - 5000 \times 7}{18} = 4150 \text{ lb. (upward)},$$

$$R_2 = \frac{10{,}000 \times 7 + 5000 \times 25}{18} = 10{,}850 \text{ lb. (upward)}.$$

With the piston force upward, we have

$$R_1 = \frac{10{,}000 \times 11 + 5000 \times 7}{18} = 8050 \text{ lb. (downward)},$$

$$R_2 = \frac{10{,}000 \times 7 - 5000 \times 25}{18} = 3050 \text{ lb. (upward)}.$$

If we assume that this crankshaft is used in a steam engine with a cut-off such that the reactions just computed obtain even in the position with connecting rod and crank arm at 90 deg. to one another, the most serious combination of moments on the crank pin will occur in the 90 deg. position. There is then at the middle of the crank pin, when the piston force is upward, a bending stress

$$s_b = \frac{8050 \times 7 \times 32}{\pi 4^3} = 9000 \text{ p.s.i.}$$

and a torsional shear stress

$$s_s = \frac{8050 \times 7 \times 16}{\pi 4^3} = 4500 \text{ p.s.i.}$$

Combined in maximum shear these stresses result in an equivalent tensile stress

$$\sqrt{9000^2 + 4 \times 4500^2} = 12{,}700 \text{ p.s.i.},$$

and an equivalent deformation stress

$$\sqrt{9000^2 + 3 \times 4500^2} = 11{,}900 \text{ p.s.i.}$$

There is also a maximum transverse shear stress at the neutral axis of

$$(4/3)(8050 \times 4)/(\pi \times 4^2) = 850 \text{ p.s.i.}$$

(see page 16), but this is small in amount and does not occur at the same place as the bending stress.

The stresses found are of a magnitude that might be permitted in Diesel engines, but would not as a rule be permitted in conservatively designed steam engines, unless the crankshaft is made of alloy steel.

It may easily be verified that the moments just investigated are the most serious ones occurring on any one of the pins or journals, and since these journals are all of the same diameter, the stresses just found will be the highest stresses occurring in any of them.

The stresses in the left crank arm may be computed exactly as those in the side crank on page 171, except that the reaction R_1 takes the place of the piston force. It is

usual to compute the bending moment to the middle of the crank pin. In the 90 deg. position it is equal to R_1r, r being the length of the crank arm. The torsion moment is $R_1(l + t)/2$, where l is the width of the bearing and t the thickness of the arm. The width of the arm should be considerably greater than the pin diameter, as breakages at the pin have occurred when the arms are no wider than the pin diameter. In our case, with a pin diameter of 4 in., the arm should be at least 5 in. wide.

The bending moment on the right arm at any section X-X at the distance x from the shaft axis is $R_1x + F(r - x) = Fr - x(F - R_1)$. Since F is greater than R_1, this expression will manifestly have a maximum value equal to Fr, where x equals zero, that is, at the shaft axis. At this point the torsion moment in the 90 deg. position is equal to $R_1(2 + 3 + 4 + 1.5) - F(2 + 1.5)$. The principle to be remembered is that the moments to a certain point of the shaft can be computed either from the forces to the left of that point or from the forces to the right, but not from forces on both sides. The fact that the shaft is bent does not change this rule.

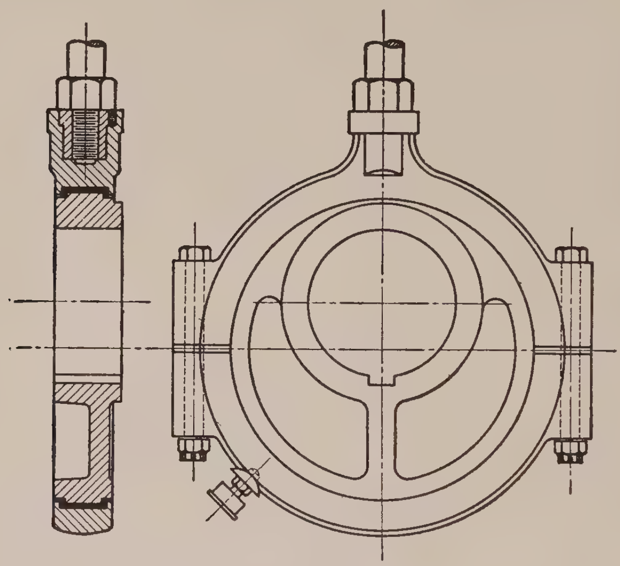


FIG. 167. ECCENTRIC.

The probable heating of the bearings should be checked by computing the pV-values for average pressure. If the pressure variation is considerable, the actual pressure diagrams must be used to obtain the average pressure. In the example we have been considering, it may be assumed that under emergency conditions the engine will be operated with almost 100 per cent cut-off. On this assumption, we may take the average of the bearing reactions as already computed. For instance, for the right bearing the average pressure is $(10{,}850 + 3050)/(2 \times 6 \times 4) = 290$ p.s.i. The circumferential velocity is $\pi \times 4 \times 150/12 = 157$ ft. per min. Hence $pV = 45{,}500$ ft. lb. per min., which is quite reasonable. On the crank pin, the average pressure would be $10{,}000/(4 \times 4) = 625$ p.s.i., and the pV-value is $625 \times 157 = 98{,}000$.

This is a very conservative value. If the value should exceed that of 133,000 listed in Table 31 for good cooling conditions, the question is whether the high

value is due to high pressure or to high speed. If the latter, it may be permissible, since high speed means that the pin passes rapidly through the air or through an oil spray and is effectively cooled. For locomotive crank pins, as much as 370,000 is listed. High pV-values due to high pressures are far less permissible. Obviously high speed does not improve the cooling conditions of the stationary bearings.

136. Eccentrics. When the radius of a crank motion is so small that the pin would not fall outside the shaft, the crank is converted to an eccentric and the strap of the eccentric, corresponding to the large end of the connecting rod, contains both the main shaft and the crank arm, as shown in Fig. 167. Eccentrics are not ordinarily used to impart motion to the shaft, but are used for valve drives, etc. Due to the large bearing diameter, the eccentric disc can be kept quite narrow for the force ordinarily transmitted. Space does not permit us to do more than furnish an illustration of the eccentric design (Fig. 167).

PROBLEMS

1. A mild steel tie rod has an effective length of 5 ft., 20 in. of which is threaded. It is subjected to a tensile load of 2000 lb., suddenly applied. Determine the necessary standard NC thread to limit the calculated induced stress to 8000 p.s.i. If the rod body diameter is the same as the outside thread diameter, determine the elongation. What is the elongation if the body diameter is reduced to the root diameter of the threads?

2. A 3/4 in. diameter tie rod, of mild steel, is to support a pull of 5000 lb. Design a strap-steel clevis on the basis of the maximum stress induced in the rod.

3. An offset link is made in the section of a T, 2 in. overall along each axis, with walls 3/4 in. thick. The line of action of tne tensile load is along the top of the T. The link is of cast iron and should have a factor of safety of 5. Determine the permissible load.

4. A rectangular tie rod, 12 ft. 6 in. long, in a truss carries an operating tensile load of 40 kips (40,000 lb.). Determine the dimensions of the cross-section and eye to limit the stress to 12,000 p.s.i. During erection, the same rod must withstand a compression load of 25 kips. Is the original design safe?

5. A piston rod on a 14 in. by 18 in. horizontal pump is made of Monel metal. The piston acts against a pressure of 500 p.s.i. Determine the necessary diameter of the piston rod if the unsupported length is 32 in. If the piston weighs 200 lb., what is the vertical deflection of the unsupported piston relative to the cross head?

6. Determine the necessary diameter of a round connecting rod for the pump mentioned in Problem 5 if the pressure is constant throughout the stroke. The ratio of connecting rod to crank length is 4. What is the maximum bending moment of the crank arm?

7. Design and make sketches for a forged connecting rod for a 24 in. by 30 in. steam engine. The connecting rod must be 75 in. long, H- cross-section, and is to transmit a maximum thrust of 120 p.s.i. of piston area at 100 r.p.m. The crank pin is 5 in. in diameter and the wrist pin 3½ in. The crank end is to be made open with a standard-type bearing cap. The wrist-pin end of the rod should have wedge adjustment for wear. Calculate the size of cap bolts, thickness of the cap, and check the H-section at the mid-point of the rod.

8. A Diesel-engine connecting-rod cap is held in place by two special bolts with standard NF threads. Select the necessary size of bolts to limit the calculated stress to 20,000 p.s.i. under a load of 9000 lb. If the pin diameter is 2½ in., and the distance between the bolt centers is equal to the pin diameter plus the bolt diameter plus 1/2 in., calculate the necessary thickness of the bearing cap of S.A.E. 2330 steel forging. The cap is 3½ in. wide.

9. The maximum connecting-rod force on a steam engine, running at 90 r.p.m., is 15,000 lb. Determine the necessary size of the overhanging crank pin. Determine the diameter of the crankshaft journal if the stroke is 36 in. and the distance between the center lines of bearing and pin is 14 in.

10. Determine for the engine of Problem 9 the dimensions of the outside crank arm for the condition of crank and connecting rod when perpendicular to each other. Crankshaft hub diameter is 14 in., crank-pin hub diameter, 7½ in. Distance x in Fig. 163 is 4 in.

11. Calculate the dimensions of the crank arm of Problem 10 at the dead-center position for the load of 15,000 lb.

CHAPTER 10

CAMS AND RATCHETS

137. Introduction. Cams are the most convenient means of producing movements of an irregular or complex character not easily obtained by linkages. With the *peripheral plate* (*disc*) type of cam, motion is imparted to the follower by the action of the outside edge of the cam. The follower may be of the roller type (Fig. 168) or of flat-

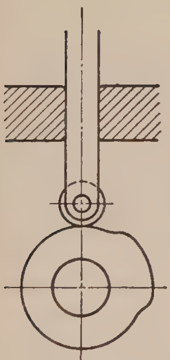


FIG. 168. PERIPHERAL PLATE CAM WITH ROLL FOLLOWER.

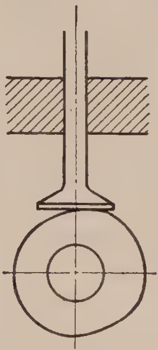


FIG. 169. PERIPHERAL PLATE CAM WITH FLAT-FACE FOLLOWER.

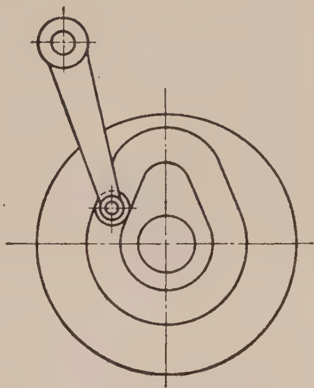


FIG. 170. FACE CAM WITH ROLL FOLLOWER.

face type (Fig. 169). In Fig. 170 a *face* cam is shown, in which either a projecting point or a roller follows a groove in the side of a revolving plate. The cylindrical type of cam (Fig. 171) transmits motion to its follower by means of grooves cut in its cylindrical surface. In the barrel type of cam (Fig. 172), the follower is actuated by formed plates attached to the surface of a cylinder rather than by grooves cut in its periphery. An end cam imparts motion to its follower by the end surfaces of a cylinder, while the follower of a spherical cam moves in a spherical path. In the toe and wiper-cam shown in Fig. 173, the actuating cam is given an oscillating, rather than a rotating motion.

In many cases there is little need to consider the forces and accelerations produced in the cam motion. There are cases, however, where trigger actions or rapid movements are necessary, while in others,

smoothness of action is essential, as excessive accelerations or impacts would be destructive to the parts. Combustion-engine valve motions provide a good example. If impacts were permitted in engines running

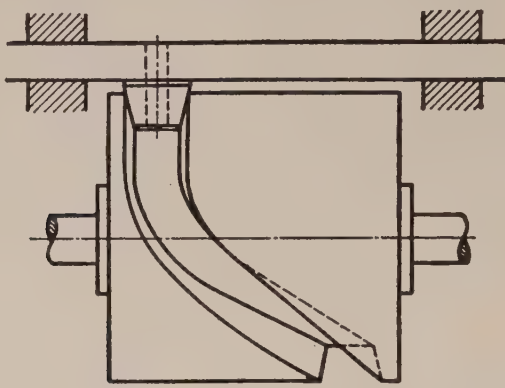


Fig. 171. Cylindrical Cam.

at 3000 to 10,000 revolutions per minute, serious results would soon occur. It is therefore necessary to investigate various cam actions from the point of view of the accelerations and forces produced.

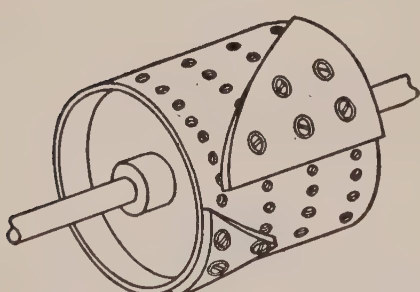


Fig. 172. Barrel Cam.

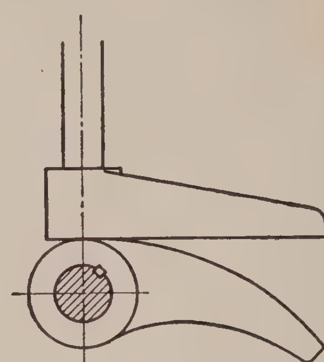


Fig. 173. Toe and Wiper Cam.

Some types of cams move the follower through its entire cycle with positive motion. This is true of all groove-type cams. Others move the follower in only one direction, the return motion being controlled by a spring, or by the force of gravity, as in peripheral cams and end cams. In such cases the returning force, whether it be a spring force or gravity, must be sufficient to produce the accelerations determined by the cam outline; otherwise the follower will not be in contact with the cam and the resultant motion will be irregular.

138. Cam Outlines, Accelerations, and Pressure Angles. If the function of a cam were to raise a follower from the level 1-1 (Fig. 174) to the level 2-2, this could be done by sliding the bar MN endwise in the direction of the arrow, but the acceleration would be theoretically infinite at the beginning and end of the motion. If friction is disregarded the force between cam and follower would be normal to the outlines at the point of contact, that is, in the direction AC. The angle CAD is called the *pressure angle.* The force BA is an undesirable side thrust which increases in magnitude with any increase in the pressure angle.

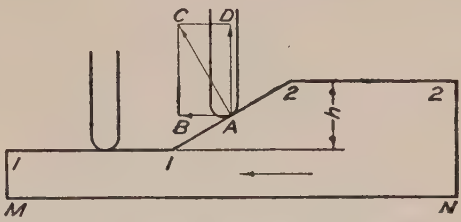


FIG. 174. FORCE DIAGRAM OF PLATE CAM AND FOLLOWER.

The acceleration at the beginning and the retardation at the end of the lift may be reduced by curving the path, as shown in Fig. 175, the radius usually being taken equal to the lift h. A *uniform acceleration* or *retardation* of the follower is obtained if the path is made parabolic and the slide or cam moves with uniform velocity. This is obvious, since the distance s moved in time t with a uniform acceleration a is $at^2/2$, where s is the lift of the cam and t in this case is proportional to the horizontal distance traveled by the slide MN.

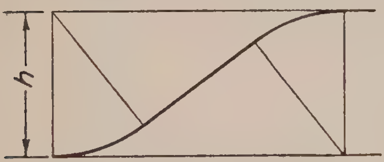


FIG. 175. MODIFIED UNIFORM MOTION.

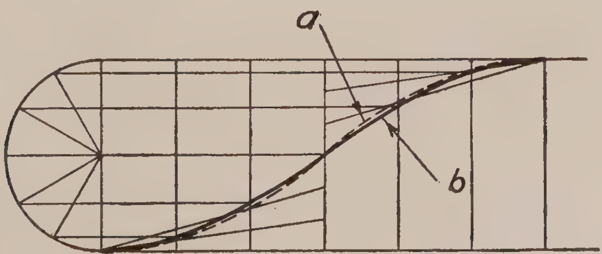


FIG. 176. COMPARISON BETWEEN UNIFORMLY ACCELERATED AND UNIFORMLY RETARDED MOTION (CURVE a) AND SIMPLE HARMONIC MOTION (CURVE b).

The parabolic path is shown in dotted line a in Fig. 176, the first half of the lift being uniformly accelerated, the second half being uniformly

retarded. The parabolic outline may be obtained by actually computing the successive positions, or by the construction shown in the figure. The lift line at the center of the diagram is divided into equal parts and straight lines are drawn from these division points to the corners of the diagram, as shown. At the intersections of these lines with corresponding equally spaced ordinate lines, representing the distance traveled by the cam, successive points are located on the parabola.

The maximum pressure angle which occurs at the point of inversion of the lift curve is, however, greater for the parabolic curve than it is for the so-called *crank motion* or *simple harmonic* curve. This latter curve, shown in full line b in Fig. 176, is obtained by projecting the successive crank pin positions for equal angular increments onto corresponding equidistant ordinate lines on the diagram.

139. Lay-Out of Cams. In order to proceed with the lay-out of a cam, data are required for the lift, descent, dwell or rest periods, and the parts of the cycle in which these actions occur.

It is helpful, particularly if the motion is at all complicated, first to lay out a developed diagram on a straight-line basis, as shown in Fig. 177. An important consideration of such a diagram is to make sure

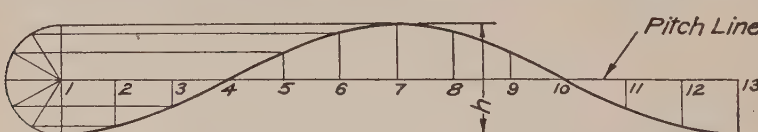


FIG. 177. DEVELOPED DIAGRAM OF CAM MOTION (HARMONIC).

that the pressure angle is not too great. If in the cam diagram the horizontal direction is taken as the x-axis and the vertical as the y-axis, the tangent of the pressure angle at any point is dy/dx, and it is easy to obtain the maximum value for the cam curve under consideration.

For instance, for a constant acceleration curve, we have $y = kx^2$; $dy/dx = 2kx$. Hence the pressure angle increases indefinitely with x and has its maximum value at the point of inflection, where acceleration stops and retardation begins. If at this point y is equal to $h/2$, where h is the lift, we have $h = kl^2/2$, or $k = 2h/l^2$, where l is the base length of the cam cycle. The corresponding value of dy/dx, that is, the maximum value of the tangent of the pressure angle, is $2h/l$, whence $l/h = 2/\tan \beta$, where β is the pressure angle. If the lift h is given, a certain length of base line distance l is necessary, if a certain pressure angle is not to be exceeded. For instance, for $\beta = 30$ deg., l/h is 3.46. The quantity l/h is called the *cam factor*.

For a simple straight-line cam profile (uniform motion), $l/h = 1/\tan \beta$. For a straight line with circular ease-offs having a radius equal to the lift, $l/h = \tan \beta + 1/\tan \beta$. For the harmonic curve, $l/h = \pi/2 \tan \beta$. For a maximum pressure angle of 30 deg. the l/h values are 1.73 for the simple straight line, 2.27 for the straight line with ease-offs of a radius equal to the lift, and 2.73 for the harmonic curve.

It will be noted that the required length of base line for a given maximum pressure is less for the harmonic curve than for the uniform acceleration curve. This is of some importance in view of the fact that in actual cam design the base line is wound up into a circle, and in consequence the cam diameter will be greater for a uniform acceleration curve than for a harmonic curve, if the pressure angle is the same. With the same cam diameter the maximum accelerative forces will be 57 per cent greater for the harmonic curve and will not be uniform, but the side pressures will be less.

It should be noted, however, that the statements just made refer only to the case where the follower moves along a line passing through the center of the cam, as shown at M in Fig. 178. It will be observed that the pressure angle β is considerable. If the center line of the follower is offset as at N, the same lift can be obtained with a very much smaller pressure angle, as the figure shows. In fact, in this case the pressure angle becomes slightly negative.

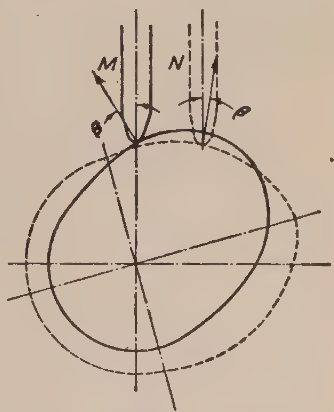


FIG. 178. EFFECT ON THE PRESSURE ANGLE β BY OFFSETTING THE FOLLOWER.

To determine the cam diameter, a *pitch circle* is drawn with a circumference equal to the length of the cam diagram. The pitch circle is taken to go through the point of inflection of the cam-motion curve. For instance, if the maximum permissible pressure angle were set at 30 deg., and a uniform-acceleration cam were used, l/h would be 3.46. Furthermore, if this part of the motion must be completed in one-third of a revolution, the length of the pitch circumference would be $3 \times 3.46 \times h$, and the pitch diameter would be $3 \times 3.46 \times h/\pi$. If h is 1/4 in., the pitch diameter = 0.82 in. A formula for the minimum pitch cam radius would be

$$r = 0.159 \frac{hf}{n}, \tag{1}$$

where $f = l/h$ and n is the fraction of a revolution in which the lift h occurs.

140. Detail Lay-Out of Peripheral Plate Cam. PROBLEM. Lay out a cam outline to lift a reciprocating follower a distance h in 90 deg. of revolution with harmonic motion; return the follower in 90 deg. with harmonic motion, and dwell for 180 deg. The center line of the follower is to pass through the center of the cam shaft.

A cam diagram with harmonic motion is constructed as in Fig. 177 with a base line having a length which will give a low pressure angle (30 deg. or less). Since the lift and return actions are confined to 180 deg., the length of this diagram will be equal to one-half the circumference of the pitch circle. By calculating the required radius, the pitch circle in Fig. 179 is drawn and the distance 1 to 13 on its circumfer-

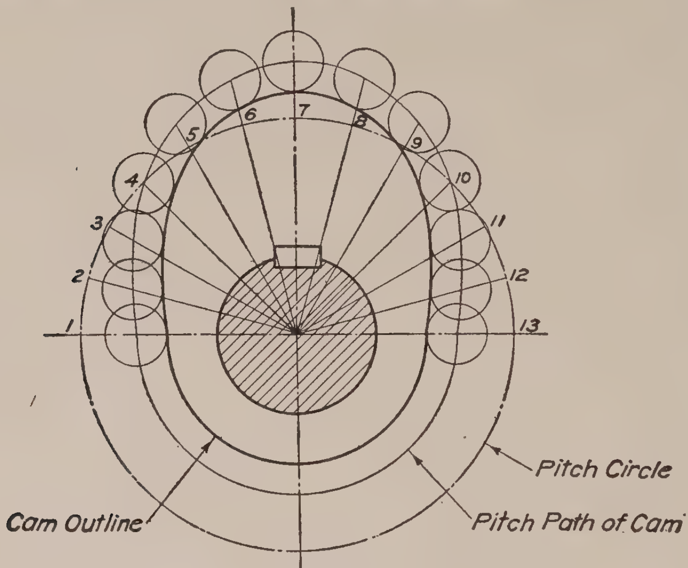


FIG. 179. LAY-OUT OF PERIPHERAL PLATE CAM.

ence is equal to the length of the cam diagram. The lift arc of 90 deg. and the return arc of 90 deg. are each subdivided into six equal parts to correspond with the divisions on the cam diagram. The points on the cam diagram, on either side of the pitch line, are transferred to corresponding radial lines on the cam drawing, as shown by identical numbers in Figs. 177 and 179. A smooth curve through these points establishes the pitch curve of the cam.

Obviously, the cam lay-out may be drawn without the aid of a diagram. In simple cases, this is usually done, but a diagram is a great aid in avoiding mistakes. A diagram is especially useful if the lay-out is to be controlled by a predetermined pressure angle. Simply to make the sequence of events clear, the diagram may be drawn to any scale, and in such cases the various points on the cam lay-out are either computed or graphically constructed.

If, at the point of contact with the cam, the follower ends in a sharp point or knife-edge, the pitch curve as just determined is the actual cam

outline. A construction of this type is impractical, however, as there would be excessive wear. If on the other hand the follower ends in a roller, circles equal in diameter to the roller are drawn with their centers on the pitch curve, and the actual cam outline is the curve tangent to these circles, as shown in Fig. 179. Where the outline is concave, as at 1-2 and 12-13, the radius of the concavity must be greater than the radius of the roller; otherwise the follower will not move along the desired pitch path.

141. Peripheral Cam. Offset Reciprocating Follower. A slight modification of the construction used in Fig. 179 is necessary in case the center line of the follower is offset with respect to the center of the cam shaft.

PROBLEM. Lay out a cam outline (Fig. 180) to lift the offset follower a distance h in 180 deg. with uniform motion, return with uniformly accelerated and retarded motion in 120 deg. and rest 60 deg.

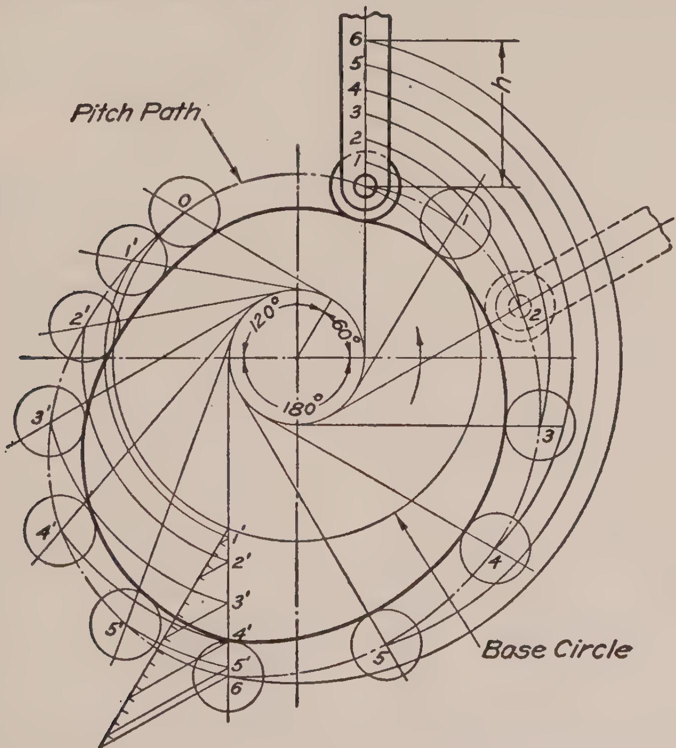


FIG. 180. LAY-OUT OF PERIPHERAL PLATE CAM WITH OFFSET ROLL FOLLOWER.

In this example a direct cam lay-out will be made by omitting the preliminary cam diagram. A *base* circle is drawn, as shown, establishing the nearest position of

the pitch line of the cam to the cam-shaft center. Since the size of the cam is controlled by the base circle, the diameter of the base circle should be large enough to obtain the required motion without causing an excessive pressure angle or producing portions of cam surface with a small concave radius.

For uniform motion, the lift h is divided into any convenient number of equal parts, six in this case, and 180 deg. of cam rotation on the base circle is divided into the same number of equal spaces. The lift points 1 to 6 are transferred with circular arcs to the corresponding lines representing the successive positions the follower will take as the cam is rotated. It should be observed that the center line of the follower is tangent to a circle which has a radius equal to the offset of the follower. In constructing the successive positions of the follower, tangents are drawn from this circle to the division points on the base circle.

It is convenient, in subdividing the distance h for the uniformly accelerated and retarded motion, to make use of the equation $S = at^2/2$. By substituting values for t, we can easily prove that, with equal units of time, the distance traversed during each successive unit will be in the ratio 1, 3, 5, 7, 9, etc. Therefore, if six units of time are considered of which three are for acceleration and three for retardation, h will be subdivided in the proportion of 1, 3, 5, 5, 3, 1. These division points numbered 1′, 2′, 3′, 4′, etc., are easily laid off by proportional division, as shown in the figure, and are then transferred to the corresponding follower positions. A circular arc drawn from point 0 to the starting point provides the 60 deg. rest period. With the pitch curve of the cam completed, the actual cam outline is the curve drawn tangent to the roller positions.

In the lay-out of many cams, as has been shown, it is often convenient to consider the follower as rotating about the cam, while the cam itself is assumed to be stationary. When considered in this manner, one of the positions the follower would take is shown in dotted lines in the illustration. Obviously, the follower is not drawn in at the various positions, but only in the portion in contact with the cam, which in this case is the roller.

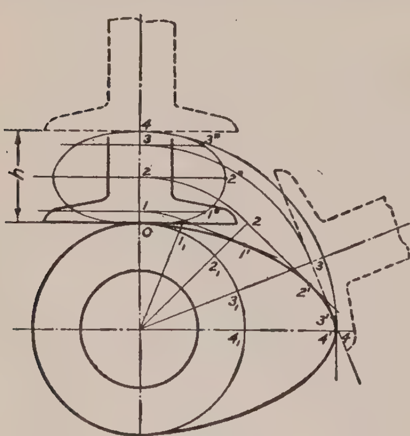


FIG. 181. LAY-OUT OF PERIPHERAL PLATE CAM WITH FLAT-FACED FOLLOWER.

142. Flat-Faced Follower. In many cases the follower is of the flat-faced or "mushroom" type as shown in Fig. 181. If the follower is to be lifted the distance h with the motion defined by the points 0 to 4, the graphical construction proceeds in the same manner as in the preceding examples. Through each successive point in the lay-out, however, a straight line is drawn to represent the position of the flat face of

the follower. A smooth curve tangent to these lines determines the actual cam outline.

With a flat-faced follower it is particularly important to ascertain the length of face required to maintain correct contact with the cam. If the distances from the design center lines to the points of contact as 1′, 2′, 3′, etc., are laid off from the original center line of the follower for the various successive lift positions, a curve 1″, 2″, 3″ may be drawn which shows the path of the point of contact. Obviously the face of the follower must be of sufficient length to comprise this curve.

If the follower face is made circular, its radius must be greater than the distance 2-2″ for the whole width of the cam to be covered. If the cam bears centrally on the follower, the surfaces are in pure sliding contact, but this is not necessarily objectionable if there is copious lubrication. A certain amount of rolling action may be introduced by offsetting the cam as in Fig. 182. This offset should not be too great since it produces a tipping moment Fe on the follower.

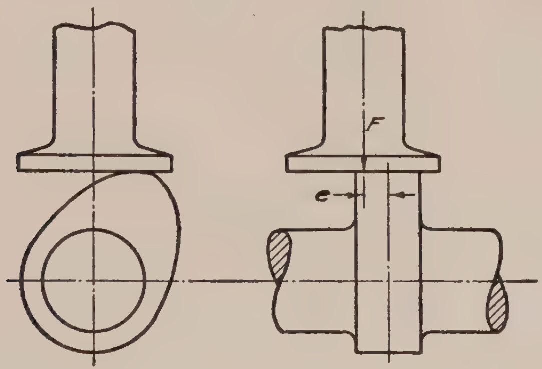


FIG. 182. FLAT-FACED FOLLOWER WITH OFFSET CAM.

A cam for a mushroom follower must be convex throughout. In view of the fact that the contact point moves back and forth, it is difficult in some cases to design a cam that will produce a prescribed movement curve or prescribed accelerations, unless the cam is made so large that the shift of the contact point may be neglected. The minimum cam diameter is reached when three successive construction tangents meet together in a point. A curve of smooth outline can then no longer be drawn and there will be excessive wear at this point. The mushroom cam has been used extensively in valve motions, but the mechanism should be checked for excessive accelerations and inertia forces.

143. Face Cams and Cylindrical Cams. A face cam (Fig. 170) is no different in principle from a peripheral cam, except that in laying out the successive roller positions, as in Fig. 180, an outer as well as an inner tangent curve is drawn, which ensures a positive return motion. The width of the groove should be slightly greater than the diameter of the roller, if free rolling motion is to be attained.

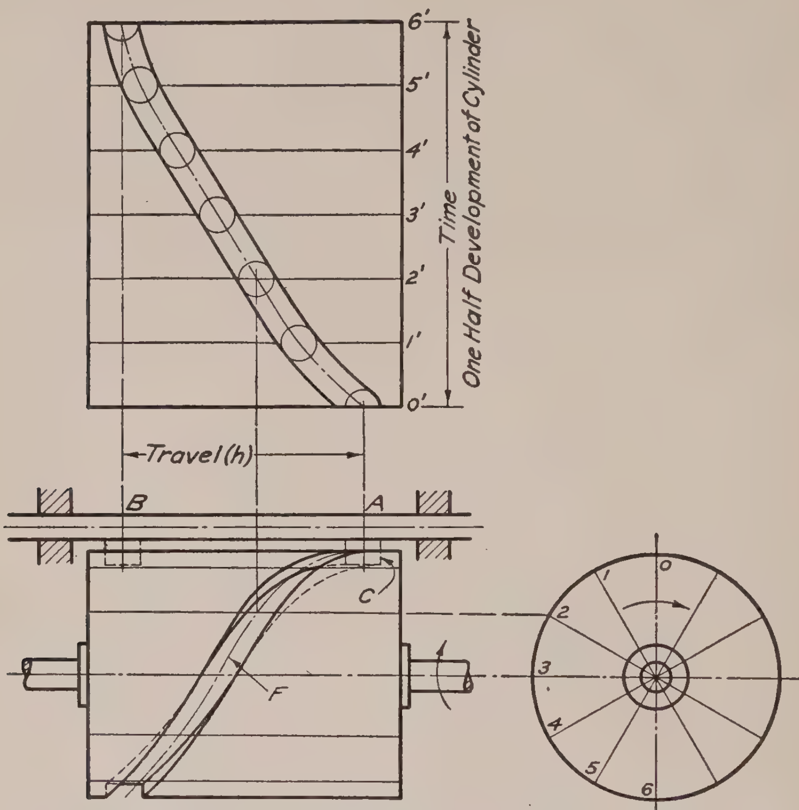


FIG. 183. LAY-OUT OF CYLINDRICAL CAM.

To illustrate the lay-out of a cylindrical cam, the slide follower AB shown in Fig. 183 is to be moved by the roller c a distance h in one-half turn of the cam.

The path of motion of the follower is shown by the curve as a time-travel diagram laid out on a development of one-half of the cylindrical surface. This curve may be a parabola, crank curve, or any other curve that will give the desired motion.

The distances 0′ to 6′ on the development are equal to the six arc divisions 0 to 6 of the circumference in the end view. Points on the pitch path of the cam curve F are found at the intersections of lines projecting points across from the end view and corresponding points down from the curve above.

144. Cams with Oscillating Arm Followers. Thus far we have assumed that the cam follower moves along a straight line. In the following example the cam actuates a follower which *oscillates* about a fixed center.

Referring to Fig. 184, the oscillating arm A is required to transmit motion from the cam to the plunger B. Assume that B is to move through a distance h with crank motion in 150 deg. of cam rotation, then return with the same motion in 150 deg., and dwell for 60 deg.

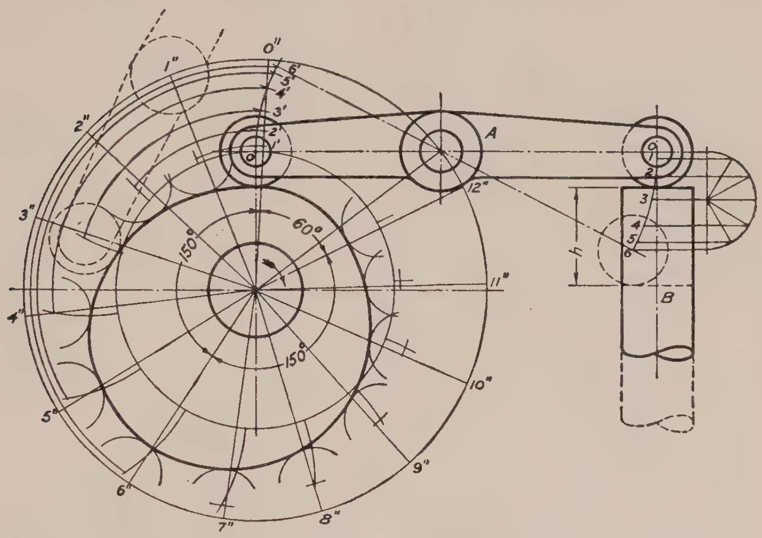


Fig. 184. Lay-Out of Peripheral Plate Cam with Oscillating Arm.

The required movement is laid out at the plunger and the various points 1, 2, 3, etc., are projected through the center of oscillation to find the corresponding positions, 1′, 2′, 3′, etc., of the cam roller at the opposite end of the arm. If the follower arm is considered as moving around the cam, a circle is drawn from the cam-shaft center with a radius extending to the center of oscillation.

This circle is subdivided, as shown, to correspond with the required cycle of events. For instance, 150 deg. of arc is divided into six equal time spaces to correspond to the six divisions of the downward travel of plunger B. By rotating the follower arm around the cam-shaft center, as shown, the various roller positions are located. The cam outline is drawn as a tangent curve to these successive positions.

145. Permissible Load between Cam and Roller. The permissible load between cam and roller is largely governed by the stress in the roller. The limiting allowable pressures are influenced by the character of the load and the lubrication. Güldner recommends a maximum allowable load of 3000 lb. per in. length of contact between cam and roller, and Hart * quotes a test with a load of 5000 lb. per in. without

* Product Engineering, January, 1935, p. 23.

failure. These values are probably high for regular use and values below 1000 lb. per in. are preferable for design. Hart gives formulas with tabulated constants to use in determining permissible loads to limit the stress in hollow rollers. A simple rule, allowing for wear, quoted by Flodin * is

$$d^2l = \frac{FV}{80{,}000}, \text{ for hardened steel,} \tag{2}$$

and

$$d^2l = \frac{FV}{35{,}000}, \text{ for unhardened machine steel,} \tag{3}$$

where d = diameter of roller, in in., l = length of contact surface between roller and cam face, in in., F = total load in lb. between roller and cam, and V = surface velocity of roller in ft. per min. The initial trial value of the roller may be taken as twice the pin diameter. The pin diameter must be sufficient to limit the shear and bending stresses.

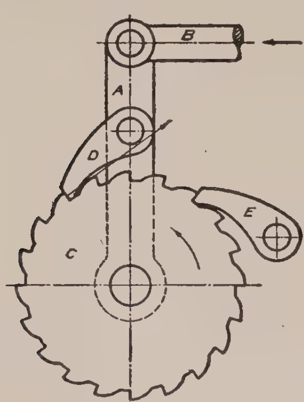


Fig. 185. Simple Ratchet Mechanism.

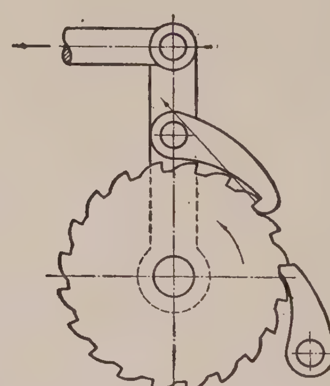


Fig. 186. Ratchet Mechanism with Pawl Arranged to Pull.

146. Ratchet Wheels and Pawls. Ratchet wheels and pawls have found considerable application in certain classes of machinery where an intermittent motion is required, especially in various types of feed mechanisms. By means of this device, reciprocating or oscillatory motion may be converted into an intermittent form of rotary motion.

A simple form of a ratchet wheel and pawl is shown in Fig. 185. The arm A may be oscillated by means of a connecting rod B, motion being imparted to the ratchet wheel C by means of the pawl D engaging

* Machine Design, April, 1932, p. 48.

with the ratchet teeth. The wheel is prevented from turning backwards by means of the click E, pivoted on a stationary fulcrum.

In the arrangement shown, the normal to the tooth surface must pass inside the pivot point, if the turning moment on the pawl is to hold it against the periphery of the wheel and thus tend to prevent it from becoming disengaged. The minimum travel of the pawl must be somewhat greater than the outside pitch of the teeth to allow the pawl to drop freely into the tooth space. Springs or weights are frequently used to hold the pawl against the ratchet.

The pawl may be arranged to pull, in which case the normal to the tooth surfaces should pass outside the pivot point as in Fig. 186. Double-acting pawls (Fig. 187) are used when it is desired to drive the wheel during both forward and backward strokes of the oscillating arm.

In the detail design of the pawl it is well not to have the inner edge too sharp, but to make it somewhat blunt, as shown in Fig. 187. The space in front of the tooth should be relieved to provide clearance for this type of nose. In the factor of safety an allowance should be made for the suddenness with which the load may be applied. If the movement is at all rapid, and if there is a decided over-travel on the return stroke, the sudden engagement really amounts to a serious impact.

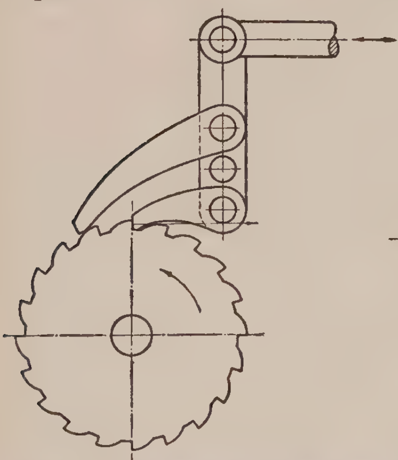


Fig. 187. Ratchet Mechanism with Double-Acting Pawls.

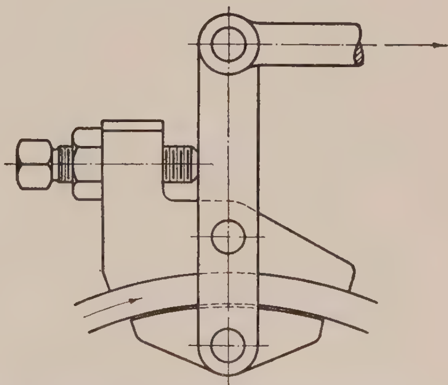


Fig. 188. Friction Ratchet (Shoe Type).

147. Friction Ratchets. There are various forms of ratchets in which the engagement depends principally upon frictional contact. These ratchets engage at any point with smooth, silent action and drive through any angle of rotation. While these properties are very

desirable, the action is not positive and slipping may occur when the parts become worn.

In Fig. 188 a friction ratchet is shown in which shoes grip and turn the band wheel when a pull is applied to the lever in the direction indicated. On the return stroke, the adjusting stop screw prevents the ratchet from turning the band wheel backwards.

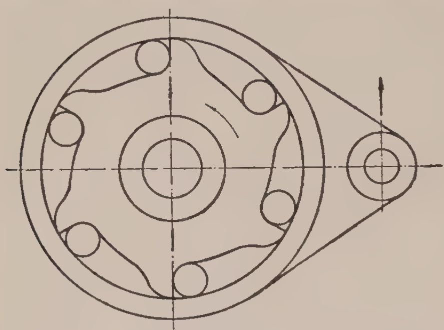


FIG. 189. FRICTION RATCHET (ROLLER TYPE).

In the type shown in Fig. 189 engagement depends upon the wedging action of the balls or rollers between the contact surfaces of the driver and the follower. This principle has found extensive use in automotive applications such as free-wheeling clutches and starting-motor drives.

PROBLEMS

1. A disc cam gives a 1¼ in. roller reciprocating follower an outward motion of 3 in. while turning through 200 deg. The outward motion is constant acceleration and deceleration having a time ratio of 2 to 1. The return is at constant velocity. Pressure angle at pitch circle should not exceed 35 deg. (a) Construct the displacement diagram. (b) Construct the cam. (c) Determine the acceleration and maximum velocity of the follower if the cam turns at 55 r.p.m.

2. A disc cam gives a flat-faced follower an upward motion of 1½ in. while turning through 180 deg. at 800 r.p.m. The follower has constant velocity except for 15 deg. acceleration and 15 deg. deceleration at beginning and end, respectively, with similar return. (a) Construct the cam. (b) The follower weighs 12 lb. What spring force is necessary to keep the follower in contact with the cam?

3. A 1½ in. soft-steel roller follower moves outward 3 in. during 80 deg. rotation of the cam at 100 r.p.m., dwells during 40 deg. and then returns. Each motion is with equal constant acceleration and deceleration. The cam is of soft steel and has a base circle of 8½ in. Follower parts weigh 120 lb. (a) Construct the displacement diagram. (b) Construct the cam. (c) What is the maximum bending force on the follower? (d) How wide should the cam be made?

4. A 1 in. roller oscillating follower is pivoted about a center located 8 in. to the right and 1 in. above the center of the cam. Construct the cam to give an outward swing of 20 deg. with simple harmonic motion during 120 deg. of cam rotation, dwell for 120 deg., and return with harmonic motion.

5. A flat-faced oscillating follower is pivoted about a point 6 in. to the right of the cam center and is horizontal in its lowest position. The normal to the contact surface from the pivot center is 1 in. long. The outward travel is 30 deg. with constant velocity during 170 deg. of cam travel. After a dwell of 30 deg. the follower returns to the initial position with simple harmonic motion. Construct the cam.

CHAPTER 11

SHAFTS, KEYS, AND PERMANENT COUPLINGS

148. General Remarks. The term *shaft*, or *shafting*, is applied to those machine members, mostly cylindrical and solid in cross-section, although sometimes hollow, used to transmit power or motion and to furnish support for rotating elements, such as pulleys, gears, or flywheels. The shaft of a steam turbine and line shafting in a factory are typical examples.

The term *axle* may be applied to the pin or shaft on which a wheel is mounted, or to the transverse member used to connect opposite wheels of a vehicle. In general, axles are subjected to transverse loads and are stressed principally in bending. A railway-car axle is a representative example.

A *spindle* is a short revolving shaft that imparts motion either to a cutting tool or to a piece on which work is to be performed. Drill press and lathe spindles are examples of each type.

149. Shaft Design and Allowable Stresses. In determining the bending moment, the shaft is considered as a beam, freely supported by the bearings. As the distribution of the bearing loads is somewhat uncertain and depends on the manner in which the shaft and bearings are fitted together, and on the deflection of the shaft, it is usually assumed that load reactions are concentrated at the middle of the bearings. This assumption is safe, as any deflection will tend to concentrate the load at the edges of the bearings, thereby reducing the bending moment.

Strength formulas applying to solid circular shafts in torsion, bending, and combined torsion and bending have already been discussed in § 9, formulas (7 to 11). In the use of these equations, a suitable factor of safety must be established in order to determine the permissible working stress. Consideration must therefore be given to the kind of material used and the nature of the loads. In this instance, we shall refer to the recommendations of the *Code for the Design of Transmission Shafting* sponsored by the A.S.M.E.

Transmission shafting for mill or factory purposes is usually purchased in the open market under the trade name of "Commercial Shafting" without any definite specifications for physical or chemical properties. It may be either open hearth or Bessemer low carbon steel

finished by cold drawing, cold rolling, turning, or grinding. The ultimate tensile strength may range from 45,000 p.s.i. for hot rolled and turned low carbon steel to upwards of 70,000 p.s.i. for cold-finished low carbon steel. Corresponding stresses at the elastic limit would be about 22,500 and 55,000 p.s.i. Grades of steel between these limits are likely to be found in commercial shafting.

The stresses recommended by the code are as follows:

s_t = 16,000 p.s.i. (max. permissible tensile or compressive stress),
s_s = 8000 p.s.i. (max. permissible shear stress).

For shafting purchased under definite physical specifications, s_t is taken equal to *60 per cent* of the elastic limit in tension but not more than *36 per cent* of its ultimate strength. The *maximum shear stress*, s_s, is taken equal to $s_t/2$, which gives a design stress *equal to 30 per cent of the elastic limit in tension and not to exceed 18 per cent of the ultimate strength in tension. The above stresses are to be reduced 25 per cent for shafts with keyways.*

150. Shock and Fatigue Factors. Under certain conditions of loading, the stresses just given must be further reduced. In order to simplify the calculations, especially when a shaft is subjected to combined torsion and bending, a *shock and fatigue factor* is used in which

K_t = numerical combined shock and fatigue factor to be applied to the computed torsional moment,
K_b = numerical combined shock and fatigue factor to be applied to the computed bending moment.

These factors have the following values:

	K_b	K_t
STATIONARY SHAFTS		
Gradually applied load	1.0	1.0
Suddenly applied load	1.5 to 2.0	1.5 to 2.0
ROTATING SHAFTS		
Gradually applied or steady load	1.5	1.0
Suddenly applied loads, minor shocks only	1.5 to 2.0	1.0 to 1.5
Suddenly applied loads, heavy shocks	2.0 to 3.0	1.5 to 3.0

The following examples will illustrate the use of the code as applied to problems in shafting.

151. Shaft Subjected to Torsion. EXAMPLE (a). A section of commercial shafting $2\frac{15}{16}$ in. in diameter is joined to another shaft by means of a keyed flange coupling. If the shaft transmits power supplied from a 2-cylinder gasoline engine and no bending action is present, what horsepower can be transmitted at 250 r.p.m.?

Referring to § 9, page 14, we find

$$s_s = \frac{16M_t}{\pi D^3}, \qquad \text{or} \qquad D = \sqrt[3]{\frac{16M_t}{\pi s_s}},$$

where s_s is the shear stress in p.s.i., M_t the torque in in. lb., and D the shaft diameter in in.

Often it is desired to express the shaft capacity as horsepower rather than as torque. If we assume a force F applied at the periphery of the shaft D, so that $Fr = M_t$, where r is the radius of the shaft in in., then the work done in one revolution $= 2\pi Fr/12$ ft. lb. If n represents the number of revolutions per minute, and hp the horsepower transmitted, then

$$hp = \frac{2\pi Frn}{33{,}000 \times 12} = \frac{2\pi M_t n}{33{,}000 \times 12},$$

$$hp = \frac{M_t n}{63{,}024}, \quad \text{or} \quad M_t = \frac{63{,}024 hp}{n}. \tag{1}$$

Combining equations and applying factor K_t, we get

$$hp = \frac{s_s \pi D^3 n}{16K_t \times 63{,}024}.$$

According to the code, the stress s_s for a shaft with keyway would be 8000 p.s.i. less 25 per cent or 6000 p.s.i. Power from a 2-cylinder gasoline engine would be impulsive, producing minor shocks; therefore, K_t may be taken as 1.0 to 1.5. If we select 1.5 as this factor, then, by substitution,

$$hp = \frac{6000\pi(2\frac{15}{16})^3 \times 250}{16 \times 1.5 \times 63{,}024} = 79.$$

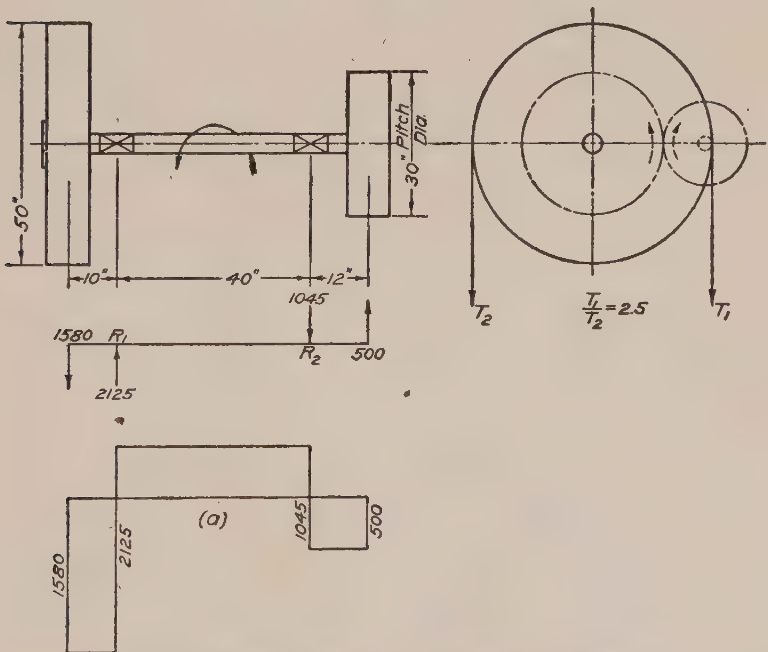


FIG. 190. SHAFT SUBJECTED TO TORSION AND BENDING.

152. Shaft Subjected to Torsion and Bending. EXAMPLE (b). In Fig. 190, 25 horsepower is to be transmitted through the shaft at 150 r.p.m. The tooth

reaction on the gear acts vertically upward as shown. The flywheel delivers the power vertically downward through a belt, the tension on the tight side (T_1) being 2.5 times that of the slack side (T_2). The gear weighs 200 lb., the flywheel 600 lb., and both are keyed to the shaft. A 0.40 per cent carbon steel is to be used that has an ultimate tensile strength of 70,000 p.s.i. and an elastic limit of 40,000 p.s.i. Loads may be considered suddenly applied, but with minor shocks only. The weight of the shaft is assumed to be negligible in comparison to the other loads.

From formula (7), § 9,

$$\frac{\pi D^3 s_{s\,\max}}{16} = \sqrt{M_b^2 + M_t^2},$$

where M_b is the bending moment and M_t the torque. Applying factors K_t and K_b, and transposing

$$D = \sqrt[3]{\frac{16}{\pi s_{s\,\max}} \sqrt{(K_b M_b)^2 + (K_t M_t)^2}}. \tag{2}$$

The maximum shear stress with allowance for keyways equals $0.30 \times 40{,}000 \times 0.75 = 9000$ p.s.i. The torsion moment $M_t = 63{,}024 \times 25/150 = 10{,}500$ in. lb. The gear tooth reaction $= 10{,}500/15 = 700$ lb. (upward). Since the weight of the gear is 200 lb., the net load $= 700 - 200 = 500$ lb.

The effective belt pull equals $T_1 - T_2$, or $10{,}500/25 = 420$ lb. Since $T_1/T_2 = 2.5$, $T_1 = 700$ lb. and $T_2 = 280$ lb. The total downward pull of the belt plus the weight of the flywheel $= 700 + 280 + 600 = 1580$ lb.

To find the reactions at the bearings, moments are taken about R_1 and R_2 (Fig. 190). We have $40R_2 = 10 \times 1580 + 52 \times 500$, or $R_2 = 1045$ lb. and $R_1 = 2125$ lb.

The maximum bending moment occurs at the left bearing, as shown in the shear diagram *a*. Hence, $M_b = 1580 \times 10 = 15{,}800$ in. lb.

If we take as shock and fatigue factors $K_b = 2.0$ and $K_t = 1.5$, we have, substituting in formula (2),

$$\begin{aligned} D &= \sqrt[3]{\frac{16}{\pi 9000} \sqrt{(2.0 \times 15{,}800)^2 + (1.5 \times 10{,}500)^2}} \\ &= 2.71, \text{ say } 2\tfrac{3}{4} \text{ in.} \end{aligned}$$

If a lower safety factor is permissible, the size of the shaft may be computed according to the deformation theory, formula (8), § 9, page 14, in which

$$\frac{\pi D^3 s_{s\,\mathrm{def}}}{16} = \sqrt{M_b^2 + 0.75 M_t^2}.$$

Then

$$\begin{aligned} D &= \sqrt[3]{\frac{16}{\pi 9000} \sqrt{(2.0 \times 15{,}800)^2 + 0.75(1.5 \times 10{,}500)^2}} \\ &= 2.67, \text{ say } 2\tfrac{11}{16} \text{ in.} \end{aligned}$$

It is apparent from formula (8), § 9, that the greatest reduction in size occurs when M_t is relatively high as compared to M_b. In this problem, M_b happens to be the larger; hence there is no appreciable difference in size.

153. Effect of the Weight of the Shaft. In important installations, especially where heavy shafts are involved, *the weight of the shaft*

should be included approximately in the first computation and checked when the shaft size has been determined.

154. Shaft Subjected to Axial Loads. A shaft may be subjected to direct axial tensile or compressive stresses, due to thrust loads, in addition to the stresses resulting from bending and twisting. Axial stresses may be added directly (algebraically) to bending stresses, since both types of loading produce tension or compression. This resultant tensile (or compression) stress should then be combined with the torsional shear stress to find either the maximum tensile stress or the maximum shear stress.

155. Mill Shafting. *Line shafts* are used extensively to deliver power supplied from a motor or an engine to various machines located at different points along its length. The power transmitted, therefore, varies from a maximum at the driving end or *head shaft* to zero at the free end. From a consideration of torsional strength alone, it is apparent that the shaft could be reduced in size at intervals along its length as power is given off. For smaller sizes, this procedure ordinarily is not followed. For larger sizes (3 in. or more), reductions may be made where the saving in cost of the shafting, bearings, hangers, etc., would offset the convenience in erection and maintenance, if all parts were uniform in size and readily interchangeable.

There are applications in practice where a shaft may be stressed in pure torsion only. In the great majority of cases, however, a bending action exists, due to the weight of the shafting, pulleys, or gears, as well as belt pulls, or gear-tooth loads. This bending action may be kept relatively low by placing these units as close to the bearings as possible and by spacing the bearings to give the shaft ample support.

156. Mill-Shafting Design. It is very difficult to determine with any degree of accuracy the magnitude of the loads on line shafting. The power requirements of individual machines are variable, belt tensions are uncertain, and a diversification exists in total power consumption of the various machines when in operation. Such computations are uncertain, and are further complicated by the fact that the shafting, since it has more than two supports, would theoretically require the use of the theorem of three moments. For these reasons, mill shafting is usually computed on the basis of a torsional strength formula, modified to allow for stiffness and such bending loads as usually exist under different types of service. The formulas given in the following table apply to mill shafting and represent common practice for ordinary applications.

TABLE 24

EMPIRICAL SHAFTING FORMULAS

Kind of Service	Load Factors Considered		Horsepower Capacity
	Torsion	Bending	
Transmission shafts in torsion only	$K_t = 1.0$	$K_b = 1.0$	$hp. = \dfrac{D^3n}{50.5}$
Line shafting with limited bending	$K_t = 1.0$	$K_b = 1.5$	$hp. = \dfrac{D^3n}{80.2}$
Head or main shafts with heavy bending loads	$K_t = 1.0$	$K_b = 2.5$	$hp. = \dfrac{D^3n}{133.7}$

In the above table, D is the shaft diameter in in., and n the r.p.m. Factors K_t and K_b are defined on page 192.

157. Line-Shaft Speeds and Bearing Spacing. Line shafts are operated at various speeds, the following being fairly representative of common practice.

Machine shops (general)	120 to 250 r.p.m.
Machine shops (high speed production)	250 to 500 r.p.m.
Wood working shops	250 to 300 r.p.m.
Textile mills	300 to 400 r.p.m.

Excessive bending of the shaft and uneven bearing wear are the factors which limit the distance between bearings of line shafting. The following recommendations of William Sellers & Co. may serve as a guide for the usual applications. These values are approximate and where unusual loads are involved, the span should be reduced accordingly.

TABLE 25

SPACING FOR LINE SHAFT BEARINGS

CENTER TO CENTER DISTANCE IN FT. AND IN.

Diameter of Shaft	Transmission Shaft Stressed in Torsion Only		Line Shaft Carrying Pulleys or Gears and Subjected to Usual Bending Loads	
	1 to 250 r.p.m.	251 to 400 r.p.m.	1 to 250 r.p.m.	251 to 400 r.p.m.
In.	*Ft.-In.*	*Ft.-In.*	*Ft.-In.*	*Ft.-In.*
$1\frac{7}{16}$	9-0	8-0	7-0	6-6
$1\frac{15}{16}$	10-0	9-0	7-6	7-0
$2\frac{7}{16}$	11-0	10-0	8-0	7-6
$2\frac{15}{16}$	12-0	11-0	8-6	8-0
$3\frac{7}{16}$	13-0	12-0	9-0	8-6
$3\frac{15}{16}$	14-0	13-0	9-6	9-0
$4\frac{7}{16}$	15-0	14-0	10-0	9-6

158. Standard Sizes of Shafting. "Commercial" shafting has been tentatively standardized by the American Engineering Standards Committee (1924) as follows:

TRANSMISSION SHAFTING. Sizes in in.: 15/16, $1\frac{3}{16}$, $1\frac{7}{16}$, $1\frac{11}{16}$, $1\frac{15}{16}$, $2\frac{3}{16}$, $2\frac{7}{16}$, $2\frac{15}{16}$, $3\frac{7}{16}$, $3\frac{15}{16}$, $4\frac{7}{16}$, $4\frac{15}{16}$, $5\frac{7}{16}$, and $5\frac{15}{16}$.

MACHINERY SHAFTING. Sizes: 1/2 in. to $2\frac{1}{2}$ in. by 1/16 in. increments, $2\frac{5}{8}$ in. to 4 in. by 1/8 in. increments, and $4\frac{1}{4}$ in. to 6 in. by 1/4 in. increments. Standard stock lengths are 16, 20, and 24 ft.

159. Methods of Manufacturing Shafting. In the cold finishing process of shafting manufacture, hot rolled steel bars may be passed through grooved finishing rolls until reduced to size by compression and elongation. This shafting is known as "cold rolled." If the reduction is made by cold drawing the bars through dies of the required diameter, "cold drawn" shafts are produced. In both of these methods there is a tendency for a hard surface layer to form on the surface of the shaft. When this scale is penetrated, as in cutting keyways, the skin tension is relieved, causing the shaft to warp. In some cases, it is practically impossible to re-straighten the shaft again. When shafting is turned or ground from rough bars, the surface hardness is relieved and there is less tendency to warp. Shafting of this type is more adaptable to keyseating and machining, and is also more accurate for size and roundness. Sizes larger than 6 in. are generally forged and then turned to size.

160. Hollow Shafts. A hollow shaft has greater strength and stiffness than a solid shaft of equal weight. In addition, the removal of the core from the center of shafts, especially large shafts, increases their reliability. Ingots from which shafting is rolled or forged may have shrinkage cavities near the center and the material in this region is likely to be less dense than in the outer portion. This condition, known as "piping," is due to the shrinkage of the interior while cooling, and to the liberation of gas.

In the rolling or forging process, this defective structure is still retained, although drawn out, and consequently reduces the strength of the shaft. As a safeguard against such structural weakness, the core may be removed from the ingot before the shaft is formed, or an axial hole may be bored throughout the length of the solid shaft. Hollow propeller shafts are extensively used for naval vessels in order to attain the greatest strength with the least weight. A common procedure is to make the diameter of the hole equal to one-half the shaft diameter. Tubular structural members and hollow shafts are regularly used in aircraft design.

161. Strength of Hollow Shafts. The torsional strength of a hollow shaft is given by the following equation:

$$M_t = s_s \frac{\pi(D_o^4 - D_i^4)}{16D_o}, \tag{3}$$

in which D_o is the outside diameter and D_i the inside diameter, while the other terms have the same meaning as in the preceding formulas. For resistance against bending only, the equation is

$$M_b = s_t \frac{\pi(D_o^4 - D_i^4)}{32D_o}. \tag{4}$$

If the shaft is subjected to combined torsion and bending, the formula becomes

$$\frac{\pi(D_o^4 - D_i^4)}{16D_o} s_{s\ max} = \sqrt{M_b^2 + M_t^2}. \tag{5}$$

Let $K = D_i/D_o$, the ratio of the diameters; then, by substitution, we find

$$D_o = \sqrt[3]{\frac{16\sqrt{M_b^2 + M_t^2}}{\pi(1 - K^4)s_{s\ max}}}. \tag{6}$$

For a hollow shaft to be equal in torsional strength to a solid shaft, their resisting moments must be equal. If both shafts are made of the same material and are stressed to the same intensity, then, if D is the diameter of the solid shaft, we have

$$\frac{\pi(D_o^4 - D_i^4)}{16D_o} s_s = \frac{\pi D^3}{16} s_s,$$

or

$$D_o^3 = \frac{D^3}{1 - K^4}.$$

If, for instance, the inside diameter is one-half of the outside diameter, $K = \frac{1}{2}$, and we find

$$D_o = 1.022D.$$

From this equation, we see that a hollow shaft of the assumed proportions would be only 2.2 per cent larger in diameter than a solid shaft of equal strength, but there would be a reduction in weight of 22.6 per cent. If both shafts are of the same diameter, the hollow shaft would be 93.7 per cent as strong as the solid shaft, yet the reduction in weight would be 25 per cent.

162. Angular Deflection of Shafts. Shafting is not always proportioned by merely considering its strength in torsion and bending. Long shafts, especially if they are relatively small in diameter, may not be *stiff* enough to transmit power with a uniform, steady motion. Furthermore, where vibration is not permissible, as in precision grinders, lathes, and similar machines, relatively short spindles are purposely made very rigid, in order that work may be performed with the proper degree of accuracy and finish. It should also be mentioned that in many cases the size of a shaft is actually controlled by the bearing proportions, and not by the strength or stiffness of the shaft itself.

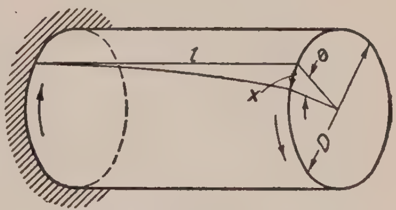


Fig. 191. Torsional Deflection of Shaft.

Referring to Fig. 191, a point on the surface of a shaft is displaced a certain distance x if the shaft is subjected to a twisting moment M_t. If the length of shaft is l in., and the displacement x is also measured in in., then, within the limits of proportionality,

$$\frac{x}{l} = \frac{s_s}{G},$$

in which s_s is the shearing stress in the shaft in p.s.i. and G is the modulus of elasticity in shear, usually taken as 11,500,000 for steel. It will be more convenient to express the displacement x in terms of the angle of twist; therefore

$$x = \frac{\pi D\theta}{360}, \qquad \text{and} \qquad s_s = \frac{16M_t}{\pi D^3},$$

where θ is the angular deflection in degrees and D is the shaft diameter in in. Substituting the values of x and s_s in the equation above,

(7) $$\theta = \frac{584M_tl}{GD^4} \qquad \text{(for solid circular shafts).}$$

By the same method we find

(8) $$\theta = \frac{584M_tl}{G(D_o^4 - D_i^4)} \qquad \text{(for hollow circular shafts),}$$

where D_o is the outside diameter and D_i is the inside diameter.

It is common practice to limit the angular deflection of line shafting to 1 degree in a length equivalent to 20 times the shaft diameter. It

is desirable to limit the permissible maximum transverse deflection of line shafting to 0.01 in. per ft.

163. Shaft Deflection. The extent of deflection of a shaft under load may be such as to interfere with its proper functioning. For this reason, methods of calculating shaft deflection must be applied in addition to the usual calculations for strength. Deflections for shafts of uniform cross-section, with loads concentrated or uniformly distributed, may be obtained from the deflection formulas given in handbooks.

The deflection of a point on a beam with unusual loading or of variable cross-section is usually obtained by double integration, or by the use of the bending moment diagram, as in the method of "elastic weights."

The method that uses double integration of the differential equation starts with the basic equation relating physical characteristics and deflection conditions of a bent beam by moment only, namely $d^2y/dx^2 = -M/EI$. This double integration involves the determination of the constants of integration from known conditions. This procedure may be a laborious one, but the method is particularly applicable where the moment of inertia and the bending moment can be expressed as a function of the distance x.

When the cross-section of the shaft is stepped in diameter or the moment is produced by a number of loads, the method of "elastic weights" offers a procedure that involves only simple arithmetical calculations, or graphical solution. This method is based upon the general principle that in a bent beam the deflection of a point from the tangent at some other point is equal to the moment of the M/EI area between the two points. Mathematically, $y = (1/E)\int(Mx/I)dx$. If, then, we take the bending moment diagram, divide by the moment of inertia of the shaft at that point, and use the resultant modified moment diagram as a loading diagram on a conjugate beam of the same length but of constant unit moment of inertia, *the deflection of any point is the bending moment of the conjugate beam divided by the modulus of elasticity*. The modified bending moment diagram may be obtained by analytical or graphical integration. Negative M/I areas are treated as loads in a direction opposite to loads of positive areas.

Example. For a simple illustration of the graphical solution by the method of elastic weights, consider the shaft shown in Fig. 192. The procedure is as follows:

First, determine the bearing reactions and bending moments at the critical points. To do this graphically lay off to a suitable force scale the loads as denoted by bc in Fig. 192b. Selecting a suitable pole point o, preferably some distance in whole inches away from the force line, draw the rays from the pole point to the ends of the forces. Then in the space diagram (Fig. 192a) construct a funicular polygon by drawing in the space B a line parallel to ray ob, and in space C a line parallel to ray oc, these lines intersecting on the force line A. To close the funicular polygon a line in the space A is necessary. A line drawn in the polar diagram (Fig. 192b) through o parallel to this closing line in space A intersects the line of forces at the point a, which establishes the values of the reactions ab and ca.

The funicular polygon (Fig. 192a) is the bending moment diagram, the bending moment at any point being equal to the vertical ordinate of the diagram multiplied by the bending moment scale. One inch of the bending moment diagram = (space scale

in inches represented by 1 in.) × (force scale in lb. represented by 1 in.) × (pole distance in in.). For this problem, the bending moment scale (M scale) = 10 × 200 × 3 = 6,000 in. lb. = 1 in. Scaling the bending moment diagram at points 1, 2, and 3, the moments are M_1 = 1800 in. lb., M_2 = 3000 in. lb., M_3 = 600 in. lb.

Second, compute the moments of inertia for the various sections.

$$I_a = 0.784 \text{ in.}^4, \qquad I_b = 3.980 \text{ in.}^4, \qquad I_c = 0.248 \text{ in.}^4.$$

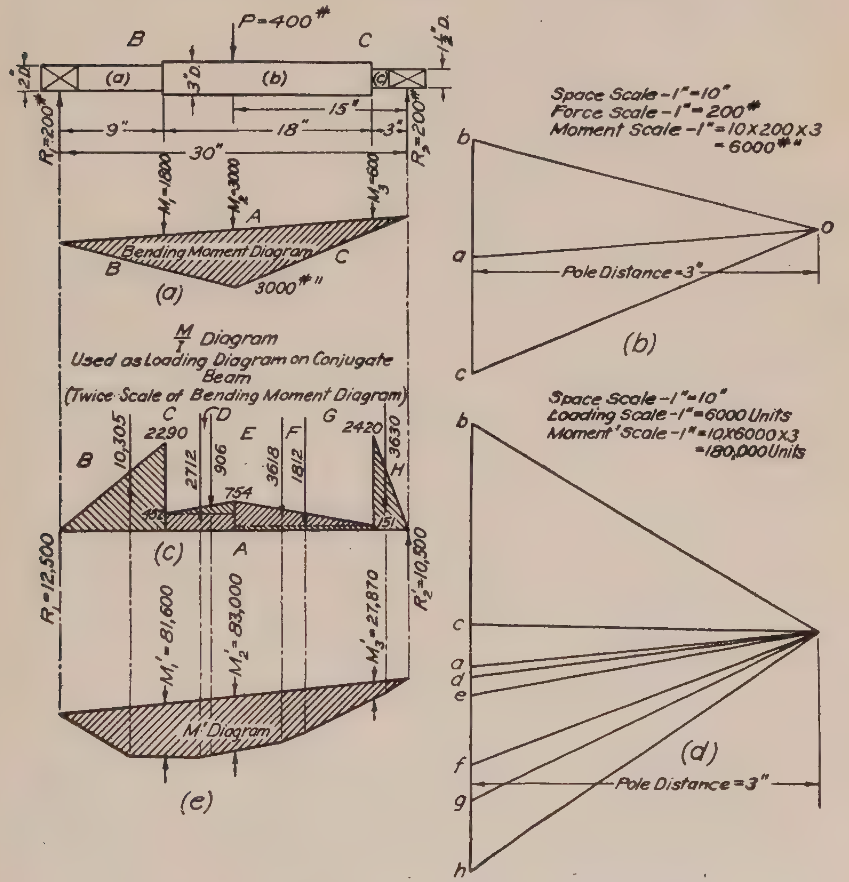


FIG. 192. GRAPHICAL SOLUTION OF SHAFT DEFLECTION.

Third, divide the bending moments by the moment of inertia of the shaft sections at that point, as 1800/0.784 = 2290, 1800/3.980 = 452, etc. These values are shown on the modified bending moment diagram (Fig. 192c).

Fourth, construct the modified bending moment diagram from the above values and use it as a loading diagram. The values of the equivalent concentrated loads are determined by subdividing the whole area of each section into convenient rectangles and triangles and considering the whole of each section to be concentrated at the center of gravity of that section. These values are indicated on diagram (Fig. 192c).

Fifth, select a loading scale (Fig. 192d) for the loading units and then find the resultant bending diagram moment (Fig. 192e) of the conjugate beam by the graphical method described above. The modified moment scale is M' scale = (space scale in inches represented by 1 in.) × (loading units represented by 1 in.) × (pole distance in in.) = 10 × 6000 × 3 = 180,000 units = 1 in.

Sixth, divide the moments found from the M' diagram (Fig. 192e) by the modulus of elasticity to determine the deflections at the various points. The values in this case are:

$$y_1 = \frac{M'_1}{E} = \frac{81,600}{30,000,000} = 0.00272 \text{ in.},$$
$$y_2 = \frac{M'_2}{E} = \frac{83,000}{30,000,000} = 0.00277 \text{ in.},$$
$$y_3 = \frac{M'_3}{E} = \frac{27,870}{30,000,000} = 0.00093 \text{ in.}$$

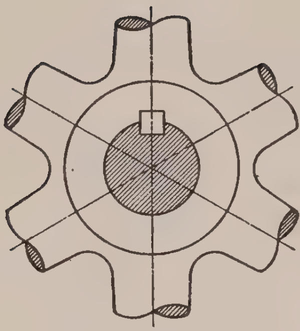


Fig. 193. Application of Key.

164. Keys. The function of a key is to prevent relative movement between two machine members. In its more general application, a key consists of a square or rectangular bar of steel inserted half in a shaft and half in the keyed-on member, such as a gear or pulley (Fig. 193). Keys have been standardized and are generally proportioned to the shaft diameter. Since they are relatively inexpensive and are easily replaced, keys are frequently designed to fail when subjected to unexpected over loads, so that the other more expensive machine members are protected.

165. Types of Keys. The *square key* (Fig. 194a) and the *flat key* (Fig. 194b) are the types most commonly used. In the plain form they are regularly made from cold-finished bar stock. Keys of this type should be accurately fitted in the keyways on the sides but will have some clearance at the top. In such applications, it is necessary to have a close fit between the hub and shaft, since the key cannot take up any looseness. If set screws in the hub are to be used to prevent axial movement, they should be applied on top of the key. This arrangement tends to prevent the key from tipping when under load.

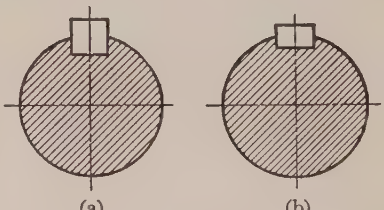


Fig. 194. (a) Square Key. (b) Flat Key.

Square and flat keys are also used in tapered form. Tapered keys have parallel sides, but the top is tapered 1/8 in. per ft. The dimensions at the large end are the same as for a plain key of the same size. The

keyseat in the shaft is straight and in the hub it is tapered 1/8 in. per ft. When driven in place the tapered key produces friction between the shaft and the hub and aids in the transmission of power. It also secures the position of the keyed member on the shaft.

Standard square and flat keys with corresponding shaft diameters, as approved by the American Standards Association, are given in Table 26.

TABLE 26

STANDARD SQUARE AND FLAT KEYS

SHAFT DIA. (in.)	SQUARE b (in.)	FLAT $b \times t$ (in.)	SHAFT DIA. (in.)	SQUARE b (in.)	FLAT $b \times t$ (in.)
1/2 to 9/16	1/8	1/8 × 3/32	2 7/8 to 3 1/4	3/4	3/4 × 1/2
5/8 to 7/8	3/16	3/16 × 1/8	3 3/8 to 3 3/4	7/8	7/8 × 5/8
15/16 to 1 1/4	1/4	1/4 × 3/16	3 7/8 to 4 1/2	1	1 × 3/4
1 5/16 to 1 3/4	3/8	3/8 × 1/4	4 3/4 to 5 1/2	1 1/4	1 1/4 × 7/8
1 13/16 to 2 1/4	1/2	1/2 × 3/8	5 3/4 to 6	1 1/2	1 1/2 × 1
2 5/16 to 2 3/4	5/8	5/8 × 7/16			

b = width of key. $t/2$ = depth of keyway.
t = thickness of key. $b = t$ for square key.

Gib-head keys (Fig. 195) are tapered 1/8 in. per ft. and are provided with a head to facilitate removal. This type is regularly used when the key must be driven in and is accessible for removal from only one end. Standard proportions are given in Table 27.

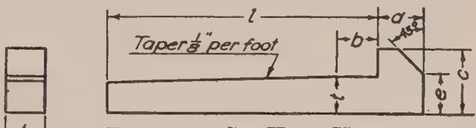


FIG. 195. GIB-HEAD KEY.

TABLE 27

STANDARD SQUARE AND FLAT GIB-HEAD KEYS

SHAFT DIAMETERS INCLUSIVE (in.)	SQUARE TYPE Dimensions in in.					FLAT TYPE Dimensions in in.				
	b	t	c	d	e	b	t	c	d	e
1/2–9/16	1/8	1/8	1/4	7/32	5/32	1/8	3/32	3/16	1/8	1/8
5/8–7/8	3/16	3/16	5/16	9/32	7/32	3/16	1/8	1/4	3/16	5/32
15/16–1 1/4	1/4	1/4	7/16	11/32	11/32	1/4	3/16	5/16	1/4	3/16
1 5/16–1 3/4	3/8	3/8	11/16	15/32	15/32	3/8	1/4	7/16	3/8	5/16
1 13/16–2 1/4	1/2	1/2	7/8	19/32	5/8	1/2	3/8	5/8	1/2	7/16
2 5/16–2 3/4	5/8	5/8	1 1/16	23/32	3/4	5/8	7/16	3/4	5/8	1/2
2 7/8–3 1/4	3/4	3/4	1 1/4	7/8	7/8	3/4	1/2	7/8	3/4	5/8
3 3/8–3 3/4	7/8	7/8	1 1/2	1	1	7/8	5/8	1 1/16	7/8	3/4
3 7/8–4 1/2	1	1	1 3/4	1 3/16	1 3/16	1	3/4	1 1/4	1	13/16
4 3/4–5 1/2	1 1/4	1 1/4	2	1 7/16	1 7/16	1 1/4	7/8	1 1/2	1 1/4	1
5 3/4–6	1 1/2	1 1/2	2 1/2	1 3/4	1 3/4	1 1/2	1	1 3/4	1 1/2	1 1/4

$t/2$ = depth of keyway.

The Kennedy or *double key system* (Fig. 196a), consisting of two square tapered keys, is employed for heavy-duty service, as in rolling mills, where power is transmitted intermittently and in both directions. The keyways should be straight and parallel except in the hub, where the bottoms should be tapered 1/8 in. per ft. Diagonals passing through key corners approximately intersect the shaft axis. When under load, the keys are in compression.

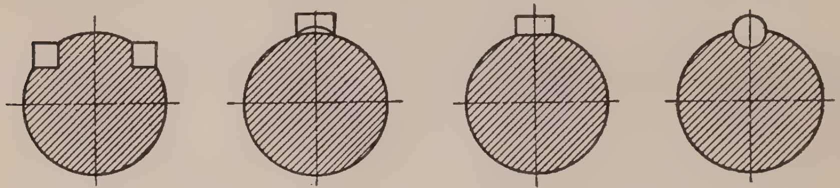


Fig. 196. (a) Kennedy Key. (b) Saddle Key. (c) Flat Key. (d) Pin Key.

A *saddle key* is shown in Fig. 196b. This type of key may be used for light service and where a keyseat in the shaft would be objectionable. Set screws are required with this key, and, in order to grip the shaft, the concavity at the bottom of the key should be greater than the curvature of the shaft. The saddle key makes it possible to fasten the hub at any position on the shaft.

A *flat* key may be used without a keyseat, as in Fig. 196c, by flatting one side of the shaft. The key should be tapered so that it may be wedged tightly in place. A *round taper* pin fitting in a reamed hole (Fig. 196d) is frequently used as a key. Both types are suitable for light loads only.

The *Woodruff key* (Fig. 197) is extensively used for light duty, especially in machine-tool construction. On account of its deep semicircular seat, this type of key minimizes the tendency of the key to tip. The circular bottomed seat permits the key to pivot and to align itself with the keyseat in the hub, and is therefore well adapted to tapered fits. Where added strength is necessary two or more keys may be used in line. The proportions of these keys have been standardized and tables may be found in engineering handbooks.

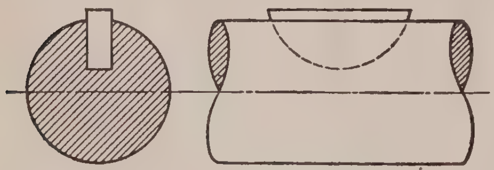


Fig. 197. Woodruff Key.

A *feather key* is used when it is necessary to slide the keyed-on member axially along the shaft. A sunk key held in place with screws

(Fig. 198) is the more common application, although the key may be anchored to the hub as in Fig. 199.

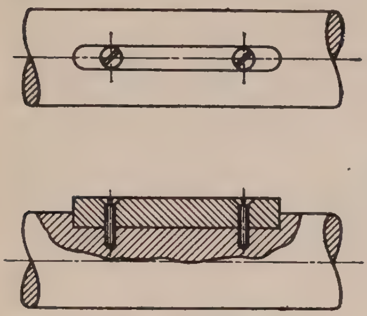


FIG. 198. FEATHER KEY INSERTED IN SHAFT.

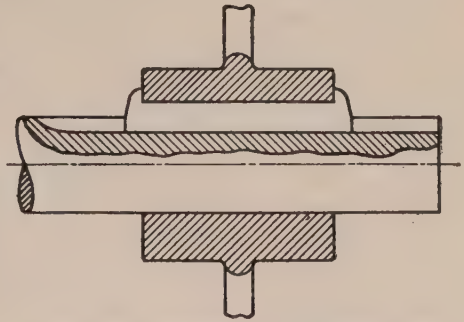


FIG. 199. FEATHER KEY INSERTED IN HUB.

166. Splined Fittings. When greater key bearing and strength are required than can be obtained by a single feather key, a *splined shaft* is used. Multiple splines (Fig. 200) cut "integral" with the shaft are regularly used in sliding gear transmissions, of automobiles and machine tools. Splined shafts are cut very accurately by the same processes as those used in generating gear teeth, while the internal spline is produced by a broaching operation. The S.A.E. standards include 4, 6, 10, and 16 spline fittings, with proportions as given in Table 28.

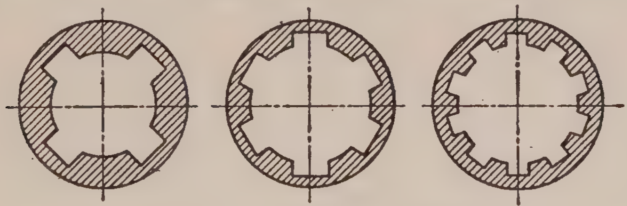


FIG. 200. MULTIPLE SPLINES.

TABLE 28

SPLINED FITTINGS

Type	Permanent Fitting			To Slide when Not Under Load			To Slide when Under Load		
	d	h	w	d	h	w	d	h	w
4 Spline..	0.850D	0.075D	0.241D	0.750D	0.125D	0.241D	—	—	—
6 " ..	0.900D	0.050D	0.250D	0.850D	0.075D	0.250D	0.800D	0.100D	0.250D
10 " ..	0.910D	0.045D	0.156D	0.860D	0.070D	0.156D	0.810D	0.095D	0.156D
16 " ..	0.910D	0.045D	0.098D	0.860D	0.070D	0.098D	0.810D	0.095D	0.098D

D = major diameter of splined fitting; d = minor diameter of splined fitting. w = width of spline; h = depth of spline.

167. Strength of Keys. The strength of a key is based principally upon its resistance to shear and compression. In Fig. 201 let l equal the length of the key, b the width, and t the thickness. In transmitting a torque, a force F_h acts against the portion of the key in the hub while a force F_s in the opposite direction acts against the portion of the key in the shaft. These forces are not equal, since they are applied at different distances from the axis. No serious error is involved, however, if an average force F is taken equal to the torque M_t divided by the shaft radius r. For resistance against shear we have

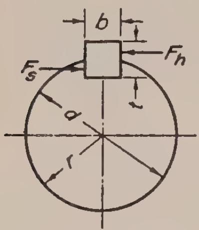


FIG. 201. FORCES ACTING ON KEY.

$$F = \frac{M_t}{r} = s_s bl, \tag{9}$$

where s_s is the shearing stress. If s_c is the compression stress, the resistance against compression is

$$F = \frac{M_t}{r} = s_c \frac{t}{2} l. \tag{10}$$

The effect of cutting a keyway in a shaft to receive a standard square key (Table 26) reduces the torsional strength of the shaft, according to formula (36), § 13, page 25, approximately 19 per cent. Keys proportioned on the basis of shaft diameter (see Tables 26 and 27) have sufficient shear and compressive strength to transmit the net shaft torque, provided the key length is approximately 1¼ times the shaft diameter, and the key and shaft materials are of equal strength. From experience it has been found that a hub length must be about 1¼ times the shaft diameter, or longer, to seat firmly on the shaft without rocking. This rule is to be observed, of course, only if the keyed member transmits practically the whole shaft torque. If it transmits only a small part thereof, narrower hubs are common.

EXAMPLE. Determine the width and thickness of key to equal the net strength of a 2 in. round shaft, if the hub length of the keyed-on member is 1¼ times the shaft diameter.

If we assume the shear stresses to be equal, and, on the basis of Moore's formula (36), page 25, for a shaft with keyway to be only 81 per cent as strong as the solid shaft, we have

$$0.81 s_s \pi \frac{D^3}{16} = s_s bl \frac{D}{2} \qquad \text{or} \qquad b = 0.51 \text{ in.}$$

If the value of shear strength is considered to be 50 per cent of the compressive strength, then

$$0.81 s_s \frac{\pi D^3}{16} = 2 s_s \frac{t}{2} l \frac{D}{2} \qquad \text{or} \qquad t = 0.51 \text{ in.}$$

According to Table 26 a 1/2 in. square key is standard for a 2 in. diameter shaft. For a flat key (Table 26) the above analysis would indicate that the key is somewhat weak in compression. A slightly longer key length would correct this condition.

A key must fit tightly in the keyway to approach the conditions outlined above. If there is any looseness the key will roll in the keyway and failure will occur from stress concentrations at the corners of the key. It should also be mentioned that bending stresses in the key have been neglected, although, according to Bach's investigations, they may be more serious than the shear stress.

168. Permanent Shaft Couplings. General Remarks. In power transmission by means of shafting, couplings are used to connect the ends of two shafts. If the connection is to be fixed, that is, broken only occasionally for repairs, and then by disassembling the unit, the coupling is considered a *permanent coupling.* When it is necessary to connect and disconnect the shafts at will, some form of *clutch* is required. Various types of clutches will be discussed separately in Chapter 16. Permanent couplings may be classified into two groups:

(1) *Rigid Couplings* for shafts having a common axis of rotation.
(2) *Flexible Couplings* for shafts having slight parallel or angular misalignment.

169. Rigid Couplings. *Flange couplings,* as shown in Figs. 202a and b, are designed to withstand severe service. They are particularly

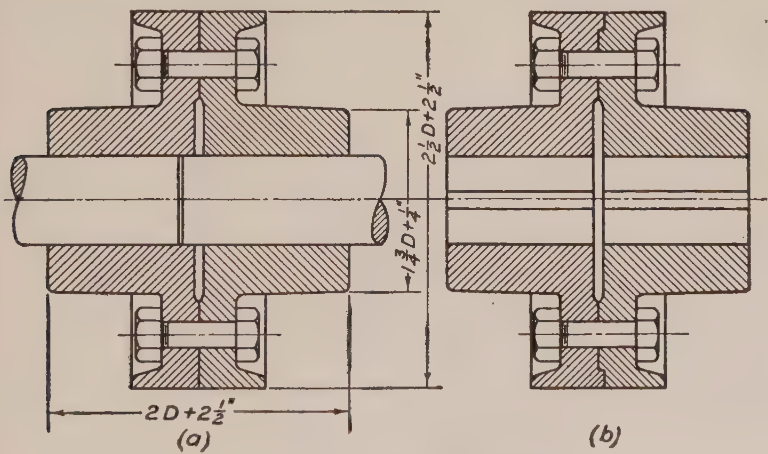


FIG. 202. FLANGE COUPLINGS.

adapted for heavy shafting and slow speeds. As a protection to workmen, the nut and bolt heads are covered by projecting flanges.

To insure true alignment, one shaft may enter the coupling of the other shaft about 3/8 in. (Fig. 202a), or a recessed joint may be used, as in Fig. 202b.

On important work, flange couplings should be forced on the shafts and keyed tightly in place. The shafts should then be straightened and the flanges faced true in place. The bolts connecting the flanges should be closely fitted in reamed holes so as to distribute the load equally. A coupling applied in this manner has great strength and rigidity. Couplings for marine or automotive propeller shafts demand a high degree of reliability and strength; for this service the flanges may be forged integral with the shafts.

The general proportions for flange couplings are given in Fig. 202a. The hub must provide sufficient length for the key so that its resisting moment about the axis of the shaft will be at least equal to the torsional strength of the shaft. Incidentally, this length will keep the flange rigidly in place and will afford an adequate area for a tight frictional grip on the shaft. The number and size of the bolts must be proportioned to transmit the full torsional strength of the shaft. The following derivation is used to determine the number and diameter of bolts, but is actually only a rough orientation leading to a rule of thumb rather than a strictly rational formula.

Let D be the diameter of the shaft, d the diameter of the bolt, r the radius of the bolt circle, all in in., n the number of bolts, and s_s the allowable shear stress in the shaft and bolts. We are assuming that the torsional shear stress in the shaft is equal to the transverse shear in the bolts.* Then

$$\frac{\pi D^3}{16} s_s = \frac{\pi d^2}{4} n r s_s,$$

or

$$d = \frac{1}{2}\sqrt{\frac{D^3}{nr}}, \tag{11}$$

where n may be taken equal to $D/2 + 3$.

In ordinary work where the bolts fit the holes with clearance, it is very unlikely that the load is equally distributed as just explained. It is commonly assumed in such cases that half of the bolts may carry the load, which gives

$$d = \sqrt{\frac{D^3}{2nr}}. \tag{12}$$

* Actually the bolts are also subjected to a bending stress which is likely to be more serious than the shear stress.

Clamp couplings (Fig. 203) are suitable for general line-shaft service. Being split longitudinally they are easily installed and removed. The ribs add strength and are a protection to the workman against injury from the revolving bolt heads and nuts.

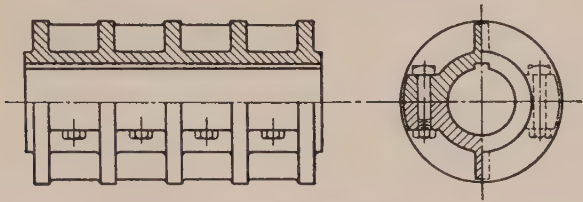


FIG. 203. RIBBED CLAMP COUPLING.

170. Self-Aligning Rigid Couplings. Cone-type compression-couplings have the advantage of being self-aligning and more easily installed than flange couplings. In the *compression coupling* (Fig. 204) two split cones, bored and keyed to fit the shaft, are wedged tightly when compressed and drawn into a double conical shell by bolts as shown. Any variation in shaft size does not distort the alignment. For smaller shafts, 3 in. in diameter or less, a similarly constructed coupling, but without keys, transmits power through friction only.

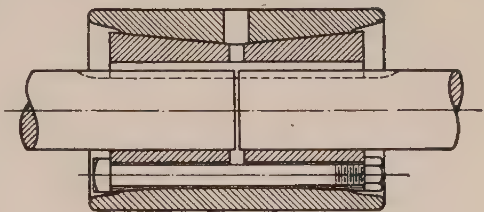


FIG. 204. COMPRESSION COUPLING.

The *Collin's coupling* shown in Fig. 205 is particularly adapted to textile mill service, as there are no recesses or projections to collect lint, dust, or dirt. Its clamping action is effected by forcing two conically bored rings over a two-piece taper sleeve.

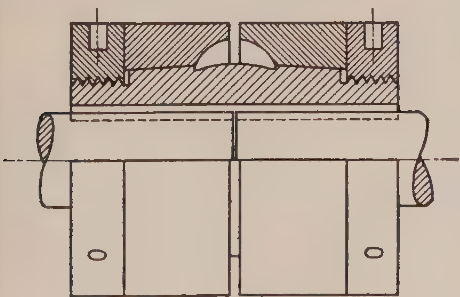


FIG. 205. COLLINS COUPLING.

171. Flexible Couplings. A flexible coupling is especially suitable for joining the shafts of direct-connected machines, particularly if they operate at relatively high speeds. It is very difficult to establish and maintain an accurate shaft alignment between two individual machines, when vibration, unequal bearing wear, expansion

due to heat, and uncertain foundations are some of the disturbing factors. To overcome this difficulty, a flexible coupling will compensate for misalignment, absorb shocks, dampen vibration, and provide free end-float of the shafts.

Motors should always be flexibly connected to direct-driven machinery. The use of a flexible coupling, however, is not an excuse for a careless installation, as their efficiency and durability are impaired through misalignment. Certain types of flexible couplings have more torsional resiliency than others. This is due to the construction and flexibility of the driving elements. A good flexible coupling should accommodate both parallel and angular displacements within reasonable limits.

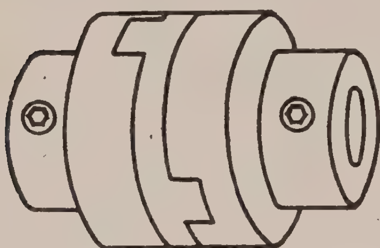
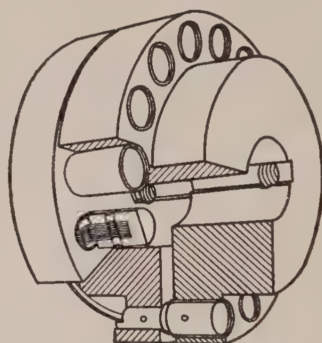


Fig. 206. Oldhams Coupling.



Smith and Serrell, Newark, N. J.

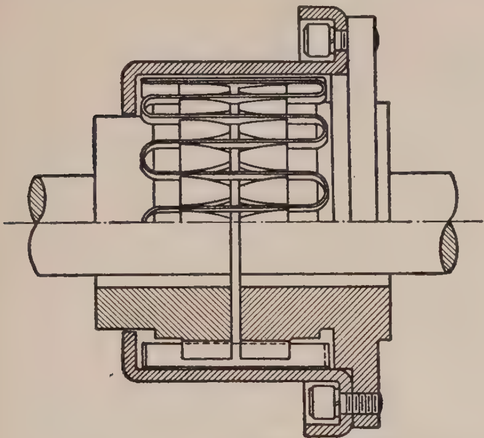
Fig. 207. Franke Flexible Coupling.

Oldham's coupling (Fig. 206) is based on the double slider-crank principle and may be used to connect shafts with parallel misalignment. A floating intermediate disc is provided with tongues on each side (at right angles to each other), which fit into corresponding slots in the flanged hubs attached to each shaft. The intermediate member is free to slide and compensate for any small misalignment between shaft axes. This coupling is suitable for heavy torques and slow speeds.

The *Francke coupling* shown in Fig. 207 consists of two flanged hubs connected by flexible laminated steel pin units. These pins are locked by a spring retaining ring in one of the flanges, and are free to slide endwise in the other flange.

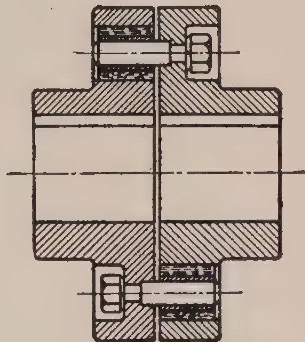
The *Falk coupling* is illustrated in Fig. 208. Two flanged steel hubs are grooved on their peripheries to receive a tempered steel spring in the form of a grid. The grooves widen inward toward each other and are so formed that the stress in the spring remains almost constant throughout the elastic range of the coupling. Under severe overloads

the spring section comes into direct shear. The coupling is encased in a steel shell and is packed with grease.



The Falk Corp., Milwaukee, Wis.

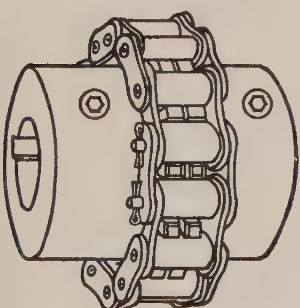
FIG. 208. FALK FLEXIBLE COUPLING.



Ajax Flexible Coupling Co., Westfield, N. Y.

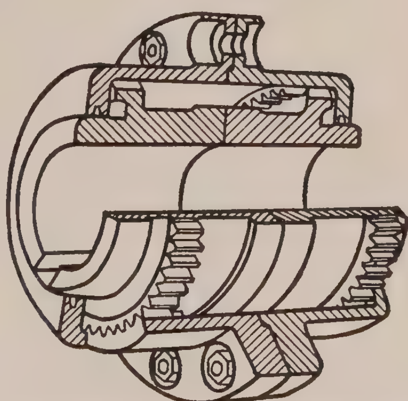
FIG. 209. AJAX FLEXIBLE COUPLING.

The *Ajax coupling* (Fig. 209) secures flexibility through drive studs which are mounted in one flange and which engage with bronze-lined rubber bushings in the other flange.



Link Belt Co., Chicago, Ill.

FIG. 210. CHAIN FLEXIBLE COUPLING.



Poole Engineering & Machine Co., Baltimore, Md.

FIG. 211. POOLE FLEXIBLE COUPLING.

The *chain coupling* shown in Fig. 210 is very rugged and simple in construction; it consists of two sprockets in parallel, coupled with a chain. The unit may be encased in a shell for lubrication.

The *Poole flexible coupling* (Fig. 211) transmits power through crowned gear teeth formed on the periphery of each flanged hub.

These gears mesh with corresponding teeth formed on the interior of a floating and connecting sleeve member, which is sealed and filled with lubricant.

172. Universal Joints. The usual method of transmitting power between two intersecting shafts, where the angle between the axes is variable, is by means of a universal joint. Both shafts end in forks, and these forks are connected by means of an annular ring, sphere, or rectangular cross, the last arrangement being shown in Fig. 212.

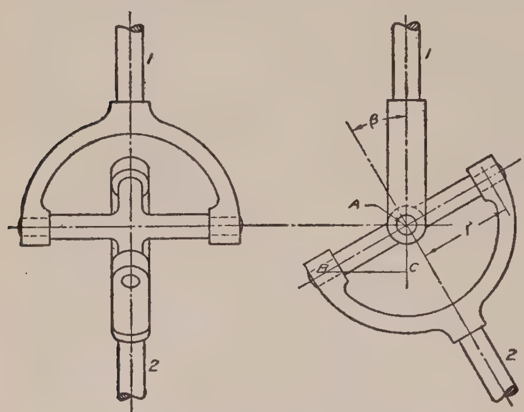


Fig. 212. Intersecting Shafts Connected by a Single Universal Joint.

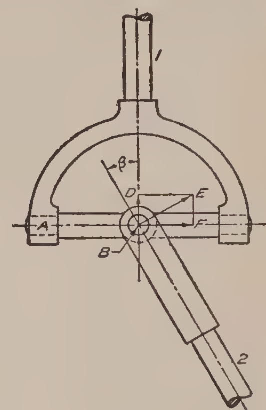


Fig. 213. Position of Universal Joint when Turned through an Angle of 90 Deg. from that Shown in Fig. 212.

It can easily be seen that in the position shown in Fig. 212, the revolving force of the cross at B is tangent to the path of the fork of shaft 2. The magnitude of the arm around the axis of rotation of shaft 1 is, however, only $BC = r \cos \beta$, where r is the length of the cross arm and β the angle between the shafts. Hence, if shaft 1 revolves with an angular velocity ω, for the position shown, shaft 2 will revolve with the velocity $\omega \cos \beta$.

On the other hand, in the position shown in Fig. 213, it is only a component BF of the tangential velocity BE of fork 2 which moves in the plane of rotation of the ends of the fork 1. Since BE is equal to $BF/\cos \beta$, in this position the angular velocity of shaft 2 is $\omega/\cos \beta$. Hence, to obtain a constant angular velocity between two shafts connected with universal joints, it is necessary to neutralize this variation in velocity ratio by the use of an intermediate shaft with both of its forks lying in the same plane, as shown in Fig. 214. Moreover,

the intermediate shaft must be inclined at the same angle to each of the main shafts.

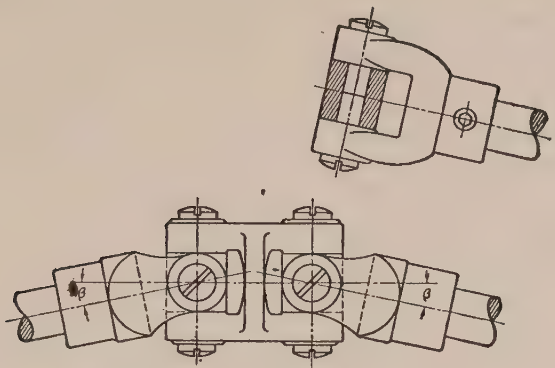
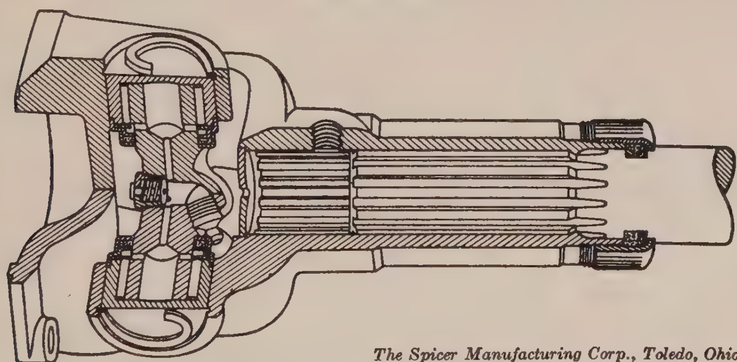


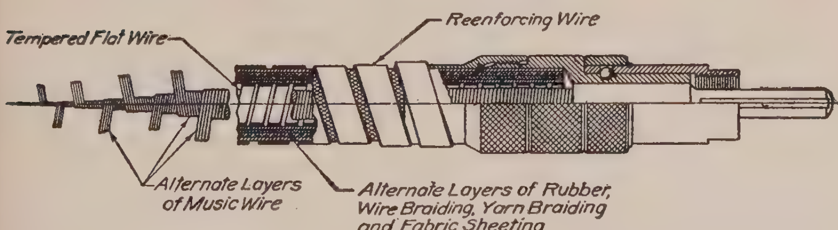
FIG. 214. DOUBLE UNIVERSAL JOINT.

The design of a modern automotive universal joint with the ends of the cross pivoted in needle bearings is shown in Fig. 215. The joint is



The Spicer Manufacturing Corp., Toledo, Ohio

FIG. 215. AUTOMOTIVE UNIVERSAL JOINT WITH TRUNNION CROSS FITTED IN NEEDLE BEARINGS.



R. G. Haskins Co., Chicago, Ill.

FIG. 216. SECTION OF FLEXIBLE SHAFT SHOWING CONSTRUCTION OF CASING AND CORE.

not easy to lubricate unless it is wholly enclosed and is provided with a grease connection, as shown in the figure.

For the transmission of comparatively light loads, such as those required to drive portable buffing and grinding wheels, dentists' drills, etc., flexible shafts may be used. A flexible shaft is shown in Fig. 216. The driving element consists of a core composed of alternate layers of music wire wound upon a wire mandrel.

173. Selection of a Flexible Coupling. In the selection of a flexible coupling, reference must be made to the manufacturer's listed horsepower ratings, which are usually based on a steady load, average good alignment, and absence of undue vibration. Consideration must also be given to the nature of the load and the type of drive; this involves the use of a "service factor." While such factors will vary to some extent with different types of couplings, the following table applying to the Falk flexible coupling illustrated in Fig. 208 is representative.

TABLE 29

SERVICE FACTORS

Type of Service	Type of Drive		
	Motor	Steam Engine	Oil or Gas Engine
Relatively smooth steady loads	1	2	4
Somewhat uneven loads	1½	2½	5
Heavy varying loads	2	3	5½
Very heavy varying loads	2½	4	6
Shock loads and reversals	3½	5	7

As an example, if a 50-horsepower steam engine is used to drive a stone crusher, a service factor of 4 would be used. Therefore a flexible coupling with a rating of 200 horsepower at the required speed would be necessary.

PROBLEMS

1. A solid coupling shaft of an electric dynamometer is to transmit 165 hp. at 1000 r.p.m. under heavy shock load conditions. Determine the necessary diameter if the shaft is made of S.A.E. 1045 and has transmission spiders keyed to the ends.

2. An automobile engine delivers 75 hp. to the rear axle at 3400 r.p.m. of the motor. The differential ratio is 4.7 to 1 and the transmission reduction in low gear is 2.8 to 1. If the axle diameter is $1\frac{3}{16}$ in. determine the maximum torsional stress in the rear axle for full-load conditions (a) in low gear, (b) in high gear. Assume a drive efficiency of 90 per cent in each case.

3. A line shaft is to transmit 25 hp. at 200 r.p.m. Determine the necessary diameter of the shaft to limit the shear stress and the angular deflection to the usual limits. What spacing should be given the bearings?

4. A valve stem is 3/4 in. in diameter with a standard 5/8 in. NC thread on one end. Determine the torque that would induce a stress of 5000 p.s.i.

5. A common formula for jack-shaft determination is

$$D = 3.36\left(\frac{hp.}{r.p.m.}\right)^{1/3}.$$

Upon what value of design stress is the formula based? What would be the torsional deflection for a length of 20 diameters?

6. A steam engine with a side crank has an m.e.p. of 80 p.s.i. at 200 r.p.m. The engine is 20 in. by 16 in. and the distance between the center lines of crank and bearing is 9 in. If the cutoff is late and the maximum effective pressure is 1.3 times the m.e.p., determine the necessary shaft diameter at the bearing. Shaft is of S.A.E. 1045 steel.

7. A commercial shaft is supported on bearings 33 in. between centers. A 24 in. gear is located 8 in. inside the left bearing and a 28 in. pulley is mounted 10 in. beyond the right bearing. The gear is driven by a pinion with a downward tangential force and the pulley drives a horizontal belt on the opposite side of the shaft. The pulley also serves as a flywheel and weighs 400 lb. The 5/16 in. belt is 6 in. wide, has a tension ratio of 3 to 1, and an allowable stress of 250 p.s.i. Determine the bending on the shaft and the necessary shaft diameter.

8. A commercial shaft is supported on bearings 30 in. apart and carries a 20 in. pulley 12 in. inside the left bearing. Overhanging the right bearing 8 in. is a 10 in. crank. A downward tangential force of 2000 lb. is applied to the crank when the crank pin is 30 deg. from its lowest position. The belt pull is vertically upward and has a tension ratio of 4 to 1. For the conditions given determine (a) the bending moments on the shaft, (b) the necessary diameter, (c) the degrees of shaft twist, (d) the size of key for a 4 in. crank hub length.

9. A hollow shaft of S.A.E. 1045 with an inside diameter of 4 in. is to transmit 1000 hp. at 90 r.p.m. Determine the necessary outside diameter. What is the twist of the shaft for a length of 20 ft.? What is the twist of a solid shaft of the same outside diameter and length?

10. A hollow shaft is to have the inside diameter one-half the outside diameter. If the torque is 5000 ft. lb. and the material S.A.E. 1035, determine the diameter for mild shock conditions.

11. A sliding gear transmits 6000 in. lb. torque to a 1½ in. shaft through a feather key whose dimensions are 1/2 in. by 3/8 in. Determine the necessary hub length.

12. A 3 in. shaft is freely supported on bearings 30 in. apart. A load of 400 lb. is carried 8 in. inside the left bearing and a load of 600 lb. 10 in. inside the right bearing. Determine the lateral deflection at the loads.

13. A 1½ in. cantilever shaft 10 in. long is loaded by 200 lb. per ft. and 1000 lb. at the end. Determine the maximum deflection.

14. A shaft freely supported at the ends is made up of a section of 1¼ in. diameter for a length of 6 in. and a section of 1 in. diameter for a length of 9 in. A concentrated load of 150 lb. is carried at the point where the section changes. Determine the deflection at the load.

15. A solid flanged coupling is keyed to a hollow shaft 4 in. outside diameter and 2 in. inside diameter with a key 1 in. wide and 3/4 in. thick. The halves of the coupling are fastened with six bolts on a 10 in. circle. If the shaft, key, and bolts are all made of the same material, which has allowable stresses of 12,000 p.s.i. in compression and 8000 p.s.i. in shear, determine the length of the key and the size of the bolts for equal strength.

CHAPTER 12

PLAIN BEARINGS AND THEIR LUBRICATION

174. Bearing Classification. A *bearing* is a machine part which supports a moving element and confines its motion. The most important as well as the most common applications of bearings are those in which the relative motion between two members is either a *sliding* action guided in a *straight* line as between a planer table and its bed; or the motion of *rotation,* as in the case of a shaft with its axis held in a fixed position by *journal* bearings, and endwise movement prevented by *thrust* bearings.

A *journal* bearing is one which forms a sleeve around a shaft and supports a load at right angles to the shaft axis. The *journal* is that part of the shaft which rotates in the bearing. A *thrust* bearing is one which takes a thrust load in the direction of the axis of the shaft. A *step* bearing is a special form of a thrust bearing used to support a vertical shaft.

Bearings for *rotating* members may be still further classified into two groups according to the type of support and the relative motion involved. Thus we have *plain bearings,* in which the shaft journal is supported directly by the bearing surface with which it is in *sliding* contact, except for the intervening film of lubricant. Another group is *ball* and *roller* bearings, in which the journal is separated from the bearing support by intervening balls or rollers. The surfaces in this case are principally in rolling contact. In this chapter, we shall deal only with *plain sliding bearings* of the *journal* and *thrust* types.

175. General Remarks. Bearings are designed on the basis of their relative importance and class of service. Consideration must therefore be given to such factors as the allowable bearing pressure, strength, distortion, heat dissipation, lubrication, adjustments for wear and alignment, materials, finish, etc. In the design of bearings, it is good practice to confine as much of the wear as possible to the bearing rather than to the journal, as it is generally more economical to replace or repair a bearing than a shaft or spindle. In this connection, it has been found that certain materials in sliding contact with each other do not wear as well as others.

It is commonly known that a soft-steel shaft in a soft-steel bearing will not wear well; in fact, it is practically impossible to lubricate such

a bearing so that the parts will not cut. On the other hand, if they are both made of hardened-steel and are accurately ground to a high finish, or, if one of the members is hardened-steel and highly finished, satisfactory service may be obtained from such a combination.

Cast iron on cast iron is the most widely used bearing combination in which two metals of the same quality are used together successfully. After the parts have been accurately fitted to each other, usually by scraping and a "running-in" period with generous lubrication, the surfaces become hard and glazed and have excellent wearing properties.

In general, however, it is better to have two materials with entirely different degrees of hardness and physical properties. A good bearing material should have a low frictional resistance and the ability to sustain the lubricating film. It should also possess sufficient strength to resist being crushed or broken under journal loads.

Shafts are usually made of steel. In order to confine the wear to the bearing, the wearing surfaces of the latter are generally made of a softer material, such as cast iron, bronze, or white-metal alloy (babbitt). In special applications wood, rubber, or other plastic compositions are used. A soft-metal bearing, if inaccurately fitted, has the property of adjusting itself to the journal during the running-in period. It possesses sufficient plasticity to "flow" or to be compressed under journal loads. Bronzes are more liable to heat under abnormal conditions, because they lack the plasticity to conform readily to irregularities of the journal and are rubbed off in service. Cast iron must be carefully fitted if high bearing loads are to be sustained.

The selection of the most appropriate material is governed to a large extent by the service requirements and the form of the bearing itself. A discussion of the design and construction of various types of bearings and their applications, as well as the selection and uses of the different bearing metals, will be more clearly understood by representative examples.

176. Solid Journal Bearings. A cylindrical hole formed in a machine member is the simplest type of a *solid journal bearing*. There is no means of adjustment for wear and the shaft must be introduced into the bearing endwise. The hole may be lined with a bearing metal, babbitt or bronze being commonly used. When worn, the bearing may be restored to its original condition by replacing the liner. The piston pin bearing in an automobile connecting rod is a familiar example of this type of bearing. For ordinary work, light loads, and moderate speeds, unlined cast-iron bearings may be used. Figure

217 shows a commercial solid cast-iron bearing, babbitt-lined. Elongated holes in the base provide for lateral adjustment.

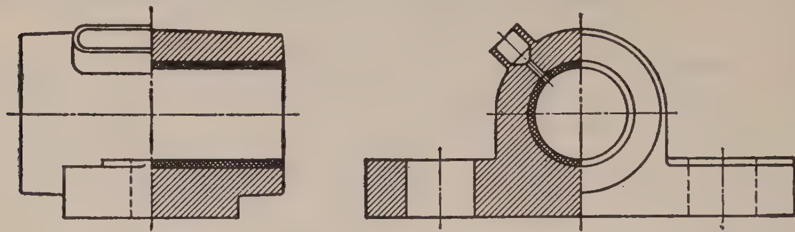


FIG. 217. SOLID JOURNAL BEARING, CAST IRON, BABBITT LINED.

177. Adjustable Journal Bearings. In order to provide an adjustment for wear, bearings may be made in two parts, as shown in Fig. 218. When construction permits, the division should be made on a plane

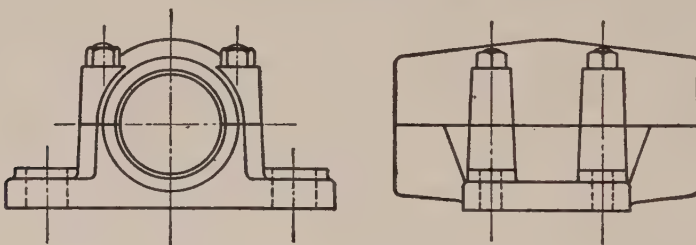
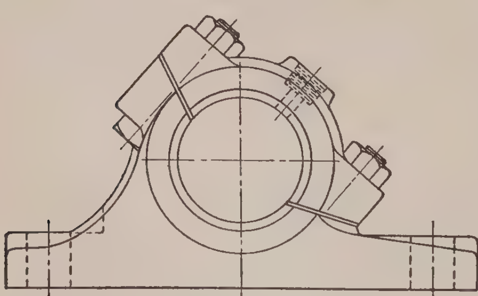


FIG. 218. SPLIT JOURNAL BEARING.

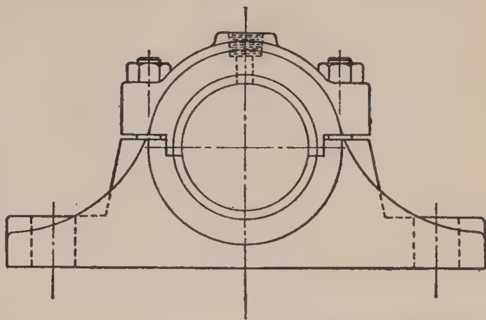
passing through the axis at right angles to the direction of the resultant load on the journal (Fig. 218). In the higher-grade bearings, and especially those for heavy duty, projections on the cap, as shown in Fig. 220, prevent horizontal movement. Adjustment for wear is made by reducing the thickness of shims placed between the cap and base, or by removing a certain amount of metal from the division surfaces. Bearings of this type may be made as individual units to be located and fastened in place, or they may be constructed as an integral part of the machine itself.



Link-Belt Co., Chicago, Ill.

FIG. 219. ANGLE JOURNAL BEARING.

Base plates are used to facilitate alignment when individual bearings are mounted on concrete foundations, or on unfinished machine members. An adjustable type, having both horizontal and vertical adjustment, is shown in Fig. 221.



Link-Belt Co., Chicago, Ill.

Fig. 220. Heavy Duty Journal Bearing.

Two-piece split bearings are extensively used in machine tools, engines, motors, and power-transmission machinery. The bearing proper is usually made of cast iron, although cast steel may be used if the added strength is needed. In certain types of machinery, for instance, lathes or punch presses, where the anticipated wear is slight, the spindle or crankshaft may be fitted directly to the bearing, no bearing liners being used. One of the most serviceable combinations is a hardened and ground steel shaft in a chilled cast-iron bearing.

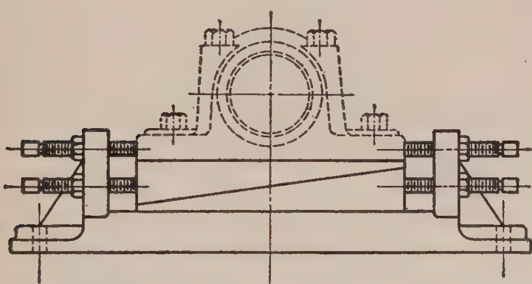
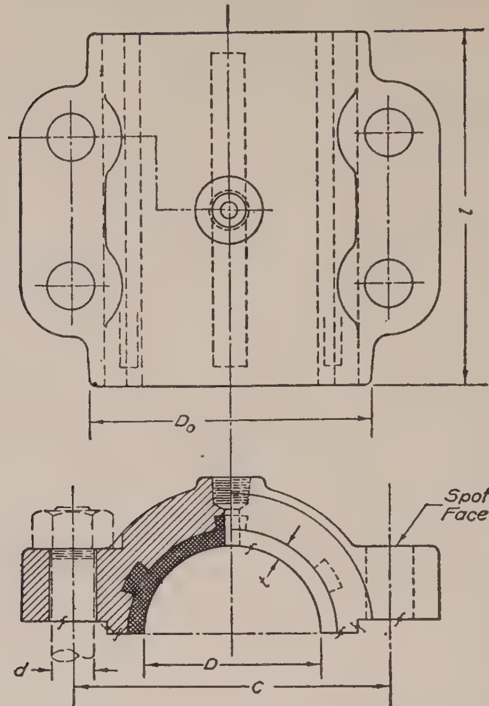


Fig. 221. Adjustable Base Plate.

A very common method of constructing a babbitt-lined bearing is to pour the molten metal directly in place, rigidly anchoring it to the bearing housing by means of recesses or dovetail slots (Fig. 222). For ordinary work, the hole is cast to size, then scraped or reamed, if necessary, to fit the shaft. In high-grade work, the hole is cast undersize, then bored and scraped to fit. In order to overcome the shrinkage which occurs while the babbitt is cooling, and to pack the metal more densely in place, the cast babbitt should be peened with a hammer or rolled under pressure before boring.

In more carefully constructed bearings, the wearing surfaces consist of removable shells, usually of bronze or cast iron. Very often these shells are babbitt-lined, and in some cases the shell is steel. Flanges at the ends (Fig. 223) are usually provided to keep the shells in position endwise, while a pin, screws, or thick shim plates may be used to prevent turning.



Principal proportions adapted from Machine Design Drawing Room Problems *by C. D. Albert*

FIG. 222. REPRESENTATIVE DESIGN OF BEARING CAP FOR SPLIT, BABBITT-LINED BEARING.

EMPIRICAL PROPORTIONS

For ordinary commercial split, babbitt-lined shaft bearings:

$D_o = 1\frac{7}{16} D + \frac{3}{8}''$,
$l = 3D$,
$d = \frac{1}{10} D + \frac{5}{16}''$,

$t = \frac{1}{16} D + \frac{1}{16}''$ but not to exceed $\frac{1}{2}''$,

$C = 1\frac{3}{5} D + \frac{1}{2}''$ to $1\frac{7}{10} D + \frac{3}{4}''$.

For split, babbitt-lined machine frame bearings:

$D_o = 1\frac{3}{4} D + \frac{1}{2}''$,
$l = 2D$ to $4D$,
$d = \frac{3}{16} D + \frac{1}{4}''$ for 4 or 2 bolts depending on length,
$t = \frac{1}{16} D + \frac{1}{8}''$ but not to exceed $\frac{1}{32} D + \frac{1}{4}''$,
$C = 1\frac{3}{5} D + \frac{1}{2}''$ to $1\frac{7}{10} D + \frac{3}{4}''$.

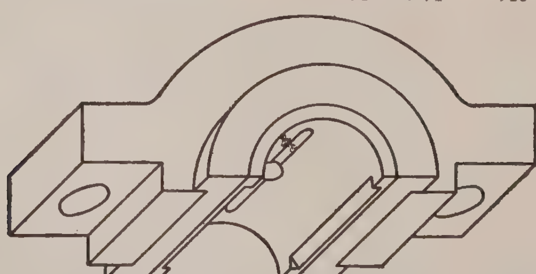


FIG. 223. BEARING CAP SHOWING OIL GROOVE AND CHAMFERED EDGES OF BEARING LINER.

178. Precision-Insert Bearing Shells. The automotive industry has been influential in the recent development of the precision-insert (interchangeable) type of bearing shell. This bearing, which may be either split or solid, consists of a very thin lining of babbitt * applied to a steel or bronze shell accurately ground on the back. The babbitt is usually applied by centrifugal casting or spinning and is broached to a mirror finish. The close tolerances observed in manufacturing these bearings (0.00025 to 0.0005 in. on important dimensions) eliminates the need of boring, scraping, or fitting during assembly, and obviously facilitates replacements.

The recommended values for the thickness of the bearing-metal lining as given by Mr. Willi † may be taken as approximately 0.010 in. per in. of shaft diameter for sizes from 0 to 6 in., with a minimum thickness of 0.020 in. No increase in thickness is required for shaft diameters from 6 to 11 in.

The thickness of the back depends upon the bearing size, material, type of bearing, and kind of service. Minimum safe values for the ratio t/D, where t is the thickness of the back and D the outside diameter of the shell, are given as follows:

Steel-back flanged (light duty)	$t/D = 0.017$
Steel-back flanged (heavy duty)	$t/D = 0.020$
Steel-back plain (light duty)	$t/D = 0.020$
Steel-back plain (heavy duty)	$t/D = 0.029$
Bronze-back flanged	$t/D = 0.034$
Bronze-back plain	$t/D = 0.050$

In the above classification, light duty is considered less than 500 p.s.i. of projected area, heavy duty over 500 p.s.i.

179. Adjustable Conical Bearings. There are various types of machine tools, especially grinding machines, in which the spindle must have a close running fit at all times, and must be free from any appreciable vibration. For this purpose, *adjustable conical bearings* of the type shown in Fig. 224 are frequently used. In this design, the bronze bushing is split longitudinally and is threaded at one end for an adjusting nut. A taper on the outside of the bushing fits a corresponding hole in the steel sleeve, which is made solid for rigidity. As the bushing is drawn into the sleeve, the spindle hole is contracted until the required fit is obtained.

* Tin-base babbitt is probably the most extensively used bearing metal for this purpose, although other special lining materials are used like copper lead, cadmium silver, cadmium nickel, and the like.

† *Industrial bearing design*, by A. B. Willi, Mechanical Engineering, May 1937, Vol. 59, No. 5.

180. Quarter-Box Journal Bearings. The ordinary two-piece split bearing is not suitable for the adjustments that become necessary on

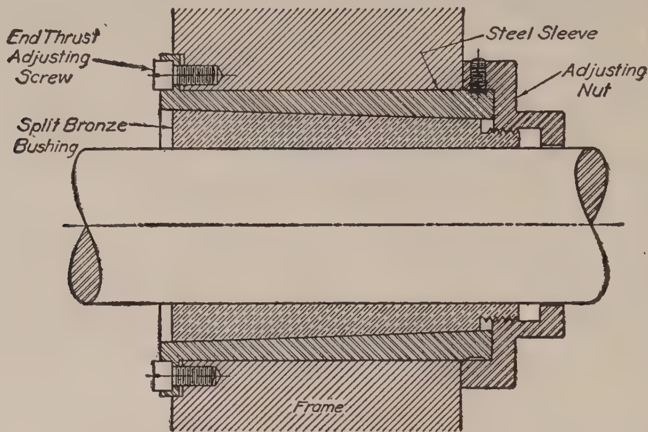
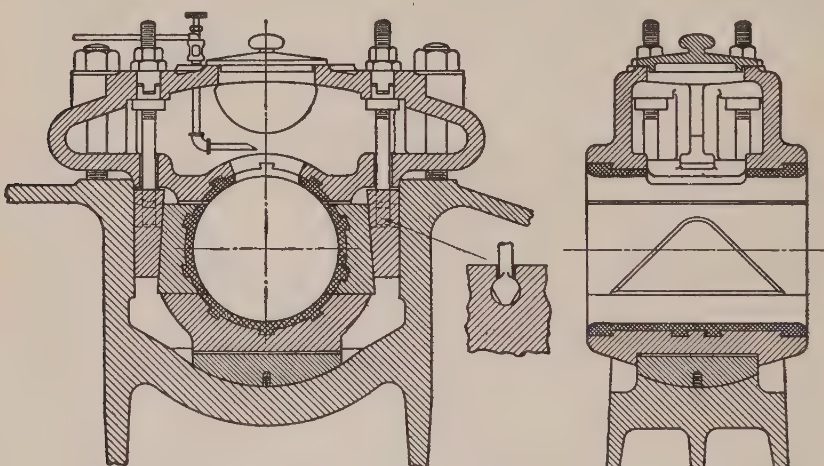


FIG. 224. SPLIT ADJUSTABLE CONICAL SLEEVE BEARING.

the important bearings of large power machinery, such as motors, turbines, and engines. When it is essential to retain the original position of a journal, if wear takes place in more than one direction,



Allis-Chalmers Mfg. Co., Milwaukee, Wis.
FIG. 225. QUARTER-BOX JOURNAL BEARING.

a *quarter-box bearing* is used (Fig. 225). In this type, vertical wear is controlled by shims, while horizontal wear is taken up by the adjusting wedges on the sides. By means of the spherical seat as shown, the bearing is self-aligning.

181. Half-Bearing Type Journal Box. If an axle carries a load which is vertical or predominately so, as in railway-car applications (Fig. 226), it is not necessary to have the bearing completely encircle the journal. In this illustration, the steel box casting is attached to the frame of the car truck, and is provided with an upper half-bearing bronze liner only, which rests on top of the axle journal. The bottom of the box, which is packed with wool waste saturated with oil, acts simply as an oil reservoir, a protection against dirt, and a means of holding the axle in place when the truck is lifted vertically.

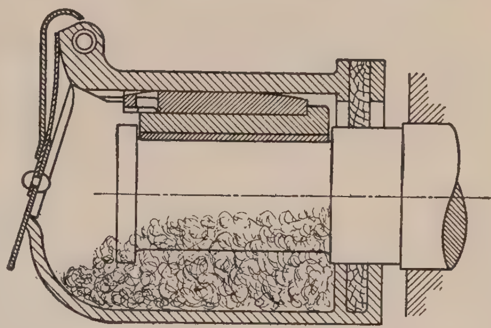
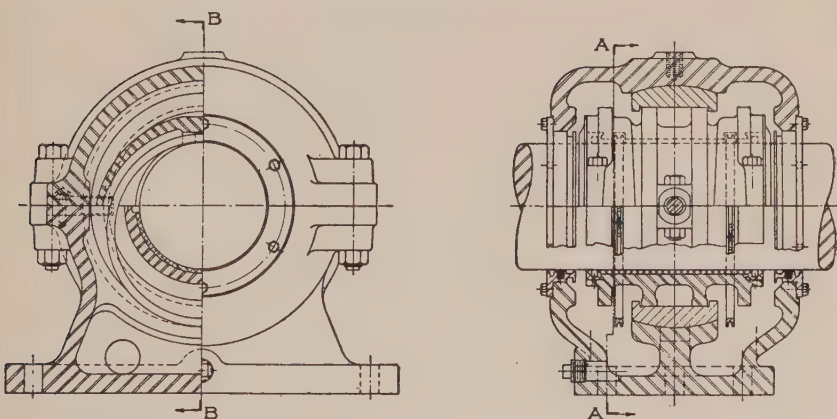


Fig. 226. Railroad-Car Journal Bearing.

182. Self-Aligning Bearings. Self-aligning bearings should be provided in cases in which it is extremely difficult to establish or maintain exact alignment, or where it is necessary to compensate for unavoidable flexure or weaving of a shaft under stress. For heavy-duty

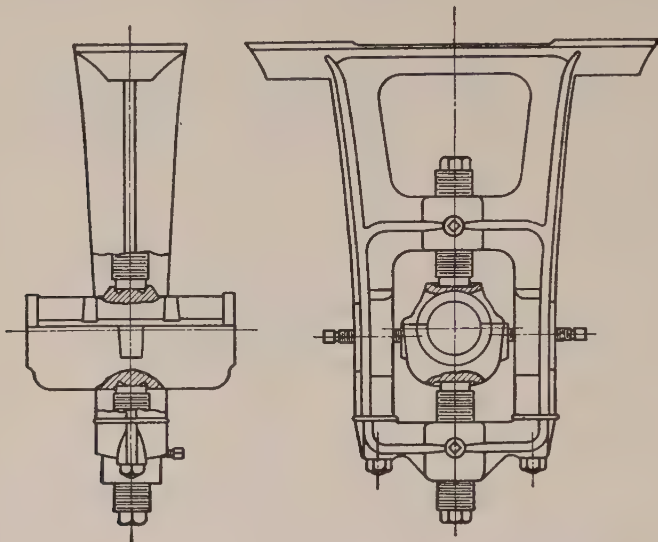


Kingsbury Machine Works, Inc., Philadelphia, Pa.

Fig. 227. Self-Aligning Journal Bearing.

service, this construction is accomplished by mounting the bearing in a housing having a spherical seat, as shown in Figs. 225 and 227, while for lighter service, various types of hangers are used.

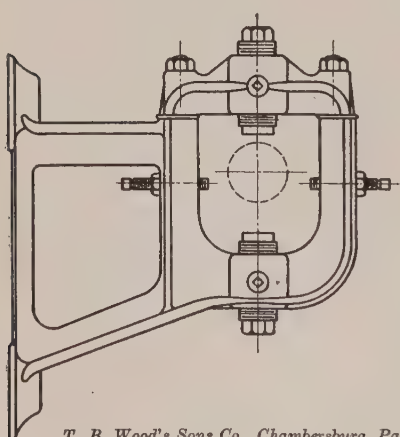
183. Hangers. Rigidly constructed, well designed hangers are important elements in the efficient transmission of power by means of



T. B. Wood's Sons Co., Chambersburg, Pa.

FIG. 228. DROP HANGER.

line shafting. Figure 228 shows a representative *drop hanger* of the type used for attachment to overhead beams or ceiling, while in Fig. 229 is shown a *post hanger*. The bearing is adjusted and held in position within the frame by means of vertical and horizontal adjusting screws. These hangers are arranged to give a ball and socket action, permitting the bearing to align itself automatically to any deflection of the shaft. Sufficient adjustment is provided to take care of any reasonable settling of the building, irregularities of supports, or improper erection.



T. B. Wood's Sons Co., Chambersburg, Pa.

FIG. 229. POST HANGER.

Ring, chain, or wick-oiled plain bearings have been used ordinarily in hangers for line shafting, but more recently, ball and roller bearings have found favor because of their efficiency and their low maintenance

cost. Hangers and bearings in all ordinary sizes and styles are commercially carried in stock. Complete dimensions and specifications may be found in manufacturers' catalogs.*

184. Methods of Lubrication. Many different methods are employed in supplying lubricant to bearings. The method used depends upon the importance of the bearing and the service expected. Oil supplied to bearings by hand is always subject to neglect by the attendant and is frequently the cause of bearing failure. At the time of application, the supply is generally excessive and results in leakage, while at other times, the supply may be insufficient to maintain the oil film. Hand oiling should be tolerated only on small unimportant bearings involving moderate loads and speeds, or, where it is impossible, for practical reasons, to employ automatic lubrication.

A number of lubricating devices have been developed to supply oil continuously and at a regulated rate. *Force-feed lubricators* of the mechanically operated type consist of an oil reservoir and one or more small pumps driven from the machine. Oil is delivered at a certain rate and under pressure, if necessary, to each bearing, but is not returned to the lubricator. This device is a very positive method of lubrication and is especially adapted to cylinder lubrication. The cost precludes its general use for ordinary applications.

Wick-feed oilers in many different forms employ the principle of the capillary action of some absorbent material in carrying oil to the bearing. They are dependable and extensively used. The railway-car journal bearing (Fig. 226) is an application of a bottom wick feed. In the siphon wick-feed oiler shown in Fig. 230, oil is drawn from the reservoir through a wick of wool yarn, the rate of flow being regulated by the number of strands of yarn.

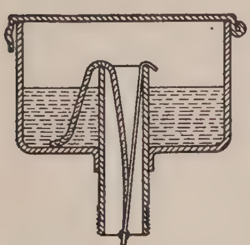


FIG. 230. SIPHON WICK-FEED OILER.

Drop-feed oilers of the type shown in Fig. 231 consist of an oil reservoir provided with an adjustable needle valve to regulate the flow. If there should be any impurities in the oil, the small aperture may become clogged, so that the feed does not operate.

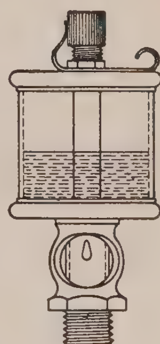


FIG. 231. DROP-FEED OILER.

185. Self-Oiling Bearings. Self-oiling bearings are provided with reservoirs which store sufficient oil to enable the bearing to run for

* See, for instance, the catalogs of the Link Belt Company, Dodge Manufacturing Corporation, T. B. Wood's Sons Company, etc.

long periods without attention. *Ring-oiled bearings*, as illustrated in Fig. 227, employ one or more rings which hang loosely over the journal and revolve with it through friction. The lower part of the rings dipping in the oil carries it to the top of the journal where it is distributed through oil grooves to the bearing surfaces. The ring-oiling method is considered one of the most reliable methods of automatic lubrication for medium and fairly high-speed journals, and is extensively used for line shafting and important machine bearings. To function properly, a surface speed of at least 50 ft. per min. is recommended by the Dodge Manufacturing Corporation. At very high speeds, the oil may be thrown off because of the centrifugal action. *Chain-oiled bearings* employ the same principle as the ring-oiled bearing, except that an endless chain is substituted for the ring. In a *collar-oiled bearing*, a collar fixed on the shaft dips in the oil and carries it upward. Chain-oiled or collar-oiled bearings are well suited for large shafts running at slow speeds.

186. Splash Lubrication. This method of lubrication is employed in machines that have cranks or gears enclosed in a housing which acts as a reservoir for oil. As the moving parts dip in the oil, a spray is formed which reaches the bearings directly, or which may be collected in channels and then distributed to the various bearings. With the oil level properly maintained, a constant supply is delivered to the bearings.

187. Oil Circulating Systems. With this system a continuous and abundant supply of oil is furnished by means of a pump. The oil under pressure may be forced directly to the bearings, or it may be delivered to an elevated tank and flow by gravity through pipe lines to the bearings. From the bearings, the oil returns by gravity to the reservoir and is recirculated by the pump. The system usually includes oil strainers and filters, and in some cases an oil-cooling device. This is the most advanced method of lubrication and is extensively used in all types of engines.

188. Grease Lubrication. Grease is a compound consisting of a soap base to which has been added a certain percentage of mineral oil. The hardness of the grease is controlled by varying either the content or viscosity of the oil. For very high bearing pressures, usually accompanied by high temperatures, greases containing certain smooth mineral solids, such as soapstone, graphite, or mica, have been used successfully. Grease is not a mobile lubricant; hence it must be applied directly to the surface to be lubricated, either under pressure or

through a breakdown of its structural formation produced by frictional or induced heat.

The lubrication of the chassis of an automobile is a well known example of the application of grease forced under high pressure through dust-tight fittings to the various bearing surfaces. Compression grease cups are also used to flow grease under pressure. In the screw-down types shown in Fig. 232, the grease is forced under the action of a screw, while in the spring compression type, a spring applies the pressure until the cup is empty.

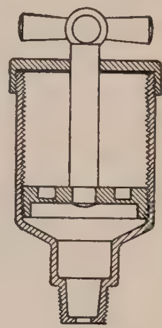


FIG. 232. SCREW TYPE COMPRESSION GREASE CUP.

The use of grease is suitable for bearings operating under intermittent service where it would be difficult to retain oil. Grease will also give more continuous lubrication in cases where oiling by hand is too infrequent or the supply of oil indefinite. A combination of low speed, high pressure, and excessive temperature is one which may require the use of a suitable grease.

189. Oil Grooves. The purpose of grooving a bearing is to assist in the distribution of oil between the rubbing surfaces. The oil should be introduced in the region of low pressure, which usually is through the cap of a split bearing. For horizontal bearings, a longitudinal groove stopping about 1/2 in. from the ends, as shown in Fig. 223, is considered a desirable construction. Complicated and elaborate grooves may be actually objectionable if they extend into the load-carrying area and destroy the continuity of the oil film. Bearings 8 in. or more in length require two or more points at which oil should be supplied to the groove to effect a quick distribution.

Oil grooves should be cut comparatively shallow, with the edges well beveled or rounded to permit the free entrance of oil between the journal and the bearing. Deep, sharp-edged grooves scrape the oil from the journal and destroy the oil film. For the same reason, the edges at the joint of split bearings should also be chamfered off, as in Fig. 223.

190. Theory of Lubrication—Introductory Remarks. Lubrication has for its object the reduction of the friction between parts that slide on one another. Lubricants may be either oils or greases. While a true lubricant should furnish a layer of moving molecules between the two sliding surfaces, there is no exact knowledge at the present time of their mode of action.

In cases where there may be whole unbroken layers of moving molecules, the condition is referred to as *perfect* lubrication, or *film*

lubrication. In other cases, the layer is broken and interrupted by roughnesses projecting from the surfaces, so that there is solid contact between portions of the moving surfaces. This condition is known as *imperfect* lubrication. Some types of lubrication produce perfect films under certain conditions of pressure, oil viscosity, and relative sliding velocity, but broken films under other conditions. Lubrication of this type is designated as *semi-perfect* or *boundary* lubrication. Copious splash, forced-feed lubrication, submersion, or flooding of the sliding surfaces are methods which give perfect lubrication. Drop oilers, wick oilers, grease cups, capillary oilers, etc., give imperfect lubrication. Ring-oiling, collar-oiling, or chain-oiling devices in journal bearings furnish the conditions for semi-perfect lubrication.

Solid lubrication is sometimes spoken of in connection with the use of graphite. Actually graphite is not a lubricant. It does possess two properties which in some cases are of great value. In the first place, it fills in cavities and depressions; hence it gives a smooth bearing surface and reduces the possibility of a ruptured oil film. In the second place, there is a considerable surface adhesion between oil and graphite. This property promotes the preservation of thin oil films, even when the supply of lubricant is much reduced, or where the motion is of a reciprocating character, so that oil is not being dragged continuously into the region of high bearing pressure. It is therefore often said that engines supplied with graphitized oil have run for hours after failure of the lubricating system. For very slow or for oscillating motion, as in spring shackles, where some friction losses are not serious, reputable firms have developed "oil-less" bearings containing graphite and probably impregnated with oil in the process of manufacture. Such bearings may be run without lubrication. Mica and soapstone are materials possessing properties similar to graphite, and, in the rubber industry for instance, they are used extensively as dry lubricants.

All bearings or sliding surfaces must be lubricated if excessive friction, heating, and scoring are to be avoided. Even ball and roller bearings must be lubricated, partly because there is sliding between balls or rollers and retaining rings or cages, and partly because it is necessary to exclude dust and gritty particles, as well as moisture, which would cause rusting.

191. Theory of Lubrication (Continued). In dealing with the theory of lubrication, it is necessary to define certain terms. In a step bearing supporting a load F, the bearing pressure p is the load divided by the actual area upon which it is supported. If a shaft of diameter D rests on its end, the bearing pressure $p = F/(\pi D^2/4)$. This

value is an actual pressure assumed to be uniformly distributed over the bearing area. In a journal bearing of diameter D and length of bearing area l, supporting a load F at right angles to its axis, the bearing pressure is defined as F/lD, that is, it is the load divided by the projected area. This quantity is not a uniform pressure distributed on that area, but is merely a convenient index of the intensity of loading.

The total friction force F' at the circumference of a journal is set equal to fF, or $fplD$, where f, strictly speaking, is merely a proportionality constant and not a friction coefficient in the usual sense of the word. The friction torque on a journal bearing is $F'D/2$. The rubbing velocity of a journal in ft. per min. is $\pi DN/12$, where D is in in. and N is the number of revolutions per minute. This rubbing velocity is usually denoted by V. In modern theory of lubrication, the rubbing velocity is sometimes given in in. per sec. This term may be denoted by U and is equal to $\pi DN/60$.

As a rule, the viscosity of the lubricating oil is determined by the velocity of outflow from various types of *viscosimeters*, such as the Saybolt, Redwood, or Engler instruments. In lubrication theory, however, the absolute viscosity z must be used. This quantity is defined as the force required to slide with unit velocity relatively to one another, two parallel surfaces of unit area separated by an oil film of unit thickness. The force is called the *viscous drag* or shearing force per unit area. If the oil film thickness is h, we have

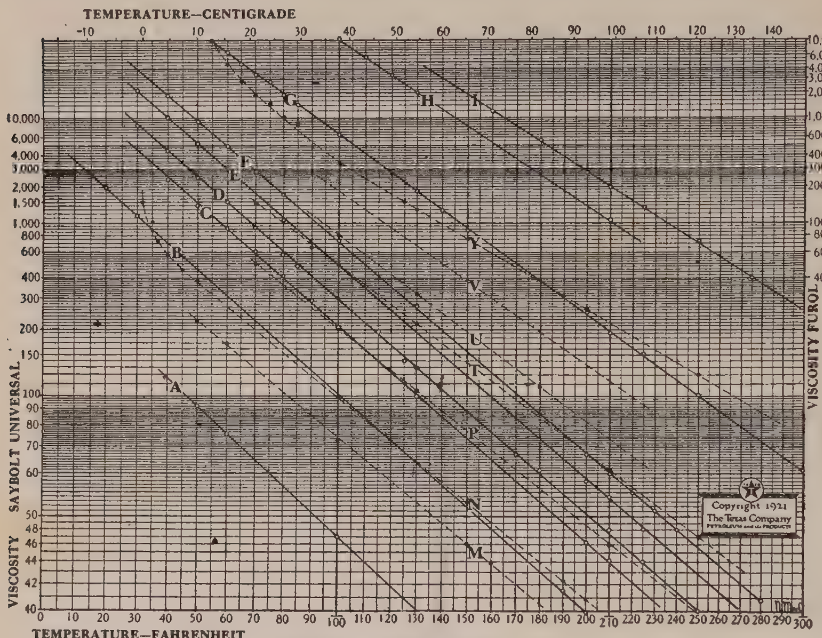
$$\text{viscous drag} = z\frac{U}{h}. \tag{1}$$

The absolute viscosity is usually given either in absolute c.g.s. units, called *poises*, or in *centipoises*, which are the hundredth part of a poise. To convert a viscosity given in centipoises to one in lb.-in.-sec. units, now often called *reyns*, the quantity is multiplied by 1.46×10^{-7}.

The chart in Fig. 233a makes it possible to convert viscosimeter readings into *kinematic viscosity*. The absolute viscosity is the kinematic viscosity multiplied by the specific gravity.* To illustrate the use of the chart, consider an oil of 100 seconds Saybolt Universal viscosity and of specific gravity 0.9. The scale at the bottom and the left combined with the curve for Saybolt Universal gives a kinematic viscosity of 20. Hence the absolute viscosity is $0.9 \times 20 = 18$ centipoises. In in.-lb.-sec. units it would be $1.46 \times 18 \times 10^{-7} = 2.63 \times 10^{-6}$.

* The specific gravity of ordinary lubricating oils at running temperatures may vary from about 0.8 to somewhat over 0.9 with 0.9 as an average satisfactory for approximate computations.

Viscosity drops rapidly with increasing temperature. It has been found that in a plotting such as shown in Fig. 233, the relation between the temperature and the logarithm of the viscosity is represented by a line that is practically straight for all commonly used oils.



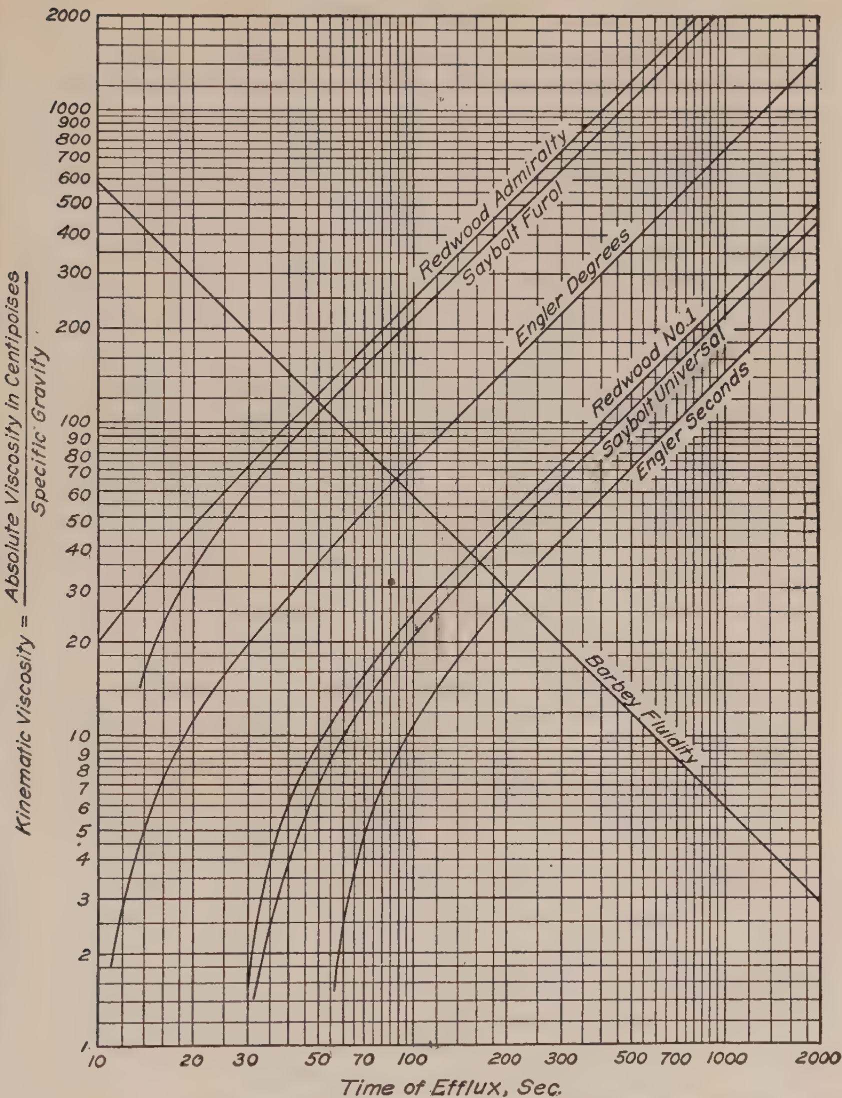
[VISCOSITY—TEMPERATURE CHART]

FIG. 233.

The lines A, B, . . Y represent the viscosity-temperature relations of individual oils. Oils of a like specific gravity (from the same crude) have the same slope.

192. The Relation of Pressure, Rubbing Velocity, Viscosity, Temperature, and Friction. The relation between bearing pressure, rubbing velocity, oil viscosity, and friction was long unrecognized, and experiments seemed to contribute little until Beauchamp Tower * in 1883 published his experiments with railway-car journals. These experiments proved that the adhesion between oil and journal was such that the journal could drag the oil from a region of low pressure into one of very high pressure and actually interpose an oil film between the bearing and journal, even in the high-pressure region. Thus it was proved that perfect film lubrication was possible and that the laws of hydrodynamics could be applied to the pressure conditions in the oil film. Osborne Reynolds † proved mathematically that in a

* Proceedings, Institution of Mechanical Engineers, 1883, p. 632, and 1884, pp. 29, 34.
† Philosophical Transactions, Royal Society, 1886, p. 157.



After The Mechanism of Lubrication *by Robert E. Wilson and Daniel P. Barnard, 4th*

FIG. 233a. VISCOSIMETER CONVERSION DIAGRAM.

For the curve marked *Engler Degrees* the horizontal scale gives 10 × *Engler Degrees*, that is, 10 × Engler Seconds/51.

For *Barbey Fluidity* the horizontal scale gives flow in cubic centimeters per hour, the time of efflux being standardized at a fixed value.

In computing kinematic viscosity the specific-gravity and the absolute-viscosity values used should apply to the same temperature. The specific gravity is specific gravity referred to that of water as unity. The same figure is obtained by expressing it in grams per cubic centimeter.

film-lubricated bearing under load, the journal assumes an eccentric position, as shown in Fig. 234; that the pressure accumulates in front of the constricted passage at H so as to produce the acceleration necessary to propel the oil through this passage; and that it is this pressure accumulation that supports the load.

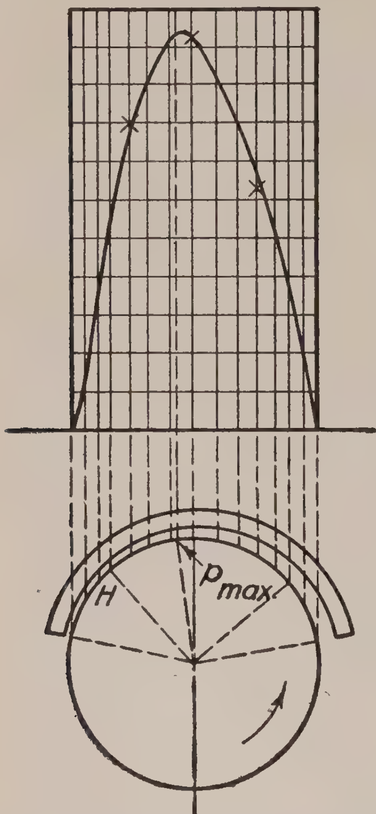


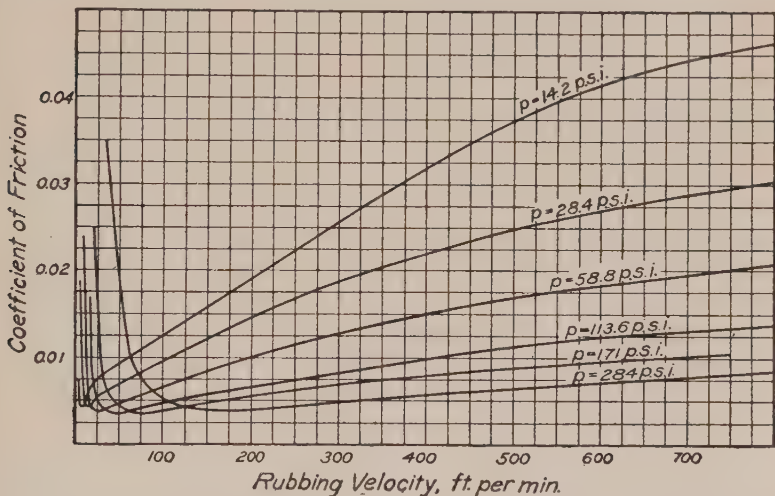
FIG. 234. ECCENTRIC POSITION OF JOURNAL AND RADIAL PRESSURE DISTRIBUTION IN RAILWAY BEARING.

The final step in developing the fundamental relations of film lubrication was brought about by Stribeck,* who experimented with ring-oiling bearings. By plotting the variation of the coefficient of friction against a varying rubbing velocity, but with constant pressure and oil viscosity, he obtained the curves shown in Fig. 235. For each pressure, when the velocity is low, there is a region in which the friction drops rapidly with increasing velocity. As the speed of the journal increases, more oil is dragged in, and more of the roughnesses are covered. Finally, perfect film lubrication is established and a minimum friction is reached. The higher the pressure, the greater is the velocity necessary to establish this minimum point, since the pressure has a tendency to squeeze out the oil at the ends of the bearing. The curves for higher pressure therefore show a minimum friction at a higher velocity than those for lower pressure. Beyond the minimum point, the friction increases because, as formula (1) shows, the viscous drag increases with the rubbing velocity. The increase is not quite linear, because the film thickness (h) also increases somewhat and reduces the drag.

The shift of the minimum point to the right with increasing pressure causes entirely different laws to obtain in the low-speed region from those in the high-speed region. In the low-speed region, the friction coefficient increases with pressure and decreases with speed;

* Zeitschrift des Vereines deutscher Ingenieure, 1902, p. 1341, etc.

in the high-speed region it decreases with pressure and increases with speed. In the low-speed region the problem is to keep the oil in the bearing, hence high viscosity reduces friction in that region. In the high-speed region, high viscosity simply means high viscous drag,



Stribeck Z. V. d. I. 1902, p. 1345

FIG. 235. DEPENDENCE OF COEFFICIENT OF FRICTION ON RUBBING VELOCITY FOR RING-OILING CAST-IRON BEARING.

hence the coefficient of friction increases with increased viscosity. Finally, since temperature reduces viscosity, high temperature reduces friction in the high-speed region, and increases it in the low-speed region. Low-speed bearings should run as cool as possible, whereas high-speed bearings may well be more than hand-hot. If the lubrication properties of oil become somewhat uncertain at 225° to 250° F., which is not true for all oils, it is advisable to keep bearing temperatures below say 180° F., or 80° C. For high-speed bearings, temperatures of 160° F. or 70° C. are beneficial.

193. Recent Theory and Working Formulas. Important research has continued on bearing lubrication in adapting the fundamental theory to the needs of practice, or in applying it to the progressive development in bearing design. Wilson and Barnard * proved that for any one bearing, all of Stribeck's curves could be reduced to a single curve by plotting the coefficient of friction not against the rubbing velocity V, but against a quantity zN/p, in which N is the r.p.m. Two such curves, one for a cast-iron bearing and one for a

* S.A.E. Journal, July, 1922, p. 49.

white-metal bearing (babbitt), are shown in Fig. 236. It will be noticed that perfect film lubrication and minimum friction are reached at a much smaller value of zN/p in the case of the white-metal than in the case of the cast-iron bearing. The reason for this condition

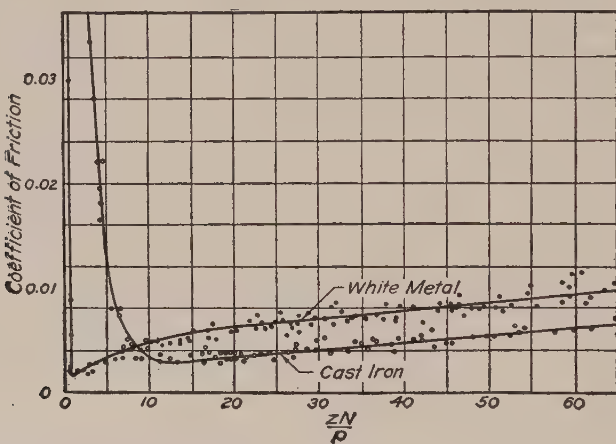


Fig. 236. Relation between Coefficient of Friction and zN/p for Cast-Iron and White-Metal Bearings. Data Obtained by Stribeck with a Steel Shaft 70 mm. (2.76 in.) in Diameter. White-Metal Bearing 137 mm. (5.40 in.) Long. Cast-Iron Bearing 230 mm. (3.06 in.) Long. (From S.A.E. Journal.)

is that the bearing surface of the white-metal bearing attains a greater smoothness when properly run in, and hence requires a smaller film thickness to cover all roughnesses and establish perfect lubrication. It is to be noted, however, that the cast-iron bearing has a lower coefficient of friction at high zN/p values. Of far greater importance is the fact that, in the case of the white-metal bearing, perfect lubrication is attained at extremely low values of zN/p, so that by proper choice of viscosity and bearing pressure it would seem possible to obtain film lubrication with such a bearing in most cases. To do so is becoming more and more the object of scientific bearing design, since, as Fig. 236 illustrates, there is an enormous increase in friction as the bearing passes into the region of imperfect lubrication.

One way to ascertain if perfect lubrication is to be attained would be to compute zN/p and make sure that the value used is several times greater than the necessary minimum value. For white-metal bearings, Fig. 236 seems to show that the minimum value is less than 2, with z in centipoises, and it has often been thought that safe minimum values of zN/p for design purposes should be 15 to 25. In Table 30 are listed

TABLE 30

Recommended Load Conditions for Bearings and Values of zN/p

Location of Bearing	Lubricant	p	N	z	zN/p	c/D*
Automobile crank shaft	Medium machine oil	300 to 700	900 to 1400	7 to 8	15 to 25	<0.0010
Aeronautic-engine crank shaft	Heavy engine oil	300 to 1800	1800 to 2000	7 to 8	15 to 25	<0.0010
Stationary gas-engine main	Medium machine oil	500 to 700	250 to 800	30	25	0.0010
Stationary gas-engine crank pin	Medium machine oil	1500 to 1800	250 to 800	50	15	<0.0010
Stationary gas-engine cross-head	Medium machine oil	1500 to 2000	250 to 800	40	10	<0.0010
Diesel engine main	Heavy engine oil	250 to 600	60 to 160	30	15	0.0010
Diesel-engine crank pins	Heavy engine oil	1500 to 4000	60 to 160	40	2 to 5	0.0010
Marine steam-engine main	Machine oil	275 to 500	180	30 to 40	20 to 30	<0.0010
Marine main crank pin	Machine oil	400 to 500	180	30 to 40	20	0.0010
Stationary slow-speed main	Heavy machine oil	80 to 400	40 to 80	70	20	<0.0010
Stationary slow-speed crank pin	Heavy machine oil	800 to 1300	40 to 80	80	6 to 8	<0.0010
Stationary slow-speed cross-head	Heavy machine oil	1000 to 1500	40 to 80	70	5	<0.0010
Stationary high-speed main	Engine oil	60 to 250	360	15	25	<0.0010
Stationary high-speed crank pin	Machine oil	400 to 1500	360	30	6 to 15	<0.0010
Stationary high-speed cross-head	Machine oil	1500 to 1800	360	25	5	<0.0010
Locomotive drive-wheel	Heavy machine oil	550	250	100	30 to 50	<0.0010
Locomotive crank pin	Heavy machine oil	1500 to 2000	250	100	5 to 8	<0.0010
Locomotive cross-head	Heavy machine oil	3000 to 4000	250	130	6 to 8	0.0010
Marine steam turbine	Light machine oil	85	2,000	10	250	0.0010
Stationary steam turbine	Machine oil	400 to 950	2,000	20	100 to 200	0.0010
De Laval 7-hp. steam turbine	Light machine oil	7 to 15	30,000	1	1500 to 3000	0.0020
De Laval 300-hp. steam turbine	Light machine oil	20 to 25	10,500	2	1000	0.0020
Railway-car axle	Heavy machine oil	300 to 450	300	100	50 to 100	
Generator and motor	Engine oil	30 to 80	150 to 500	25	200	0.0010
Rolling-mill main	Hot neck grease	1800 to 2500	60	—	<1	—
Cotton-mill spindle	Spindle oil	1	8,000 to 12,000	2	10000	0.0050
Gyroscope		750 to 850	800 to 1,500	60 to 30	55	0.0013

* The symbol c denotes the clearance, that is, the difference between the journal and the bearing diameter. D is the bearing diameter.

values actually found in practice. It would seem that the rolling-mill main bearing is about the only one for which perfect lubrication would be out of the question. In all other cases, perfect lubrication would seem attainable, at least with babbitt-lined bearings, although in the case of the Diesel-engine crank pin and several other applications, the conditions may be on the border line.

194. Computation of Bearing Constant. Efforts are being made constantly to make possible the computation of pressures and friction forces from the hydrodynamic relations in the oil film and the dimensions of the bearing. In Germany, E. Falz * has proposed a comparatively simple system which involves some rather arbitrary and conventionalized assumptions with regard to the influence of end-leakage, the ratio of bearing length to diameter, etc. More painstaking investigations have been undertaken in America by Kingsbury, Howarth,† and others on the influence of end-leakage and various bearing arrangements.

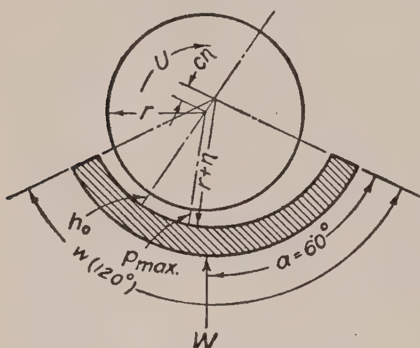


FIG. 237. GEOMETRICAL REPRESENTATION OF 120 DEG. CENTRALLY SUPPORTED JOURNAL BEARING WITH RUNNING CLEARANCE.

These efforts were brought to a definite, although possibly only temporary, conclusion in a paper by Sydney J. Needs ‡ of the Kingsbury organization. While Needs confined his research to centrally supported bearings of 120 deg. arc width (see Fig. 237), it is no doubt safe to assume that pressure and viscosity conditions which give perfect film lubrication in this type of bearing, will also give the same conditions in bearings of greater arc width. A smaller arc width needs hardly to be considered.

The radial clearance is denoted by η, that is, the difference between the journal radius r and the bearing radius $(r + \eta)$ (see Fig. 237). The distance between the center of the bearing and the center of the journal is denoted by $c\eta$, c being the *eccentricity factor*. The film thickness h_0 at a point of closest approach is then equal to $\eta - c\eta = \eta(1 - c)$.

* *Grundzüge der Schmiertechnik*, by E. Falz, Berlin, Julius Springer, 1926.

† See Kingsbury, Transactions A.S.M.E., 1931, APM–53–5, p. 59; Howarth, Transactions A.S.M.E., 1935, MPS–57–2, p. 169, 1934, MPS–56–2, p. 891, and many other A.S.M.E. papers.

‡ *Effect of side leakage in 120 deg. centrally supported journal bearings*, Transactions A.S.M.E., 1934, APM–56–16, p. 721.

Two characteristic functions are introduced, one the *load function* $Wh_0^2/(zUr^2)$, in which W is the load per unit bearing length ($W = F/l$); and one the *friction function* fr/h_0, where f is the coefficient of friction. In Figs. 238 and 239, there are given two sets of curves that show the

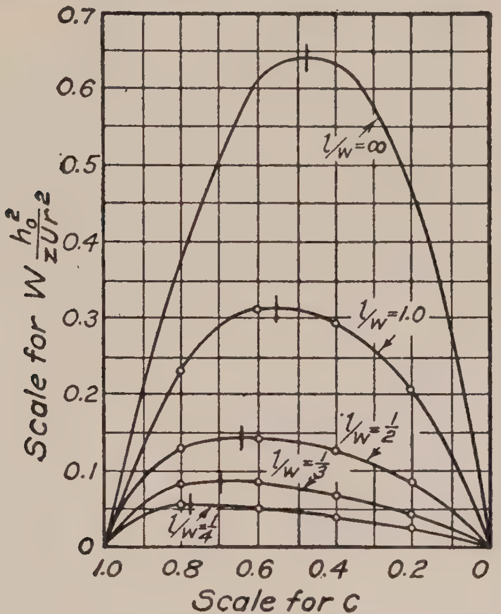


FIG. 238. EFFECT OF ECCENTRICITY ON LOAD FUNCTION. Viscosity z is in reyns.

dependence of these functions on the eccentricity factor c for various values of l/w, where w is the arc width of the bearing. (For 120 deg. bearings $w = \pi D/3$.)*

It will be observed that for bearings which are not too short, the curves for the friction function run very flat, and for l/w ratios from 1/2 to ∞ the c-values for optimum load function vary only from a little more than 0.6 to a little less than 0.5. The maximum load function for the "square" bearing ($l/w = 1$) lies at 0.55, and this value also gives a very low friction function. A very wide bearing is apt to give a high total of friction work, while a very narrow bearing can withstand only a low load. An approximately square bearing is a good compromise. For a first approach to a bearing problem such a bearing may well be assumed. An example will now illustrate how the permissible bearing pressures and the probable friction work for such a bearing may be obtained.

* In Need's original article, w and l are interchanged, w standing for the axial dimension. The more common practice, however, is to denote the bearing length by l.

EXAMPLE. Assume for instance that the bending moments require a shaft diameter of 6 in. and that the speed is 1000 r.p.m. We will assume also that the bearing has a c value of 0.55, and that a load function of 0.31 and a friction function 5.6 will be used in the computation. We then have $U = \pi \times 6 \times 1000/60 = 314$ in. per sec. Assume, with Needs, an oil having an absolute viscosity of 3.4×10^{-6} in in.-lb.-sec. units. (The viscosity in centipoises is $3.4 \times 10^{-6}/1.46 \times 10^{-7} = 23.3$. With a specific gravity of say 0.9, the kinematic viscosity would be $23.3/0.9 = 25.9$, and from the chart in Fig. 233a, the Saybolt viscosity about 130 seconds.) Assuming a medium fit with a maximum radial clearance of 0.003 (see page 7), we have $h_0 = 0.003(1 - 0.55) = 0.00135$. From the load function,

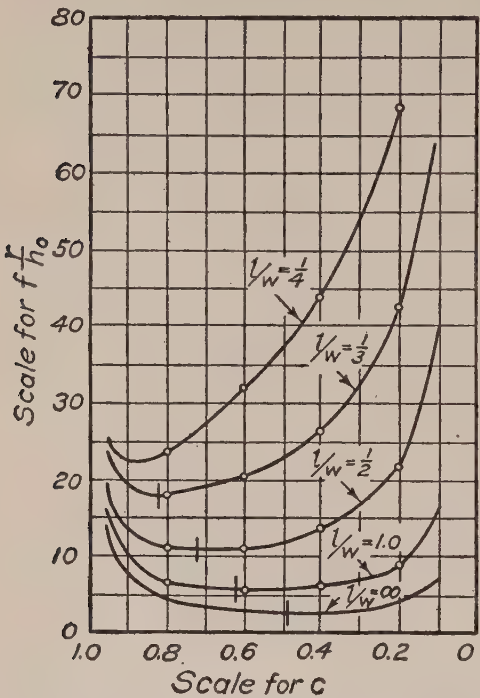


FIG. 239. EFFECT OF ECCENTRICITY ON FRICTION FUNCTION.

$$\frac{Wh_0^2}{zUr^2} = 0.31,$$

we find

$$W = \frac{0.31zUr^2}{h_0^2}$$

$$= \frac{0.31 \times 3.4 \times 10^{-6} \times 314 \times 3^2}{0.00135^2}$$

$$= 1620 \text{ lb.}$$

This value is the load per in. of axial length. The pressure p is $W/D = 1620/6 = 270$ p.s.i. For the friction function we have $fr/h_0 = 5.6$, or

$$f = 5.6 \times \frac{h_0}{r} = 5.6 \times \frac{0.00135}{3} = 0.0025.$$

The expression for the permissible bearing pressure is easily derived from the load function, assumed to be constant. We have

$$(2) \qquad p = \frac{W}{2r} = \text{const.} \frac{zUr}{h_0^2}.$$

The value of the constant is about 0.31/2 or 0.155 for $l/w = 1$ and $c = 0.55$.

For constant speed and shaft diameter, the permissible pressure can be increased by increasing the oil viscosity z and by decreasing the minimum film thickness h_0, that is, the clearance. It can also be increased by lengthening the bearing, as shown by the curves in Fig. 238.

It may seem strange that the friction function fr/h_0 for optimum conditions does not contain the oil viscosity. Actually, the attainment of the minimum oil film thickness h_0 is dependent on the viscosity. By introducing the value for h_0 from equation (2), we find

$$f = Q\sqrt{\frac{zU}{pr}}, \tag{3}$$

where Q is a factor dependent on the eccentricity factor c. Since U/r is the angular velocity of the journal, the expression under the root sign is in fact identical with the function zN/p, which Wilson and Barnard found in determining the coefficient of friction.

In the example computed above, we used a good compromise for the values of c, the load function and the friction function. It may happen that the permissible bearing pressure so derived is not high enough to carry the load with the dimensions established for the bearing. As formula (2) shows, the pressure can be increased by reducing h_0, that is, increasing c, if the clearance is not reduced. Stribeck's original curves in Fig. 235 show that in the high-speed region the coefficient of friction decreases with increased pressure. In fact the product pf remains practically constant, so that the friction loss in the bearing is not increased by an increase in pressure.

The reduction of the minimum film thickness, however, increases the danger of having imperfect lubrication. Designers in general, at least in this country, have probably preferred to judge this danger by the determination of zN/p values, as listed in Table 30. As already stated, the maximum permissible bearing pressure is very often determined by taking established values from successful designs, as listed in Table 30, instead of relying upon a theoretical derivation.

On the other hand, the practice as recommended by Falz in Germany is to assume a minimum permissible oil-film thickness. For ground journals and excellently finished bearings, Falz puts this thickness at 0.01 mm., that is, 0.0004 in. He also assumes that the best practical compromise for load capacity and low friction is to put the eccentricity factor c equal to 0.5, and hence the minimum film thickness equal to one-half of the radial clearance, or one-fourth of the diametral clearance.

The assumed eccentricity factor agrees well with the one adopted here on the basis of Needs' investigations, which are far more searching than those of Falz. We may, however, ascertain what capacity and coefficient of friction would be attained if Falz's minimum film thickness were adopted.

We have

$$f = \frac{5.6h_0}{r} = 5.6 \times \frac{0.0004}{3} = 0.00075.$$

With the oil viscosity already chosen, we have, from equation (2),

$$p = \frac{0.155 \times 3.4 \times 10^{-6} \times 314 \times 3}{0.0004^2} = 3100 \text{ p.s.i.}$$

This is an enormous pressure. If it is considered that the minimum film thickness is not safe for bearings of ordinary finish, the pressure must be reduced very appreciably, or else the viscosity of the oil must be increased. If the film thickness is doubled, the permissible pressure for the same oil viscosity is reduced to one-fourth, that is, to somewhat less than 800 p.s.i.

195. Heating of Bearings. With the notation used, the friction force at the journal circumference is $fF = fplD$, and the friction work per minute is $fplDV$, where V is the rubbing velocity in ft. per min. The heat resulting from this friction work must be conducted and radiated away from the bearing, if excessive temperature rises are to be avoided. In this connection, the conduction of the heat through adjoining metal sections is less of an obstacle than the transmission of the heat from the surface of the bearing to the surrounding atmosphere or surrounding objects. Radiation constants have been determined for a variety of materials, but the outer surface of the bearing is often of rather involved shape; the texture and finish are uncertain; heat dissipation depends not on the bearing alone but also upon the metallic masses which it adjoins, as well as on the amount of air in motion around it. In addition, the amount of heat to be radiated is dependent upon a coefficient of friction of uncertain value. For the same type of bearing, the radiating surface can be assumed to be approximately proportional to the projected area of the journal. For equilibrium between heat generation and radiation we may therefore set $fplDV$ equal to a constant $\times lD$, or $pV = \text{constant}/f$, or simply equal to a constant, since f may not vary a great deal. Bearing computations based on pV values may still be useful for rapid orientation and a list of such values which have been regarded as permissible in the past is therefore given in Table 31.*

The values in the table (page 241) can be much increased if the bearings are well run in and are given careful attention. Especially can these values be increased if the bearings are cooled by being subjected to a rapid movement through the air or by direct fan ventilation. Thus, for passenger-car journals, pV values up to 118,000 are in use, and for

* From Bach, *Maschinenelemente*, 7th ed., p. 385, and Halsey's *Handbook for Machine Designers, Shop Men, and Draftsmen*, p. 11.

TABLE 31

PERMISSIBLE pV VALUES

CLASS OF BEARING OR JOURNAL	pV VALUE
Mill shafting, with self-aligning cast-iron bearings, grease or imperfect oil lubrication, maximum value	12,000
Mill shafting, self-aligning ring-oiled babbitt bearings, maximum	24,000
Self-aligning ring-oiled bearings, continuous load in one direction	35,000 to 40,000
Crankshaft journals with bronze bearings	22,000
Crankshaft bearings with babbitted bearings, max.	59,000
For excellent radiating conditions	133,000

locomotive journals values up to 220,000. For locomotive crank pins, the values may even go to 370,000.

Falz * increases the radiating constant 2.3 times for an air velocity of 100 ft. per min. and 9.2 times for a velocity of 2400 ft. per min., that is, for crank pins moving with this velocity through the air.

With the advent of film lubrication and scientific insight into the nature of such lubrication, attempts were made to establish the radiating capacity of bearings on a more scientific basis. Stribeck's curves showed that in the region of film lubrication the coefficient of friction dropped with the pressure, so that for any given temperature the product pf was very nearly constant. Lasche † gave for this product the expression

$$pf = \frac{51}{t - 32}, \tag{4}$$

where t is the oil temperature in degrees Fahrenheit. For ordinary high-speed bearing temperatures, this formula gives a pf value close to 1/2 p.s.i. With this friction force per square inch of projected journal area quite definite, there was no reason why an attempt should not be made also to allow with some degree of accuracy for the radiating capacity. Lasche's values for the radiating capacity of different types of bearings are represented by the curves in Fig. 240.

An attempt has been made to refer the radiating capacity, based on the projected area of the journal, to the actual radiating surfaces of the bearing and its adjoining parts.‡ Since it is difficult to determine the actual radiating surface with any degree of accuracy, and since experimental data for evaluating this surface are yet rather incomplete, it is perhaps desirable for the present to adhere to Lasche's curves.

EXAMPLE. To illustrate the use of these curves, let us investigate whether a 4 in. turbine journal running at 3600 r.p.m. requires artificial cooling in order not to

* *Grundzüge der Schmiertechnik*, Springer, 1926, p. 130.

† Zeitschrift des Vereines deutscher Ingenieure, 1902, p. 1881.

‡ See Karelitz, *Performance of oil-ring bearings*, Transactions A.S.M.E., 1930, APM-52-5, p. 57.

overheat. We may assume a permissible bearing temperature of 160° F. and a room temperature of 100° F.

We have from formula (4), $pf = 51/(160 - 32) = 0.40$. The rubbing speed is $\pi \times 4 \times 3600/12 = 3770$ ft. per min. The friction work per sq. in. of projected area per min. is $0.40 \times 3770 = 1508$ ft. lb. Since the available temperature difference is 60° F., the necessary radiation constant is $1508/60 = 25$, approximately. It will be seen that this value is very much higher than even the highest curve shown in Fig. 240.

The bearing must therefore be cooled, for instance by imbedding water coils in the babbitt bearing lining, or by circulating the lubricating oil through a cooler. Under the conditions given, the maximum permissible value of pfV without artificial cooling is about 4.5. This value would mean a maximum rubbing speed of $4.5 \times 60/0.40 = 675$ ft. per min. At 3600 r.p.m. this speed would correspond to a shaft diameter of somewhat less than 3/4 in.; at 1800 r.p.m., one of about 1½ in. It will be seen that in the case of motors, generators, centrifugal pumps, steam turbines, or similar high-speed machinery, only machines of small size can be run without artificial cooling of the bearings.

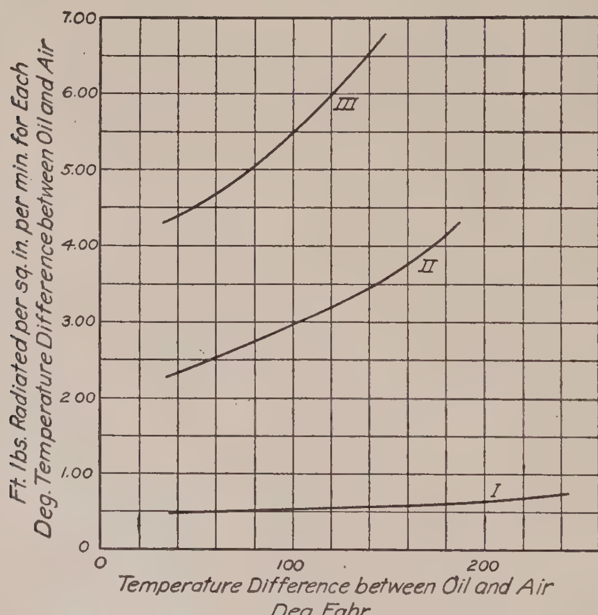


FIG. 240. RADIATING CAPACITY OF DIFFERENT TYPES OF BEARINGS AS DETERMINED BY LASCHE.

I. Thin shells, not joined to radiating masses. *II*. Curve for average bearings. *III*. Bearings with exceptionally large radiating surface, or joined to large radiating masses.

196. Procedure in Bearing Design. Where it is considered better to follow the proportions of other successful designs, rather than to rely upon computations for establishing the conditions for perfect film

lubrication, the first consideration is to assume a bearing pressure and a ratio of bearing length to bearing diameter. Of the latter we may say simply that the ratio varies from 1 and 1.5 for crank pin and crankshaft bearings, to 3 and 4, or perhaps even more, for machine-tool and line-shaft bearings.

When the bearing alignment is difficult to establish or to maintain, as in connecting rods, the bearing length must be relatively short or the pressure will not be uniformly distributed. On the other hand, if the alignment is easily attained, as with spherically seated bearings, the length may be increased to suit the requirements of the design.

It has been considered particularly important to keep the pressures low for cast-iron bearings by making the bearing long. This practice might well be adhered to, particularly for machine tools with slow-running shafts, since it takes considerable speed to establish perfect lubrication with such bearings.

Bearing pressure values representative for a number of conditions are listed in Table 30. For line shafts with cast-iron bearings, pressures as low as 15 to 25 p.s.i. have been in use (see Halsey, *Handbook for Machine Designers and Draftsmen*, p. 8). At the other extreme, pressures for very low speed and intermittent service, as in turn tables and bridges, may run as high as 7000 to 9000 p.s.i., and in shears and punches 3000 to 4000 p.s.i. and higher.

In Germany, and perhaps elsewhere, it has been customary to make bearings narrower than in America; even with imperfect or semi-perfect lubrication, pressures as high as 900 p.s.i. for unhardened steel on babbitt are regarded as generally applicable, while for steel on cast iron, pressures of 350 to 450 p.s.i. are considered permissible. In justification of this practice it might be stated that in very long bearings there may be a bending of the journal, so that excessive localized pressures may occur at the edges.

Assuming the load on the bearing to be F, the calculations now proceed as follows. We have $l/D = n$, where n is the ratio of length to diameter. Consequently, $F = pnD^2$, or $D = \sqrt{F/np}$, and $l = nD$. The diameter D should be checked for bending and torsion if this has not already been done. The velocity $V = \pi D(\text{r.p.m.})/12$ should be computed and the pV value checked. An example will illustrate the design procedure.

Example. Compute the main bearing of a Diesel engine crankshaft running at 300 r.p.m. An overhung flywheel weighing 4000 lb. is attached to the shaft and the distance from the center line of the flywheel to the center of the bearing is 8 in. The maximum load on the main bearing is estimated at 8000 lb. and the maximum torque transmitted is 20,000 in. lb.

The bending moment of the shaft is 8 × 4000 = 32,000 in. lb. From a standpoint of strength, and with an allowable tensile stress of 8000 p.s.i., according to formula (9), § 9, on page 15, the shaft diameter D would be

$$\sqrt[3]{32\sqrt{32{,}000^2 + 20{,}000^2}/(\pi \times 8000)} = 3.65,$$

say 3¾ in. If we assume l/D to be 1.5 and the bearing pressure 600 p.s.i. maximum, then the required diameter based on load carrying capacity would be

$$\sqrt{8000/(600 \times 1.5)} = 3 \text{ in.}$$

Hence the diameter of 3¾ in. is sufficient.

The rubbing velocity is $\pi \times 3.75 \times 300/12 = 295$ ft. per min. If the pV value is computed for the maximum bearing pressure it will be 600 × 295 = 177,000. This value is high for a stationary bearing and would not be permitted without some sort of artificial cooling. However, the load fluctuates, owing to the variation of the piston force, and would probably not average much over 5500 lb., so that the actual pV value would be about 5500 × 177,000/8000 = 121,000. This value is still high. If some sort of pump lubrication or flooded lubrication is provided, film lubrication might be established. For an approximate check on what radiating constant would be required, we might assume a friction force of 0.5 lb. per sq. in. of projected area. The energy to be radiated per sq. in. per min. would be 0.5 × 295 = 147 ft. lb. With a temperature difference of 60° F., the radiating constant that would be required would be only 147/60 = 2.4 ft. lb. per sq. in. per deg. Fahr. per min. From the middle curve in Fig. 240, this would seem entirely attainable.

To check our results and to ascertain what oil would be required, let us assume a minimum film thickness of 0.0008 in., which is twice the minimum recommended by Falz for excellently finished bearings. With a journal radius of 1.875 in. the friction coefficient would be 5.6 × 0.0008/1.875 = 0.0024. With a pressure of 600 p.s.i., the friction force per sq. in. would be 0.0024 × 600 = 1.4 lb., that is, considerably higher than the value assumed. If we consider that the average bearing pressure would be around 5000 × 600/8000, say 375 p.s.i., instead of 600 p.s.i., the average friction force would be about 0.9 p.s.i. To bring this value down to 0.5 we should have to reduce the minimum film thickness to 0.0008 × 0.5/0.9 = 0.00045 in., which would still be permissible according to Falz. With an eccentricity of 0.55, this would mean a radial clearance of 0.00045/(1 − 0.55) = 0.001 in., and a diametral clearance of 0.002 in., which is very nearly the minimum allowance for a medium fit on a 3¾ in. shaft, although it is much below the maximum allowance.

Consequently, if perfect film lubrication can be established, there is apparently no insurmountable difficulty about either cooling capacity or clearance. We must ascertain, however, what oil viscosity would be required to establish such lubrication. From formula (2) on page 238 we have $z = ph_0^2/(\text{const. } Ur)$. Let us assume the minimum film thickness of 0.00045 in. For the constant, let us assume the value 0.155, although for $l/D = 1.5$ the value would probably be somewhat higher, and the required viscosity in consequence not quite so high as computed. The value thus derived for z will then be on the safe side. Factor U is the circumferential velocity in in. per sec. and is equal to 295 × 12/60 = 59. For the maximum pressure we have

$$z = (600 \times 0.00045^2)/(0.155 \times 59 \times 1.875) = 7.1 \times 10^{-6} \text{ reyns},$$

or

$$= 7.1 \times 10^{-6}/1.46 \times 10^{-7} = 48.6 \text{ centipoises.}$$

The corresponding kinematic viscosity, with a specific gravity of 0.9, is 48.6/0.9 = 54, and the Saybolt viscosity from the diagram, page 231, is about 250. As the chart

on page 230 shows, this is not an unattainable viscosity, even at a working temperature of about 200° F., yet it requires an unusually heavy oil. The ordinary automobile oils are not nearly so heavy as this.

In automotive engines, however, the circumferential velocity is apt to be higher. For instance, with a 3 in. crankshaft running at 1500 r.p.m. the Saybolt viscosity at the running temperature could be less than 100 sec. even with a bearing pressure of 600 p.s.i. This viscosity falls within the range attainable with ordinary automobile oils, and shows that perfect film lubrication is possible in the automotive type of engines.

197. Bearing Metal Alloys. The important bearing metals consist of the bronzes, babbitts, and copper-lead alloys. Certain brasses, that is, copper-zinc alloys, have been used to only a limited extent for bearing purposes and are relatively unimportant. Brass is cheaper, and for this reason it has been substituted in some instances for bronze for light service.

198. Bearing Bronzes. The copper-tin alloys, that is, bronzes, are used extensively for bearing liners. The bronzes in general are especially suitable for bearings subjected to heavy loads and severe working conditions. They possess a high resistance to pounding, and are therefore used for such applications as locomotive connecting-rod

TABLE 32
BEARING BRONZES

S.A.E. Spec.	No. 63 Leaded Gun Metal	No. 64 Phosphor Bronze	No. 66 Bronze Backing for Lined Bearings	No. 67 Semi-Plastic Bronze
Copper	86.0–89.0	78.5 –81.5	83.0–86.0	76.5 –79.5
Tin	9.0–11.0	9.0 –11.0	4.5– 6.0	5.0 – 7.0
Lead	1.0– 2.5	9.0 –11.0	8.0–10.0	14.5 –17.5
Phosphorus	0.25 max.	0.05– 0.25	——	——
Zinc	——	0.75 max.	2.0	4.0 max.
Antimony	——	——	——	0.4 max.
Iron	——	——	——	0.4 max.
Impurities	0.50 max.	0.25 max.	0.25 max.	1.0 max.
Ultimate strength p.s.i.	30,000	25,000	25,000	20,000
Yield point p.s.i.	12,000	12,000	12,000	——
Elongation in 2 in.	10%	8%	8%	10%

*S.A.E. Spec.—63.** Combines strength with fair machining qualities. A general utility bronze, especially good for bushings subjected to heavy loads and severe working conditions.

S.A.E. Spec.—64. An excellent composition for bearings, standing up exceedingly well under heavy loads and severe usage.

S.A.E. Spec.—66. An inexpensive alloy suitable for bronze-backed bearings.

S.A.E. Spec.—67. A soft bronze with good anti-friction qualities.

* S.A.E. Specifications taken from the Society of Automotive Engineers Handbook, 1937 edition.

bearings and rolling-mill bearings. Under abnormal conditions, bronze is more liable to heat than babbitt, because it lacks the plasticity to conform to any irregularities of the journal. Bronze-lined bearings are easily renewed and, to facilitate replacements, finished bushings in a wide range of sizes are commercially carried in stock.

While there are many different compositions of bearing bronzes, a number of which have been developed by certain industries for their own particular needs, only a few standardized alloys with their specifications and physical properties will be given here. (See Table 32.)

199. Babbitt. The genuine babbitt metals are alloys of tin, copper, and antimony. The cheaper alloys of lead, tin, and antimony are also commercially known as babbitts. Alloys of this type consist of relatively hard and soft microscopic grains intimately mixed together. It has been held that such composition is essential for good bearing metals, but this idea has recently been seriously criticized. (See *Automotive Industries*, 7-24-'37, p. 121.) In general, tin hardens and increases the compressive strength of the babbitt, copper adds toughness, antimony prevents shrinkage, while lead contributes to plasticity. The babbitts have better anti-friction properties than the bronzes and are extensively used for bearing liners.

For best performance, babbitt subjected to high speeds, heavy pressures, impact loads, and vibration should be backed up solidly by a metal of higher compressive strength, such as bronze or steel. A thin layer of high-tin babbitt thoroughly fused to a tinned bronze or steel

TABLE 33

White Bearing Metals (Babbitts)

S.A.E. Spec.	No. 10	No. 11	No. 12	No. 13
Tin	90.0	86.0	59.5	4.5 – 5.5
Copper	4.0–5.0	5.0–6.5	2.25– 3.75	0.50 max.
Antimony	4.0–5.0	6.0–7.5	9.5 –11.5	9.25–10.75
Lead max	0.35	0.35	26.0	86.0
Iron max	0.08	0.08	0.08	——
Arsenic max	0.10	0.10	——	0.20
Bismuth max	0.08	0.08	0.08	——

S.A.E. Spec.—10. A very fluid babbitt, suitable for bronze-backed bearings, particularly for thin linings.

S.A.E. Spec.—11. A rather hard babbitt suitable for linings subjected to heavy pressures; wiping tendency is very slight.

S.A.E. Spec.—12. A relatively cheap babbitt for large bearings subjected to moderate loads.

S.A.E. Spec.—13. A cheap babbitt for large bearings and light service.

All of the above babbitts are also suitable for die casting.

shell has exceptional load-carrying capacity and impact resistance. As babbitt cannot be fused dependably to cast iron, a lining for a cast-iron bearing should be anchored in place by means of dove-tail slots or drilled holes. While the melting point of babbitt varies between 358° and 473° F., depending upon its composition, the pouring temperature must be above the point of complete liquidity. For instance, babbitt of S.A.E. Spec. No. 10 has a melting point of 433° F. and a pouring temperature of 825° F.

Composition and uses of several babbitts are given in Table 33.

200. Copper-Lead Alloys. Recently, alloys containing a high percentage of lead and known as copper-lead alloys have found considerable use as a bearing material. Straight copper-lead alloys of this type have only about half the strength of the regular bearing bronzes.

According to Hensel and Tichvinsky,* the main advantage of the copper-lead alloy over babbitt is in the increased tensile strength at elevated temperatures. Typical compositions of copper-lead alloys contain about 25 per cent lead and 75 per cent copper, and have a melting point of approximately 1800° F. The tensile strength of these alloys is approximately 11,000 p.s.i.,—about the same as high-grade babbitt. Most babbitts have a lower melting point, however, ranging from 437° to 464° F., and they lose practically all tensile strength around 400°. A copper-lead alloy at this same temperature still possesses a strength of approximately 5000 p.s.i.

201. Other Bearing Materials. An extremely hard wood of great density, known as *lignum vitae*, has been used for bearing applications. With water as a lubricant and cooling medium, its anti-friction properties and rate of wear compare favorably with bearing metals. In step bearings of vertical water turbines, paper-mill machinery, marine service, and even roll-neck bearings of rolling mills,† lignum vitae has been used with fairly satisfactory results.

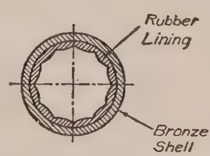


Fig. 241. Rubber Bearing.

More recently, soft vulcanized *rubber* bearings have been developed for service where the use of water as a lubricant is necessary, especially if sand and grit are present. A soft, tough, resilient rubber acts as a yielding support, permitting grit to pass through the bearing without scoring the shaft or the rubber. Longitudinal grooves, as shown in Fig. 241, allow free passage of the

* F. R. Hensel and L. M. Tichvinsky, *Straight copper-lead alloys versus leaded solid-solution bronzes for heavy duty bearings*, Transactions A.S.M.E., 1932, IS–54–3, p. 11.

† K. W. Atwater, *Lignum-vitae bearings for roll necks of medium-sized rolling mills*, Transactions A.S.M.E., 1932, IS–54–1, p. 1.

cooling water with any foreign matter present. With feathered edges, these grooves are also very effective in forming constricted passages in front of which the supporting pressure is built up in the fluid film.

According to Busse and Denton,* the coefficient of friction of soft-rubber bearings, under proper conditions, compares favorably with that of roller bearings. Loads of 600 to 800 p.s.i. may be carried, provided the shaft is very smooth and the load is applied only after the shaft reaches a peripheral speed of about 500 ft. per min. The cooling water temperature must always be under the boiling point. In some cases, rubber bearings have been found to give as much as ten times the service as bearings of metal or lignum vitae.

Rubber bearings have been used successfully in centrifugal and deep-well pumps, sand washers, and similar applications where water must be used as a lubricant. The resiliency and cushioning properties of rubber may lead to a more extensive use of such bearings in reducing vibration of high-speed shafts.

Bearings have been made from several varieties of *phenol* derivatives. A hard, dense, tough substance is produced by impregnating sheets of woven fabric such as linen or cotton duck with the resin, superimposing one sheet upon another to obtain the required thickness, and subjecting the laminated mass to heat and very high pressure. Bearings made of this material are resilient and can withstand severe impact loads without permanent deformation. While water is generally the lubricant, oils or greases improve the performance of such bearings appreciably. The admixture of graphite improves the material for certain purposes. In many instances, according to Arthur J. Schmitt,† phenol bearings under the trade name of Celoron, applied to roll necks of rolling mills, have carried loads of 1000 to 2500 p.s.i.

202. Thrust Bearings. Some form of a thrust bearing must be provided when a shaft is subjected to a load applied in the direction of the axis of the shaft. Relatively light loads may be carried by the hubs of gears or by collars fastened to the shaft and bearing against the ends of journal bearings. Bearings especially designed to carry heavy thrust loads may be broadly classified into three groups: *step bearings, collar bearings*, and *pivoted-segment bearings*.

203. Step Bearings. *Step bearings* or *pivot bearings* are used to support the lower ends of vertical shafts. A simple form of such bearings is shown in Fig. 242, with the shaft supported by a hardened-steel disc.

* W. F. Busse and W. H. Denton, *Water-lubricated soft-rubber bearings*, Transactions A.S.M.E., 1932, IS–54–2, p. 3.

† Arthur J. Schmitt, *Application of impregnated fabric bearings to roll necks*, Transactions A.S.M.E., 1932, IS–54–4, p. 25.

The wear on the surface of a flat pivot is proportional to the rubbing velocity and reaches a maximum at the outer edge. As a result of this

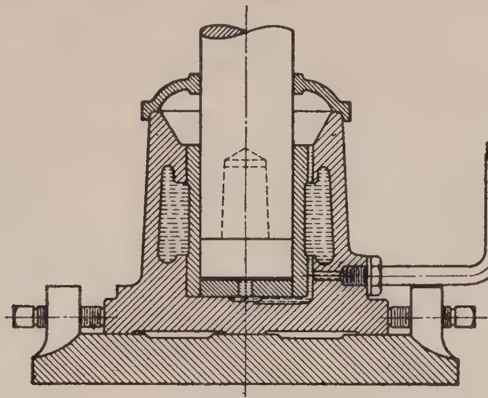


FIG. 242. STEP BEARING.

uneven wear, the pressure becomes concentrated toward the center and bearing failure may occur. This condition can be corrected to a certain extent by removing the bearing surface near the center, as shown. Increasing the outside diameter and thereby enlarging the bearing area reduces the unit bearing pressure, but as the friction radius also increases, the energy lost in friction is correspondingly greater. Consequently there is a limit to this method of reducing the unit pressure.

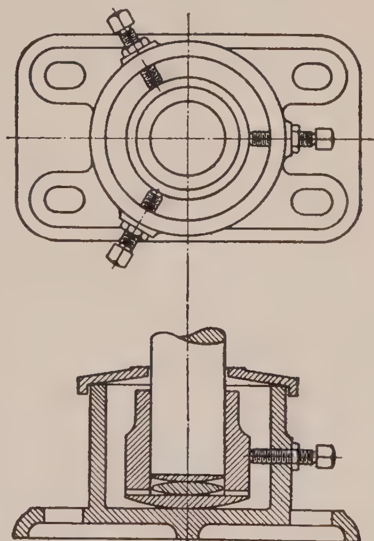


FIG. 243. SELF-ALIGNING STEP BEARING.

Another form of a step bearing is shown in Fig. 243. The lenticular hardened-steel washer and the spherical bottom of the socket permit the shaft to align itself and establish a better support.

A further improvement may be effected by placing a number of washers between the end of the shaft and the bearing, which results in a lower relative rubbing velocity. If faulty lubrication causes any pair of washers to seize, sliding continues on those free to move. These washers may be flat or lenticular, and are usually of bronze and steel,

alternately arranged. If the discs are lubricated from the center, centrifugal action tends to force the oil across the bearing surfaces.

Ordinary step bearings are never perfectly lubricated even if they are submerged in a bath of oil. There is obviously no action tending to separate the rubbing surfaces by the formation of a wedge-shaped oil film, as in journal bearings. Instead, the tendency is to scrape and to force the oil out from between the surfaces. With a forced circulation of oil, however, very heavy pressures at high speed may be attained. For slow and intermittent service with oil bath lubrication, pressures of 1500 p.s.i. may be used. If the average velocity of the rubbing surface is 200 ft. or more, a bearing pressure of only 50 to 60 p.s.i. is regarded as permissible.

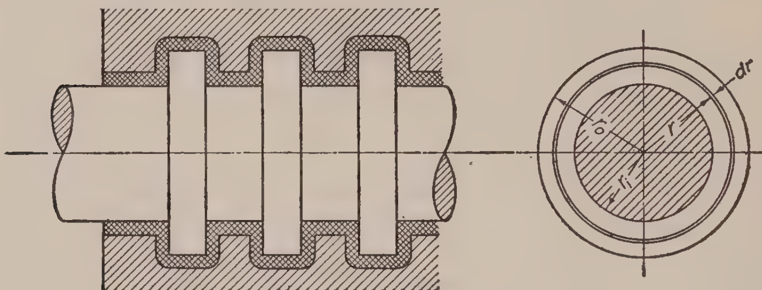


FIG. 244. COLLAR THRUST BEARING.

204. Collar Bearings. *Collar thrust bearings* (Fig. 244) take a thrust load on one or more collars cut integral with the shaft. These collars engage with corresponding bearing surfaces in the thrust block. This type of bearing is used if the load would be too great for a step bearing, or if a thrust must be taken at some distance from the end of the shaft. In ordinary work, the bearing surfaces may be machined directly in the thrust block, or they may be babbitt lined, as shown. A somewhat better construction would consist of inserted replaceable bronze rings. In any case, the bearing surfaces should be scraped so that the load may be uniformly distributed over all collars.

Figure 245 shows a horseshoe-shaped thrust shoe as used in propeller thrust bearings for marine service. These units are frequently cored for water circulation and may be adjusted along the shaft to fit the collars. Individual shoes may be easily removed without disturbing any other part of the bearing.

Collar thrust bearings of the above types involve the same problem of lubrication as that encountered in step bearings. Oil should be introduced between the collars where they join the shaft; the centrifugal

force will tend to throw the oil outward across the bearing surfaces. Bearing pressures of 50 to 75 p.s.i. are considered permissible for ordinary collar thrust bearings. In order to avoid uneven wear, the diameter of the collars is usually made from 1.4 to 1.8 times the diameter of the shaft.

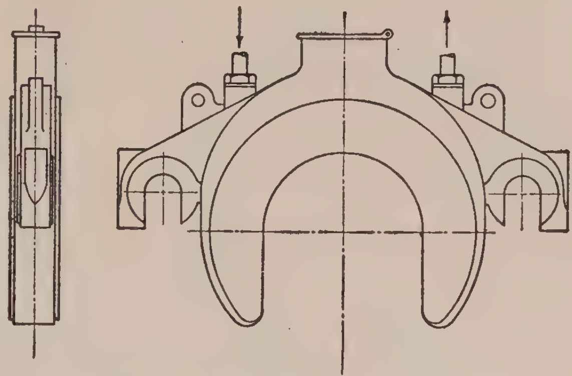


FIG. 245. HORSESHOE-SHAPED THRUST SHOE.

Due to the compactness in design and the higher unit bearing loads that may be carried, pivoted-segment type thrust bearings are now displacing the multiple-collar thrust bearings in many applications.

205. Work Absorbed in Collar Thrust Bearing. If an axial thrust is considered to be uniformly distributed over the bearing surfaces of the collars of a thrust bearing (Fig. 244), then

$$p = \frac{F}{\pi(r_o^2 - r_i^2)n},$$

where p is the unit bearing pressure in p.s.i., F the total load on the thrust bearing in lb., r_o the outside radius in in., r_i the inside radius of the collars in in., and n the number of collars. The pressure on an elementary ring of radius r equals $2\pi rp\,dr$. If the coefficient of friction is f, the friction moment on this elementary ring $= 2\pi r^2 fp\,dr$, and the friction moment on the entire surface of the collar is equal to

$$(5) \qquad 2\pi fp \int_{r_i}^{r_o} r^2\,dr = \frac{2\pi fp(r_o^3 - r_i^3)}{3}.$$

The friction moment on the whole thrust bearing (n collars) is

$$(6) \qquad \frac{2fF(r_o^3 - r_i^3)}{3(r_o^2 - r_i^2)},$$

and the friction work done in ft. lb. per min. equals

$$(7) \qquad \frac{2\pi N}{12} \times \frac{2fF(r_o^3 - r_i^3)}{3(r_o^2 - r_i^2)} = \frac{\pi NfF(r_o^3 - r_i^3)}{9(r_o^2 - r_i^2)},$$

where N is the number of revolutions per minute.

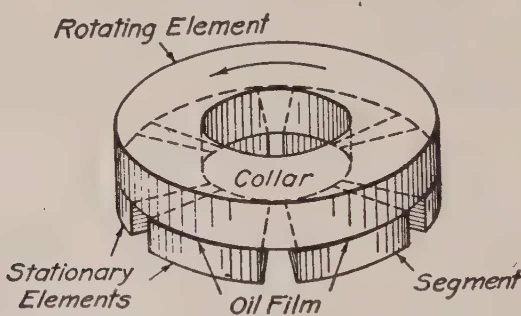
In the step bearing $r_i = 0$, and formula (7) becomes $\pi NfFr_o/9$.

Example. Determine the number of thrust collars required to sustain a load of 8000 lb. on a shaft 6 in. in diameter if the allowable bearing pressure is 50 p.s.i. If the coefficient of friction is 0.03 and the shaft revolves at 200 r.p.m., compute the horsepower absorbed in friction work.

We will assume the collar diameter as 1.5 times the shaft size, or 9 in. The permissible bearing load per collar equals $50 \times \pi(4.5^2 - 3^2) = 1765$ lb. The number of collars required $= 8000/1765 = 4.5$, say 5 collars. The horsepower absorbed in friction work

$$= \frac{\pi \times 200 \times 0.03 \times 8000(4.5^3 - 3^3)}{9 \times 33{,}000(4.5^2 - 3^2)} = 2.9 \text{ hp.}$$

206. Pivoted-Segment Thrust Bearings. (Kingsbury Bearings.) It has been shown, both mathematically and by experiment, that the "natural" condition of oil film lubrication is that in which the film takes a slightly wedge-shaped form, with the thick end at the entering side. Wedge films occur naturally in journal bearings under favorable conditions, through a slight lift and lateral displacement of the shaft. In a thrust bearing whose working surfaces are held rigidly parallel the wedge cannot form. The free entry of oil, and the consequent



The Kingsbury Machine Works, Inc., Philadelphia, Pa.

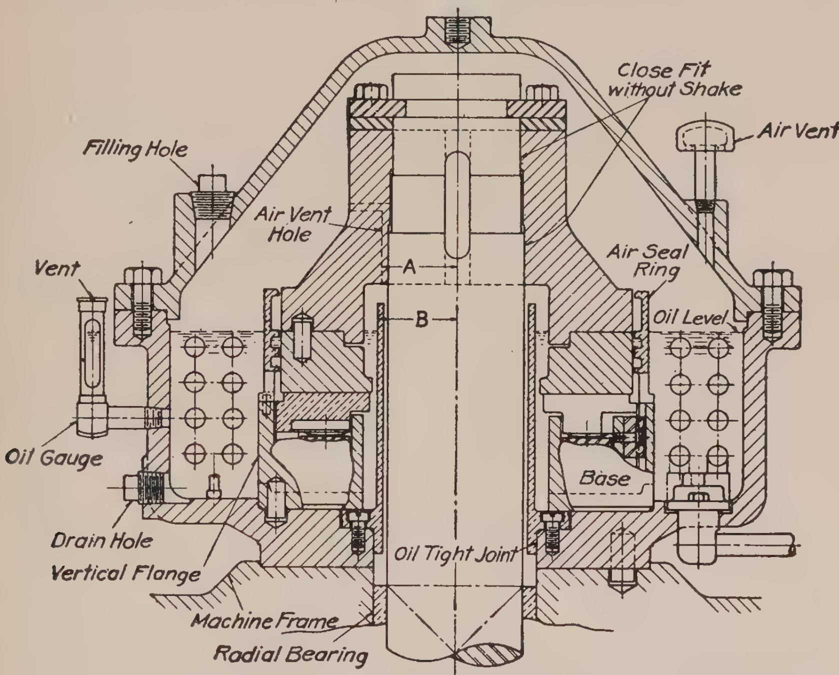
Fig. 246. Arrangement of Collar and Segments of Kingsbury Bearing.

formation of continuous films in thrust bearings, is permitted by the pivoted-segment type, invented by Albert Kingsbury in 1898 and known in this country as the *Kingsbury bearing.**

* Pivoted-block thrust bearings were developed independently and approximately simultaneously with the Kingsbury developments by the Australian engineer Michell, who was given the English patent. In Europe the bearings are therefore often called Michell bearings.

In this bearing, the stationary element is divided into segments (Fig. 246). Each segment is pivoted and free to tilt slightly, radially and tangentially. The collar and the segments are submerged in or flooded by oil, as shown in Fig. 247. As long as the shaft turns, the bearing faces are completely separated by the wedge-shaped films of oil.

The Kingsbury thrust bearing has a very low coefficient of friction, usually from 0.001 to 0.005, depending on unit load, speed, and vis-



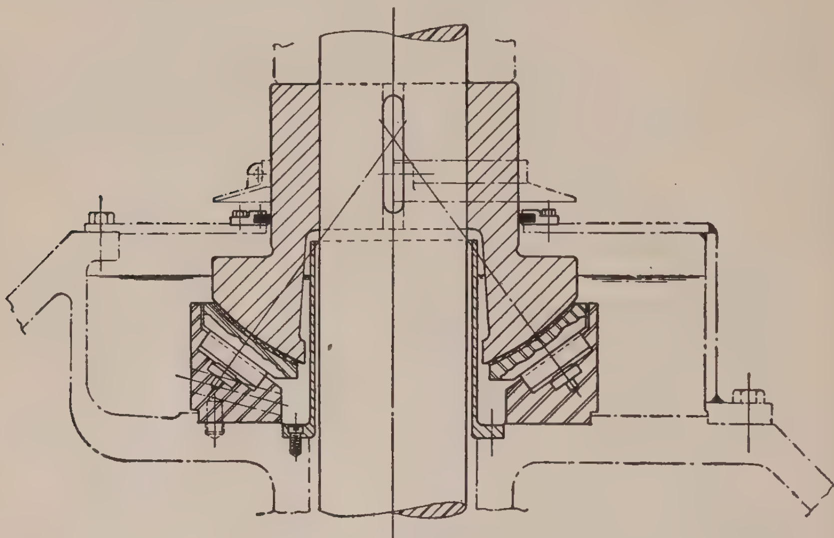
The Kingsbury Machine Works, Inc., Philadelphia, Pa.

FIG. 247. KINGSBURY VERTICAL THRUST BEARING.

cosity of oil. Tests have shown it to have about one-tenth of the friction of a horse-shoe multi-collar thrust bearing. Unit pressures of from 250 to 400 p.s.i. are easily sustained with light oils, whereas the multi-collar marine type is usually loaded to 50 p.s.i. or less. With proper lubricant and cooling system, unit loads of 2000 p.s.i. or more may be carried by Kingsbury bearings. Even with loadings of 400 p.s.i., only one collar is ordinarily required; in vertical hydro-electric generators, loads of 1,500,000 lb. or more are carried on a single collar with pressures ranging from 400 to 500 p.s.i. Where the load and speed require, the oil bath is cooled, either by a water coil inserted

in the oil reservoir or by circulating the oil through an external cooler.

Kingsbury thrust bearings, either separate or combined with journal bearings, are built in forms for both vertical and horizontal shafts, and may be arranged to take thrust in either or both directions. Unlike most bearings, the load capacity of these thrust bearings is greater at higher speeds. Speeds as high as 11,000 r.p.m. for a 6¼ in. diameter collar have been attained in service.



The Kingsbury Machine Works, Inc., Philadelphia, Pa.

FIG. 248. KINGSBURY SPHERICAL BEARING.

Typical installations include hydro-electric generators, marine screw-propeller thrusts, steam turbines, deep-well pumps, centrifugal pumps for all purposes, including boiler feed and oil cracking, rolling mills, plate-glass grinders, and similar applications.

A recent development is the Kingsbury spherical thrust bearing shown in Fig. 248. It replaces in one unit the usual flat thrust bearing and adjacent guide bearing. Unlike cylindrical guide bearings, it requires no radial clearance; hence it brings about perfectly true running of the shaft at the sphere center, thereby avoiding the gyrating action that is noticeable at the upper end of the shaft of a hydro-electric generator mounted in the usual guide bearings.*

* Various bulletins published by the Kingsbury Machine Works, Philadelphia, Pa., give complete specifications and rated load capacities of these bearings. The authors are indebted to Mr. H. A. S. Howarth of the Kingsbury Machine Works for his suggestions and cooperation.

PROBLEMS

1. A gear running at 800 r.p.m. is carried midway between two bearings by a 2 in. shaft. The oil to be used has a Saybolt viscosity of 320 sec. at 100 deg. F. Assuming an operating temperature and a radial clearance of 0.001 in., determine the length of the bearings for a gear load of 1200 lb. Are the clearance and oil satisfactory?

2. A sleeve bearing 5 in. long supports a load of 6000 lb. from a 3 in. shaft turning at 500 r.p.m. Select a suitable oil and clearance, and check the conditions for satisfactory operation.

3. The end bearing of an electric generator is 12 in. in diameter and 28 in. long. The load is 16,500 lb. at 1800 r.p.m. Select and check suitable clearance and oil. The bearing temperature is not to exceed 130° for a room temperature of 90° F. Will the bearing need artificial cooling? How many ft. lb. per min. must be removed artificially?

4. A steam-turbine bearing 4 in. in diameter and 6 in. long supports a load of 9750 lb. The operating speed is 1800 r.p.m. (a) Select an oil to give a (zN/p) value of about 100. (b) What heat is generated under these conditions? (c) To limit the operating temperature to 175° F., is artificial cooling necessary for a room temperature of 100° F.?

5. A Diesel-engine main bearing, 6 in. in diameter, carries a load of 21,000 lb. at 175 r.p.m. Select the proportions of the bearing and an oil to give a proper (zN/p) value at 150° F.

6. A spherical-seated mill-shaft bearing carries a load of 1200 lb. from a $2\frac{1}{2}$ in. shaft. Fix upon the length according to the usual proportions and check the (pV) value at 350 r.p.m.

7. A babbitted crankshaft bearing carries a load of 6500 lb. at 1500 r.p.m. Fix upon the diameter and length of the bearing by the usual proportions and check the (pV) value.

8. A Diesel-engine crankshaft operates at 240 r.p.m. A flywheel weighing 6000 lb. overhangs the bearing by 10 in. The maximum torque on the shaft is 28,000 in. lb. Average load on bearing is about 9000 lb. and s_s may be taken as 4000 p.s.i. (a) Fix upon the proportions of the shaft. (b) Check the (pV) value. (c) Select an oil and clearance. (d) Calculate the heat generated.

9. A single-screw ship is driven at 17.5 knots (22 m.p.h.) by a turbine delivering 6600 hp. (a) If the propeller efficiency is 2/3, what is the load on the collars? (b) The load is carried by U-shaped supporting collars, each contacting a collar on the shaft. The U-collars have an inside diameter of 17 in. and an outside diameter of 26 in. One-third of the effective shaft-collar area is lost because of the openings in the U-collars. How many collars are required to limit the bearing pressure to 50 p.s.i.? (c) At 125 r.p.m. what horsepower is lost in friction if the coefficient of friction is 0.04?

10. A $2\frac{1}{8}$ in. shaft running at 750 r.p.m. carries a thrust load of 5000 lb., which is supported by 4 collars. (a) What must be the outside diameter of the collars to limit the unit pressure to 40 p.s.i.? (b) What is the power loss if the coefficient of friction is 0.03?

CHAPTER 13

BALL AND ROLLER BEARINGS

207. General Remarks. The chapter on plain bearings explained the importance of separating the journal from the bearing surface by a film of oil, and the necessity of maintaining this film if friction and wear are to be avoided. An even more positive way to avoid sliding friction between journal and bearing is to introduce rolling elements between these sliding surfaces. Ball bearings were developed along with the bicycle toward the end of the last century; roller bearings appeared even later; yet today these machine elements are of tremendous importance. Continuous progress in the metallurgy of steel, improved methods of precision manufacture, extensive research, and large-scale production are the factors which have contributed to their success and extensive use.

208. Characteristics of Ball and Roller Bearings. While ball and roller bearings are commonly known as anti-friction bearings, and have certain advantages in reducing friction, they have other properties of as great importance.

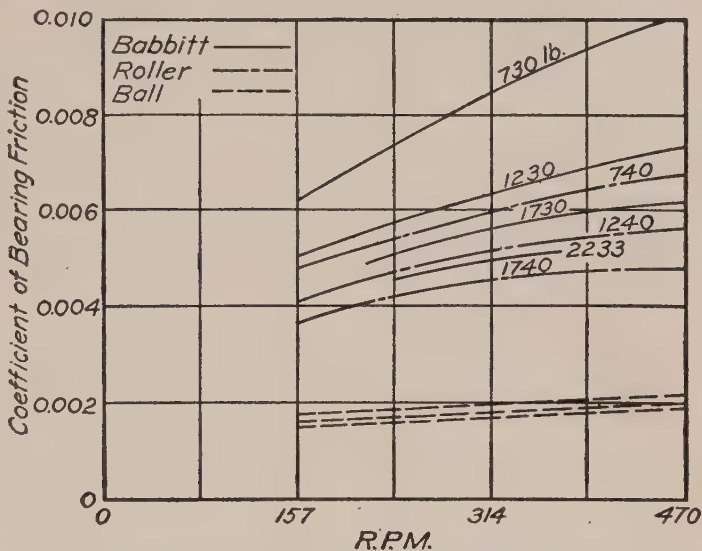
Dependability in service is perhaps their outstanding characteristic. They are less exacting with respect to lubrication than plain bearings. If well filled with grease and protected from dirt and moisture, they will run from three months to a year without attention; then need only additional lubricant. Where maintenance is troublesome and apt to be neglected, a bearing will continue to give satisfactory service even if the supply of lubricant is temporarily exhausted.

Another characteristic of anti-friction bearings is their ability to support heavy overloads for a considerable period of time without sudden failure. Furthermore, due to the virtual absence of wear, as long as they are in good condition, they have the distinct feature of maintaining the exact position of parts, so that adjustments are unnecessary.

There is no extremely great superiority from a standpoint of low operating friction of ball or roller bearings over plain bearings, provided the latter are *perfectly* lubricated. This is clearly shown by the curves in Fig. 249 taken from a paper by Thomas, Maurer, and Kelso.* It is to be noted that for the higher loads, the coefficient of friction

* Transactions A.S.M.E., 1913, p. 613.

for roller bearings and plain bearings is practically the same. For ball bearings, the coefficient of friction is lower and remains fairly constant under different loads and speeds. The frictional resistance in all cases is relatively so low that the actual difference is of little importance, except in rare cases where bearing friction absorbs a large percentage of the power input.



Thomas, Maurer, and Kelso, Transactions, A.S.M.E. 1913

FIG. 249. COEFFICIENTS OF FRICTION FOR BABBITT, ROLLER, AND BALL BEARINGS AT VARIOUS LOADS AND SPEEDS.

With *imperfect* lubrication, however, which is the case for many plain bearings, a considerable reduction in friction is obtained by the use of ball or roller bearings. Even plain bearings with perfect lubrication have relatively high coefficients of friction when starting from rest. In comparison, ball and roller bearings have practically the same friction at starting as when running. This is a valuable property when starting torque is an important consideration, as in railway service.

In general, ball and roller bearings are somewhat larger in diameter than plain bearings, a condition which may be objectionable in some cases. On the other hand, with the exception of certain types of roller bearings, they are considerably shorter in length than plain bearings, which is usually an advantage.

209. Fatigue Life of Anti-Friction Bearings. Research by Stribeck and subsequent investigations indicate that the normal life of anti-friction bearings is limited by "fatigue" failure of the metal. The load-

carrying surfaces, that is, the raceways and rolling elements, are subjected to a repetition of stresses, the effect of which eventually causes the formation of microscopic cracks. As this condition progresses, the surfaces begin to flake off, an indication that the useful life of the bearing has ended. Failure of any other type, such as the actual fracture of parts, is considered abnormal, and may be avoided by correct design, manufacturing control, or proper maintenance.

The first really scientific investigations on the carrying capacity of ball bearings were carried out by Stribeck in Germany,* and ball bearings the world over have been standardized on the basis of Stribeck's recommendations. Since then, manufacturers have conducted extensive investigations on the load capacity and service life of various sizes of bearings at various speeds. Recommendations based on these investigations are contained in their various data books,† which designers should use as the basis of design. Roller-bearing data were never established in any other way.

210. Design of Ball Bearings. The modern ball bearing, with the exception of special types, is an assembled, non-adjustable unit consisting of an inner and outer race with the intervening balls. The load on the journal is transferred from one race to the other through the balls. While load-carrying capacities are always determined on the basis of the number and size of the balls, actually the life of the bearing is controlled by the inner race. This member is subjected to a greater number of stress repetitions than could ever occur at any point on a ball, and eventually it breaks down.

Owing to the elasticity of the material, the contact between the ball and the race is not a point but a small elliptically shaped area, the size of which depends upon the load and the curvature of the race. The closer this curvature conforms to the ball, the larger this contact area becomes and the greater may be the load imposed upon the bearing. The radius of the race of many deep-grooved bearings is made within a few percent of the ball radius. Such bearings are well suited for shock loads and large thrust capacities, but require very careful alignment. Obviously, a more open curvature is required where a shaft may be flexed under load.

According to the New Departure Manufacturing Company, compression stresses at the contact areas of balls and races may reach

* Stribeck's work was originally published in the Zeitschrift des Vereines deutscher Ingenieure, 1901, pp. 73 and 118.

† See, for instance, SKF Engineering Data Sheets, SKF Industries, Inc., New Departure Handbook, The New Departure Manufacturing Co., Timken Engineering Journal, The Timken Roller Bearing Company, and publications by other manufacturers of ball and roller bearings.

values from 200,000 to 300,000 p.s.i. under normal loads. To withstand such stresses a hard tough steel is required with high fatigue-resisting properties, since as a matter of fact, the stresses on the small central area may exceed the elastic limit. The SKF Industries use an alloy steel specially developed for ball bearings, the analysis of which shows approximately 1 per cent carbon, 1.4 per cent chromium, and extremely little sulphur or phosphorus. In the heat-treated condition it has a Brinell hardness of 600 to 650 with 5 mm. ball and a scleroscope reading of 75 to 85. The races and balls are hardened throughout, and are accurately ground to exceedingly close limits. Balls are held within limits of plus or minus 0.00005 in. for sphericity and size.

The balls are introduced in the raceway either through a filling slot ground in the sides of both races (Fig. 250), or by inserting as many

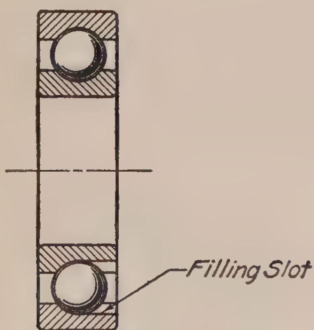


Fig. 250. Filling Slot Type Bearing.

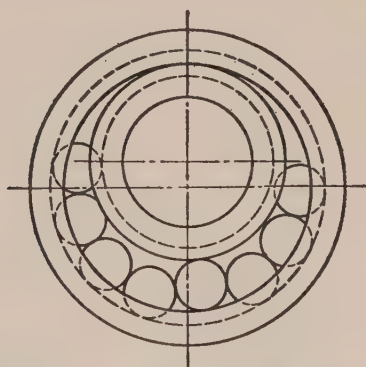


Fig. 251. Eccentric Displacement of Inner Race.

balls as possible in the crescent shaped space formed by the eccentric displacement of the inner race (Fig. 251). Owing to the larger number of balls, the filling slot type provides the maximum radial load capacity. Generally speaking, this construction is used for medium and large sized bearings. The type without filling slot, with its uninterrupted raceway, is preferred for the smaller sizes operating at high speeds. A retainer or cage of pressed steel or bronze completes the assembly and provides a means of equally spacing and holding the balls in place.

211. Load-Carrying Capacity of Radial Ball Bearings. Stribeck concluded from his researches that the carrying capacity of a ball varies as the square of its diameter, expressed as follows:

$$F_0 = KD^2,$$

where F_0 = permissible static load, D = the diameter of the ball, and K = a constant for the material and shape of raceway.

For average bearings containing 10 to 20 balls, Stribeck gives

$$\frac{W}{F_0} = \frac{n}{4.37},$$

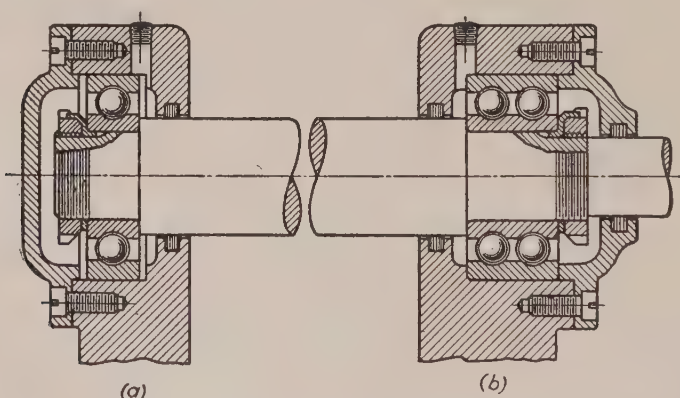
where W = load capacity of the bearings in lb., and n = number of balls. Combining these two equations, we have

$$W = \frac{KnD^2}{4.37}.$$

In view of a slight play and deformation of the races, the formula is modified for purposes of computation to

$$W = \frac{KnD^2}{5}. \tag{1}$$

According to the above equation, the load carrying capacity of a bearing, as found by Stribeck, varies directly as the square of the ball diameter. It is the conclusion of the SKF Industries, for instance, after an extensive experimental investigation, that this assumption is true for static condition only, and that when rotation of the bearings and fatigue life are taken into consideration the load capacity varies approximately as the 3/2 power.



The Fafnir Bearing Co., New Britain, Conn.

Fig. 252. (a) Single-Row Radial Bearing. (b) Double-Row Radial-Thrust Bearing.

212. Different Types of Ball Bearings. Operating conditions vary to such an extent and the nature of the loads is so different from case to case that a large number of types of bearings is required to meet all conditions satisfactorily. While it is not possible to go into the matter

in great detail, the characteristic features of some of the more important types will be discussed briefly.

The single-row *radial* ball bearing illustrated in Fig. 252a is the most widely used of all types. For a given number and size of balls its thrust capacity in relation to radial load depends principally on the curvature of the raceway. It may vary from 50 per cent to 100 per cent or more of the radial capacity.

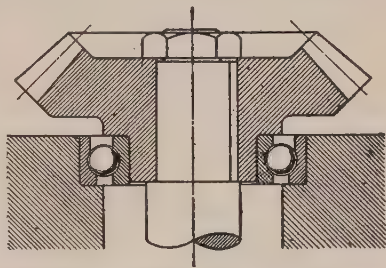
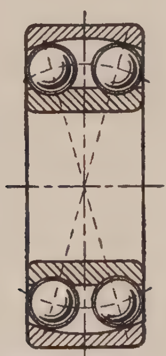


FIG. 253. APPLICATION OF ANGULAR-CONTACT BALL BEARING.

The *angular-contact* ball bearing shown in Fig. 253 is particularly adapted for combined radial and thrust loads where the thrust component predominates and is beyond the capacity of ordinary single-row radial bearings. Thrust is taken against the high shoulder of the outer race and in one direction only. For a thrust load in either direction the bearings may be mounted in opposed pairs.

A *double-row radial-thrust* bearing is shown in Fig. 252b. This is a compact bearing capable of resisting heavy combined radial and thrust



SKF Industries, Inc., Philadelphia, Pa.

FIG. 254. SELF-ALIGNING BALL BEARING.

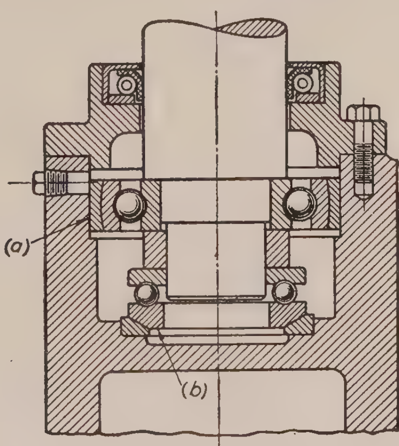
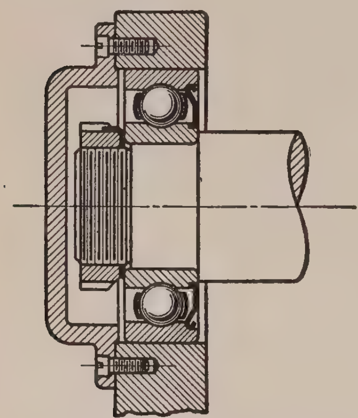


FIG. 255. (a) SELF-ALIGNING RADIAL BEARING. (b) SINGLE THRUST BEARING.

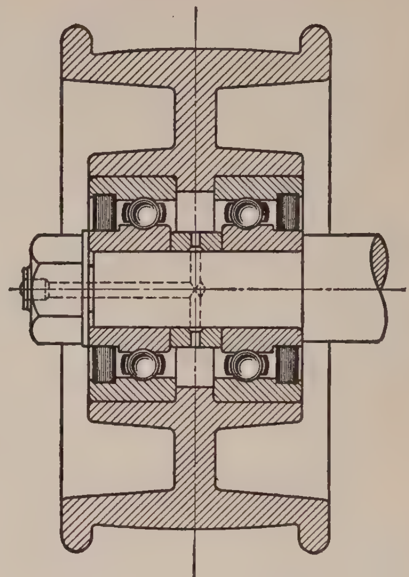
loads in either direction. It has practically double the capacity of a single bearing. Rigidity is secured by preloading the bearing at the time of manufacture, with the result that any external load will cause less displacement of the shaft. This type is particularly recommended where an axial position must be maintained under heavy thrust loads.

There are various types of *self-aligning* ball bearings, one of which, the SKF type, is shown in Fig. 254. Its characteristic feature is a spherical outer race, which allows the shaft to align itself freely within rather wide limits, yet has sufficient thrust capacity to locate a shaft axially and quite posi-



The New Departure Manufacturing Co., Bristol, Conn.

FIG. 256. SHIELDED BEARING.



Norma-Hoffmann Bearing Corp., Stamford, Conn.

FIG. 257. FELT-SEALED BEARINGS.

tively, provided there is no special thrust load. Other types employ ordinary plain radial bearings mounted in spherical housings (Fig. 255a).

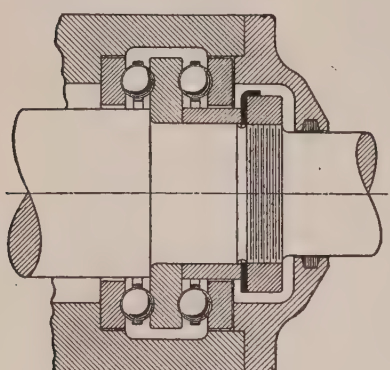


FIG. 258. TWO-DIRECTION THRUST BEARING WITH FLAT SEATS.

A *shielded* bearing is used in certain applications where it is desirable to prevent dust, metal chips, or abrasives from entering the bearing. This shielding is accomplished by pressing a steel disc into the side of the bearing, as shown in Fig. 256.

Ball bearings are also provided with virtually grease tight *felt seals* as an integral part of the bearing itself (Fig. 257). This closure will retain lubricant often for the life of the bearing and exclude foreign matter.

Thrust bearings are designed to carry thrust loads exclusively and at relatively low speeds. They consist essentially of a row of balls spaced in a retainer and held between two grooved washers. While the single thrust bearing shown in Fig. 255b is self-aligning due to the spherical seat and aligning washer, there is also available a rigid type having a flat seat. For a thrust load that occurs in either direction, a two-direction thrust bearing is employed, as in Fig. 258. At high speeds, the centrifugal force tending to throw the balls out of the raceway greatly impairs their efficiency. For such applications, radial bearings capable of carrying heavy thrust loads are recommended.

213. Standardization of Ball Bearings. Ball bearings in general have been standardized with respect to bore, outside diameter, and width. These dimensions are given in millimeters, whereas the ball size, which varies with different manufacturers, is measured in fractions of an inch* (see Tables 34, 35, and 36).

A number of the different styles of bearings are made in three series, namely: *light*, *medium*, and *heavy*. A comparison of their relative sizes is shown in Fig. 259. For a common bore diameter, there is a choice

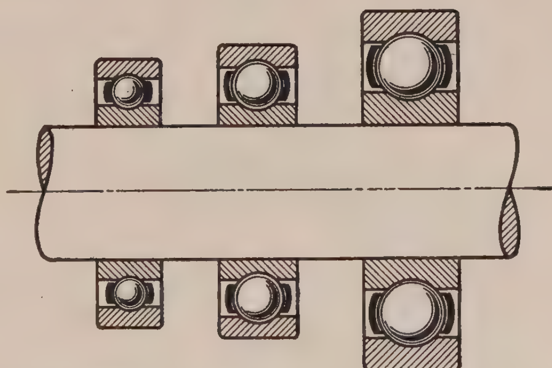


FIG. 259. COMPARISON OF LIGHT, MEDIUM, AND HEAVY BALL BEARINGS.

then of three different bearing sizes (Tables 34, 35, and 36). Each series is proportioned to meet certain load and speed conditions as well as space requirements.

214. General Remarks on Roller Bearings. The basic principles of ball-bearing design also apply to those bearings in which rollers are used in place of balls. The "line" contact of a roller under load, however, provides a greater area of support than the "point" contact of

* This is due to the far-reaching influence of Stribeck's investigations on ball bearings, carried out in Germany. Balls, originally made in England, have retained the English unit of measurement.

a ball. Consequently, roller bearings have correspondingly greater load-carrying capacity and greater resistance to shock or overloads. The friction of roller bearings is somewhat higher than that of ball bearings.

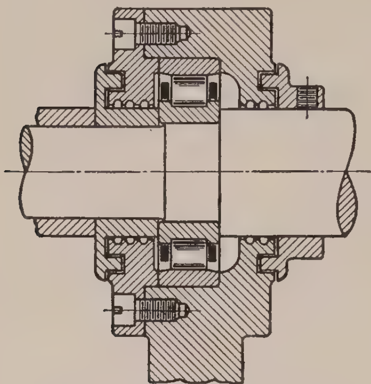
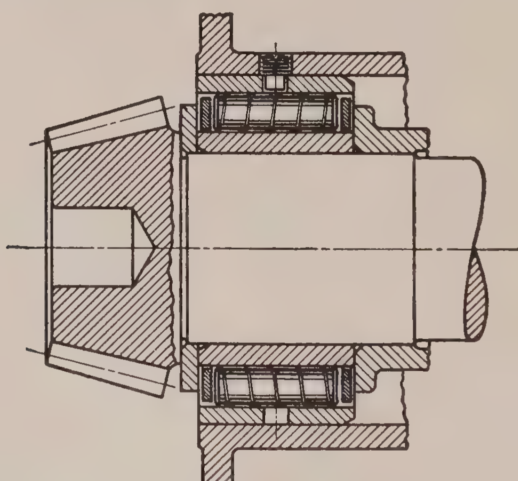


Fig. 260. Cylindrical Roller Bearing.

215. Cylindrical Roller Bearings. Figure 260 shows a bearing with *solid cylindrical* rollers whose principal dimensions correspond to those of single-row radial ball bearings. Such bearings are interchangeable with the latter in the light, medium, and heavy series. While they have greater radial-load capacity than ball bearings, straight cylindrical roller bearings have no thrust capacity. The rollers are held in position endwise by the groove cut in one or both of the races.

Another type known as the *Hyatt* roller bearing is shown in Fig. 261. The characteristic feature of this bearing is the hollow rollers formed by

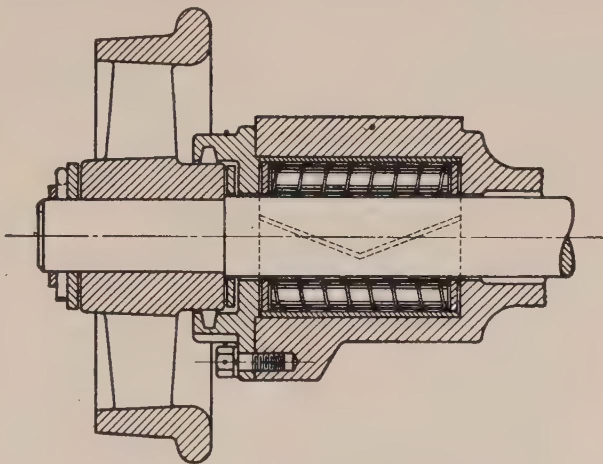


Hyatt Roller Bearing Co., Newark, N. J.

Fig. 261. Hyatt Roller Bearing for Heavy Service.

helically wound strips of alloy steel which are cut to length, heat-treated, and ground. Rollers of this construction are resistant to shocks, and, having some flexibility, adjust themselves to a better load

distribution. Bearings are assembled with alternate right and left hand wound rollers insuring a good distribution of lubricant. For heavy duty and precision work, solid races of hardened steel are recommended. For lighter service, the outer race may be of a split type, rolled from strip steel (Fig. 262). In the latter case, the inner race is

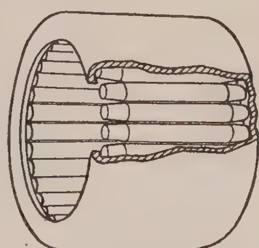


Hyatt Roller Bearing Co., Newark, N. J.

FIG. 262. HYATT ROLLER BEARING FOR LIGHT SERVICE.

eliminated, the rollers bearing directly on the shaft, which may be hardened. The rolls in this type may be 8 to 10 diameters long.

216. Needle Bearings. The demand for an anti-friction type bearing of smaller proportions radially, one which would require little if any more space than a bronze or babbitt bushing, has led to the introduction of so-called *needle bearings* (Fig. 263). The characteristic features of this bearing are the large number of very small cylindrical rollers which completely fill the annular space, and the absence of a cage or separator. Rollers 1/16 in. to 1/8 in. in diameter are generally used for shaft diameters of 1/2 in. to 2½ in., respectively.



The Torrington Co., Torrington, Conn.

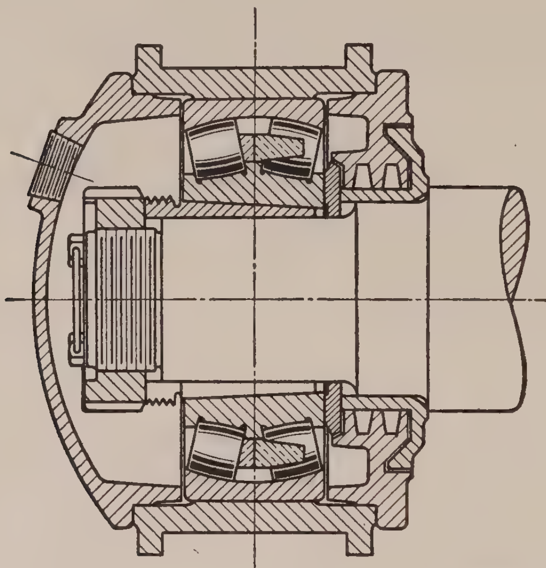
FIG. 263. NEEDLE BEARING.

Bearings are obtainable with both inner and outer raceways, but where it is possible to harden the shaft, the inner race may be eliminated and the rollers run directly on the shaft. They have very high radial load-carrying capacity, and in limited spaces may be used to advantage

to replace other types of bearings which have proven inadequate. For this reason, needle bearings are being adapted for use with universal joints, valve rocker arms, cam rollers, large piston pins, and similar members.

217. Curved Roller Bearings. Several types of bearings have been developed with curved rollers that are capable of carrying combined radial and thrust loads and are self-aligning.

The *SKF* spherical roller bearing is shown in Fig. 264. The outer race has a spherical seat, and there are two rows of barrel shaped rollers,



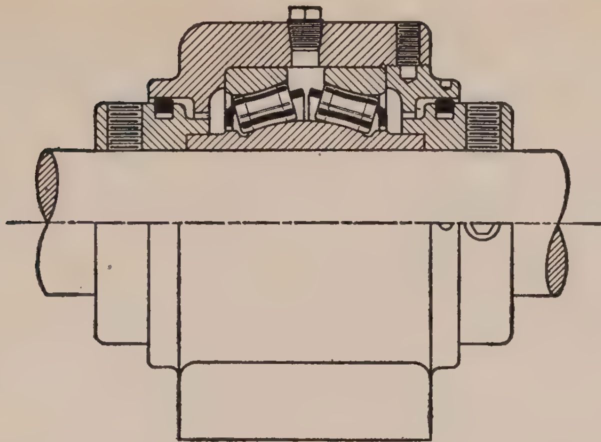
SKF Industries, Inc., Philadelphia, Pa.

FIG. 264. APPLICATION OF SKF SPHERICAL ROLLER BEARING.

which run in a grooved inner race. Thrust can be taken in either direction and is rated about 25 per cent of the radial load. This bearing has about twice the capacity of a ball bearing for the same general dimensions.

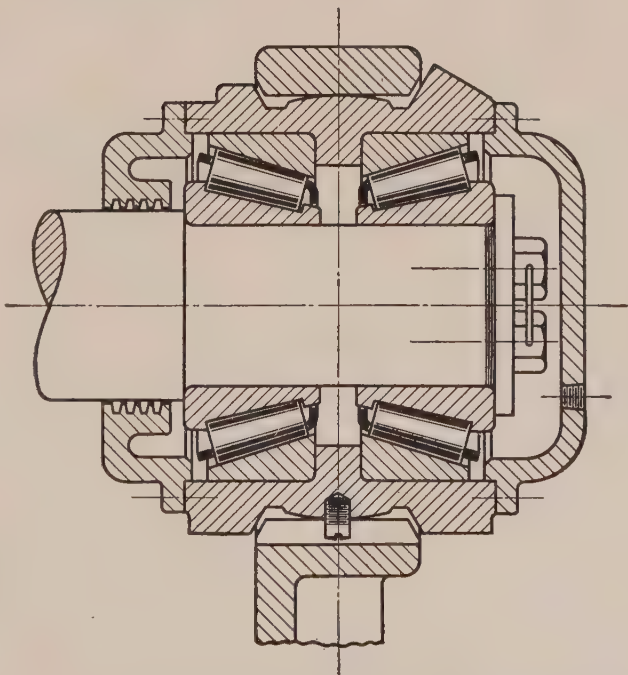
The *Shafer* roller bearing (Fig. 265) has a spherical seat on the inner race and the rollers are hourglass shaped. The angular position of these rollers permits high thrust loads. The single-row bearing has a thrust rating approximately 100 per cent of its radial capacity whereas the double row has about 50 per cent.

218. Tapered Roller Bearing. The Timken roller bearing in its standard form is shown in Fig. 266. The rollers are conical, with the



Shafer Bearing Corp., Chicago, Ill.

FIG. 265. SHAFER SELF-ALIGNING ROLLER BEARING MOUNTED IN PILLOW BLOCK.

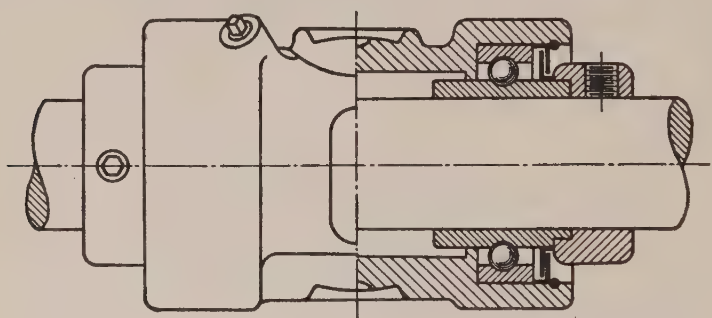


Timken Roller Bearing Co., Canton, Ohio

FIG. 266. APPLICATION OF TIMKEN TAPERED ROLLER BEARINGS.

elements of the tapered surfaces intersecting in a common point on the shaft axis. The rollers are held in correct alignment by end contact against a shoulder on the inner race. The tapered rollers and their angular position allow this bearing to carry both radial and thrust loads. Standard types have thrust ratings varying from about 60 per cent to 90 per cent of their radial capacities. In the larger sizes and multiple units, these bearings have been applied successfully to railway journal boxes and roll necks in steel mills.

219. Applications of Anti-Friction Bearings. While their most extensive application is in the automotive industry, the dependability and low maintenance cost of anti-friction bearings has led to their increasing use in all types of machinery and industrial equipment. The application of roller bearings in a self-aligning pillow block is shown in Fig. 265. The ball-bearing hanger box illustrated in Fig. 267 is interchangeable with ring-oiled plain bearings in standard shaft hangers.



The Fafnir Bearing Co., New Britain, Conn.

FIG. 267. BALL-BEARING HANGER BOX.

220. Interpretation of Load Ratings. There is considerable variation in the load capacities of similar bearings as given by different bearing manufacturers. In comparing bearings of the same proportion, it would be unfair to assume, without further investigation, that the bearing having the highest load rating at a given speed is necessarily of better quality than a bearing having a lower rating. Where differences exist, it is mainly due to the consideration given to the service life of the bearing. Some manufacturers establish their ratings on a shorter life basis and consequently give correspondingly higher load capacities. Therefore, to have ratings comparable, it is necessary to convert the load capacities at a given speed to the same *average life expectancy;* otherwise the results may be misleading.*

* In bearing manufacturers' catalogs, conversion factors are given by which the load ratings may be converted to various periods of life expectancy.

According to the SKF Industries

$$\text{(2)} \qquad \text{Hours life} = \frac{C}{(\text{Speed}) \times (\text{Load})^3},$$

where C is a constant determined experimentally, and is dependent upon the size and type of bearing involved. From this equation, it is to be noted that the life of the bearing varies *inversely* with the *speed* and *inversely* with the *third power* of the *load*. When the *load* is *doubled*, the life is reduced to *one-eighth*. When the *load* is reduced to *one-half*, the life is increased *eight* times.

It is apparent that a very careful analysis must be made to determine the loads imposed upon a bearing, if it is to be economically selected or to give the desired service. Furthermore, it is evident that a slight increase in bearing size will result in a substantial increase in bearing life.

221. Selection of an Anti-Friction Bearing. The type of bearing to select for a certain purpose depends to a large extent upon the magnitude and direction of the loads imposed and the nature of the application. In the preceding discussion, the characteristics of various types were briefly considered. In the determination of the bearing size, reference must be made to the load capacities as given in the catalogs of bearing manufacturers. An illustrative example will demonstrate the procedure followed in the selection of bearings.

Example. The spindle of a wood-working machine revolving at 1000 r.p.m. is to be mounted on two single-row radial ball bearings. Bearing (A) is subjected to a radial load of 500 lb. and a thrust load of 417 lb. Bearing (B) is subjected to a radial load of 500 lb. only. The machine is to be used approximately 8 hrs. per day and an average service life of 10 years is desired. In the design of the machine, the spindle diameter is not to exceed 2 in. Select suitable bearings.

The *dimensions* and *radial load ratings* of single-row ball bearings as given by the Norma-Hoffman Bearing Corporation may be taken as fairly representative, and are given in Tables 34, 35, and 36.

Note. Where the outer ring of a bearing is rotated (with the inner ring stationary) the speed should be multiplied by a factor of 1.6 for either ball or roller type,* and the bearing chosen upon the basis of this revised speed.

The ratings given in Tables 34, 35, and 36 are based upon *an average of 10,000 hours of operation under uniform and steady load.* If, however, the required life of the bearing differs from 10,000 hours, then the actual load on the bearing should be modified by a factor as given in Table 37, so that the subsequent bearing selection will be made upon the basis of the resultant, revised load.

* This regulation is according to the recommendations of the Norma-Hoffman Bearings Corporation and may vary to some extent with other bearing manufacturers on certain types of bearings.

TABLE 34—SINGLE-ROW BALL BEARINGS
(Non-Filling Notch Type)—Bronze Retainers—*Light Series*

Bearing Number	S.A.E. No.	Bore		O.D.		Width		Ball Diam.	Load in Lb. at Speed in R.P.M.								
		mm.	in.	mm.	in.	mm.	in.	in.	100	300	500	1000	1500	2000	3000	4000	5000
110	200	10	.3937	30	1.1811	9	.354	7/32	330	230	190	150	130	120	110	100	90
112	201	12	.4724	32	1.2598	10	.394	7/32	390	270	220	180	150	140	120	110	100
115	202	15	.5906	35	1.3780	11	.433	7/32	450	310	260	210	180	160	140	130	120
117	203	17	.6693	40	1.5748	12	.472	1/4	560	390	330	260	220	210	180	160	150
120	204	20	.7874	47	1.8504	14	.551	9/32	690	480	400	320	280	250	220	200	180
125	205	25	.9843	52	2.0472	15	.591	9/32	780	540	450	360	320	280	250	220	210
130	206	30	1.1811	62	2.4409	16	.630	5/16	1050	720	610	480	420	380	330	310	280
135	207	35	1.3780	72	2.8346	17	.669	3/8	1450	1010	850	670	590	530	470	410	390
140	208	40	1.5748	80	3.1496	18	.709	7/16	1720	1190	1000	790	690	630	540	490	460
145	209	45	1.7717	85	3.3465	19	.748	7/16	1910	1320	1110	880	760	700	610	550	510
150	210	50	1.9685	90	3.5433	20	.787	7/16	2100	1450	1220	960	840	770	660	600	560
155	211	55	2.1654	100	3.9370	21	.827	1/2	2640	1830	1540	1220	1070	960	840	770	
160	212	60	2.3622	110	4.3307	22	.866	1/2	2640	1830	1540	1220	1070	960	840	770	
165	213	65	2.5591	120	4.7244	23	.906	9/16	3240	2250	1890	1500	1320	1190	1040	940	
170	214	70	2.7559	125	4.9213	24	.945	9/16	3540	2460	2070	1640	1440	1300	1130		
175	215	75	2.9528	130	5.1181	25	.984	5/8	3920	2710	2280	1810	1580	1440	1250		
180	216	80	3.1496	140	5.5118	26	1.024	5/8	4270	2950	2490	1980	1730	1570	1370		
185	217	85	3.3465	150	5.9055	28	1.102	11/16	5040	3490	2940	2340	2030	1850			
190	218	90	3.5433	160	6.2992	30	1.181	3/4	5380	3730	3140	2490	2170	1980			
195	219	95	3.7402	170	6.6929	32	1.260	13/16	6180	4280	3620	2870	2500	2280			
200-H	220	100	3.9370	180	7.0866	34	1.339	7/8	7060	4890	4120	3280	2850				
205-H	221	105	4.1339	190	7.4803	36	1.417	7/8	7060	4890	4120	3280	2850				
210-H	222	110	4.3307	200	7.8740	38	1.496	7/8	7710	5330	4500	3570	3110				
220-A	224	120	4.7244	215	8.4646	40	1.574	1	8910	6180	5210	4130	3610				
230-A	226	130	5.1181	230	9.0551	40	1.574	1 1/16	8910	6180	5210	4130					
240-A	228	140	5.5118	250	9.8425	42	1.654	1 3/16	12030	8340	7030	5580					

TABLE 35—SINGLE-ROW BALL BEARINGS
(Non-Filling Notch Type)—Bronze Retainers—*Medium Series*

Bearing Number	S.A.E. No.	Bore		O.D.		Width		Ball Diam.	Load in Lb. at Speed in R.P.M.								
		mm.	in.	mm.	in.	mm.	in.	in.	100	300	500	1000	1500	2000	3000	4000	5000
310-H	300	10	.3937	35	1.3780	11	.433	1/4	560	390	330	260	220	210	180	160	150
312-H	301	12	.4724	37	1.4567	12	.472	1/4	650	450	380	300	260	240	210	190	170
315-H	302	15	.5906	42	1.6535	13	.512	9/32	810	560	470	370	330	290	260	230	210
317-H	303	17	.6693	47	1.8504	14	.551	5/16	980	670	560	450	390	350	310	280	260
320	304	20	.7874	52	2.0472	15	.591	5/16	1120	770	650	520	450	410	360	330	300
325	305	25	.9843	62	2.4409	17	.669	3/8	1550	1080	910	720	630	570	500	450	420
330	306	30	1.1811	72	2.8346	19	.748	7/16	2040	1410	1190	940	820	750	650	590	550
335	307	35	1.3780	80	3.1496	21	.827	1/2	2560	1780	1500	1190	1040	940	820	750	690
340	308	40	1.5748	90	3.5433	23	.906	9/16	3150	2190	1840	1460	1280	1160	1010	920	850
345	309	45	1.7717	100	3.9370	25	.984	5/8	3800	2630	2220	1760	1540	1400	1220	1110	1030
350	310	50	1.9685	110	4.3307	27	1.063	5/8	4270	2950	2490	1980	1730	1570	1370	1240	
355	311	55	2.1654	120	4.7244	29	1.142	11/16	5040	3500	2940	2340	2020	1850	1620	1470	
360	312	60	2.3622	130	5.1181	31	1.220	3/4	5220	3620	3050	2420	2110	1920	1680		
365	313	65	2.5591	140	5.5118	33	1.299	13/16	6750	4670	3990	3140	2730	2490	2160		
370	314	70	2.7559	150	5.9055	35	1.378	7/8	7700	5340	4500	3570	3110	2830	2470		
375	315	75	2.9528	160	6.2992	37	1.457	7/8	7700	5340	4500	3570	3110	2830	2470		
380	316	80	3.1496	170	6.6929	39	1.535	15/16	8680	6010	5070	4020	3530	3190			
385	317	85	3.3465	180	7.0866	41	1.614	1	9720	6730	5680	4510	3930	3570			
390	318	90	3.5433	190	7.4803	43	1.693	1	9720	6730	5680	4510	3930	3570			
395	319	95	3.7402	200	7.8740	45	1.772	1 1/8	11940	8280	6980	5540	4830	4390			
400	320	100	3.9370	215	8.4646	47	1.850	1 1/8	11940	8280	6980	5540	4830				
405-H	321	105	4.1339	225	8.8583	49	1.929	1 1/4	14360	9960	8400	6660	5820				
410-H	322	110	4.3307	240	9.4488	50	1.969	1 1/4	14360	9960	8400	6660	5820				
420-H	324	120	4.7244	260	10.2362	55	2.165	1 3/8	16980	11770	9930	7880	6890				
430-A	326	130	5.1181	280	11.0236	58	2.284	1 1/2	19780	13720	11600	9160					
440-A	328	140	5.5118	300	11.8110	62	2.441	1 5/8	22750	15780	13300	10500					

TABLE 36

Single-Row Ball Bearings

(Non-Filling Notch Type)—Bronze Retainers—*Heavy Series*

Bearing Number	S.A.E. No.	Bore		O.D.		Width		Ball Diam.	Load in Lb. at Speed in R.P.M.								
		mm.	in.	mm.	in.	mm.	in.	in.	100	300	500	1000	1500	2000	3000	4000	5000
517	403	17	.6693	62	2.4409	17	.669	3/8	1550	1080	910	720	630	570	500	450	425
520	404	20	.7874	72	2.8346	19	.748	9/16	2360	1640	1380	1095	960	870	755	690	635
525	405	25	.9843	80	3.1496	21	.827	9/16	2755	1915	1610	1275	1120	1015	885	805	745
530	406	30	1.1811	90	3.5433	23	.906	5/8	3325	2300	1945	1540	1345	1225	1065	970	900
535	407	35	1.3780	100	3.9370	25	.984	11/16	3920	2720	2290	1820	1585	1445	1260	1145	1060
540	408	40	1.5748	110	4.3307	27	1.063	13/16	4500	3120	2630	2090	1820	1655	1445	1310	1220
545	409	45	1.7717	120	4.7244	29	1.142	13/16	5250	3640	3070	2440	2125	1935	1690	1530	
550	410	50	1.9685	130	5.1181	31	1.220	13/16	6000	4160	3510	2790	2430	2210	1930	1750	
555	411	55	2.1654	140	5.5118	33	1.299	7/8	6850	4750	4000	3180	2770	2520	2200		
560	412	60	2.3622	150	5.9055	35	1.378	15/16	7720	5350	4510	3580	3140	2840	2480		
565	413	65	2.5591	160	6.2992	37	1.457	1	8640	5990	5050	4010	3500	3180			
570	414	70	2.7559	180	7.0866	42	1.654	1 1/8	10620	7360	6210	4930	4300	3910			
575	415	75	2.9528	190	7.4803	45	1.772	1 3/16	11670	8100	6820	5420	4730	4300			
580	416	80	3.1496	200	7.8740	48	1.890	1 1/4	12770	8860	7470	5930	5180				
585	417	85	3.3465	210	8.2677	52	2.047	1 5/16	13900	9640	8130	6450	5640				
590	418	90	3.5433	225	8.8583	54	2.126	1 3/8	15100	10470	8825	7010	6120				
595		95	3.7402	250	9.8425	55	2.165	1 1/2	17590	12200	10280	8160	7130				
600		100	3.9370	265	10.4331	60	2.362	1 5/8	20230	14030	11825	9390	8200				

TABLE 37

LOAD CONVERSION FACTORS

DESIRED AVERAGE EXPECTED LIFE IN HOURS	MULTIPLY ACTUAL LOAD BY FOLLOWING FACTOR BEFORE SELECTING BEARING FROM LOAD TABLE
2500	0.63
5000	0.80
7500	0.91
10,000	1.00
20,000	1.25
30,000	1.43
50,000	1.67
80,000	2.00

In our problem, if we assume the machine is used 300 working days per year, then for 10 yrs. at 8 hrs. per day,

$$\text{Desired Life} = 10 \times 300 \times 8 \text{ hrs.} = 24{,}000 \text{ hrs.}$$

From Table 37 we would use a factor of approximately 1.34.

222. Combined Radial and Thrust Loads. We will consider next the question of *combined radial* and *thrust loads*. According to Norma-Hoffman Bearing Corporation, a single-row closed-type ball bearing with uninterrupted uniform race sides will carry, simultaneously with the radial load, thrust loads of varying magnitude. Such combined loads are converted to an equivalent radial load by the formula *

$$F = \frac{F_r + 3F_t}{2}, \tag{3}$$

where F is the equivalent radial load on the basis of which the bearing selection should be made, F_r is the calculated radial load, and F_t is the calculated thrust load.

In considering single-row closed-type ball bearings of the maximum type using filling notch (or slot), the same formula may be used provided the bearing is not subjected to a thrust in excess of one-half of the radial load.

Referring again to our problem, bearing (A) is subjected to a radial load of 500 lb. and a thrust load of 417 lb. Then, for this bearing,

$$F = \frac{500 + 3 \times 417}{2} = 875 \text{ lb. (equivalent radial load).}$$

223. Shock Load Factors. Finally, consideration must be given to the application or nature of the load. Since published load ratings are based strictly upon a uniform or steady load, where shock loads are encountered, certain modifying factors must be used. The following table of *shock-load factors* covers varying conditions, and indicates the factor by which the calculated load should be multiplied before making the bearing selection.

* This formula applies only where the thrust exceeds one-third of the radial load; if less than this percentage it may be disregarded.

TABLE 38

SHOCK-LOAD FACTORS

Type of Service	Multiply Calculated Load by Following Factors	
	Ball Bearings	Roller Bearings
Uniform and Steady Load	1.0	1.0
Light Shock Load	1.5	1.0
Moderate Shock Load	2.0	1.3
Heavy Shock Load	2.5	1.7
Extreme and Indeterminate Shock Load	3.0	2.0

In our problem, we could safely assume that moderate shock loads would be encountered; therefore a shock load factor of 2.0 would be used. By combining these various factors, we would select the bearings from Tables 34, 35, and 36 as follows:

Bearing (*A*) 1.34×875 lb. $\times 2.0 = 2345$ lb.

At 1000 r.p.m., the nearest size bearing to meet our requirements would be No. 545 (S.A.E. No. 409) Heavy Series. This bearing has a load rating of 2440 lb. and a bore of 1.7717 in. Any other bearing having the load capacity, in either the Light or Medium Series, would have a bore in excess of 2 in., and therefore could not be considered.

Bearing (*B*) 1.34×500 lb. $\times 2.0 = 1340$ lb.

The nearest bearing in this case is No. 340 (S.A.E. No. 308) Medium Series, having a load rating of 1460 lb. and a bore of 1.5748 in. As this bore is slightly small, the next larger size bearing, No. 345 (S.A.E. No. 309) Medium Series, may be more appropriate. This bearing has excess load capacity (1760 lb.), but the bore is 1.7717 in., the same as selected for bearing (*A*). This arrangement would make a well balanced design.

It should be stated that it is the policy of bearing manufacturers to submit, without obligation, recommendations on designs involving the application of ball or roller bearings. The facilities and experience of their specialized engineering service is available to those considering the use of anti-friction bearings. Consequently, the bearing manufacturers' guarantee is dependent upon an approval of the bearing selection and application.

224. Mounting Ball or Roller Bearings. In general, the rotating ring should be mounted with a light press fit and the stationary ring with a close push fit. In the majority of applications, the shaft is the rotating member and the housing is stationary. In such cases the inner ring should be pressed on the shaft, seating against a shoulder, as in Fig. 252. It should be further secured by means of a nut-locking device. With the outer ring having a push fit in the housing, it will be free to adjust itself to the inner ring and to creep slowly in its seat, with the result that all parts of the race will be uniformly utilized.

When two ball bearings are mounted on a shaft some distance apart, only one may be located in a fixed axial position. See Fig. 252. The other must be free to adjust itself to any expansion or contraction of the shaft due to temperature changes, inaccuracies in machining, or shaft deflection.

Housings should provide ample reservoir space for a generous supply of oil or grease. A variety of housing closures are available which prevent leakage of the lubricant and at the same time exclude foreign matter and moisture. Flexible materials like felt, cork, or leather in light sliding contact with the rotating parts are very effective and extensively used. See Figs. 252, 255, and 258. Grease grooves (Fig. 266) packed with a heavy grease and slingers (Fig. 264) are other types in common use. A labyrinth closure shown in Fig. 260 is an effective seal against acid fumes, liquids, or abrasives. It is used mainly for very high speed.

225. Lubrication of Anti-Friction Bearings. The function of a lubricant for an anti-friction bearing is to reduce friction within the bearing itself, to prevent rust and corrosion, to dissipate heat, and to aid in the protection against the intrusion of foreign matter or moisture. Oil or grease may be used, depending largely upon design requirements. A neutral mineral-base product is preferred to animal or vegetable oils, as the latter may become rancid and develop free acid which tends to corrode the finished surfaces.

Grease is more easily retained in the housing and is recommended where the bearing will receive a minimum of attention. Oil is more suitable for higher speeds. Lubricants compounded with solids, such as graphite, talc, or mica, are objectionable on account of their tendency to cake and clog the bearing.

PROBLEMS

1. Select ball bearings for the wood-working machine spindle mentioned in this chapter if the speed is 3000 r.p.m., the loading being 500 lb. radial and 250 lb. thrust.

2. A 2 in. shaft is supported on ball bearings 28 in. apart. A pulley overhanging one bearing by 9 in. causes a vertical force of 600 lb. Select the proper bearings for an operating speed of 2400 r.p.m. and 20,000 hr. of life expectancy. Load is steady. Give the dimensions of shaft and bore of inner race.

3. A ball bearing supporting a bevel-gear shaft is subjected to a radial load of 350 lb. and a thrust load of 120 lb. Select the proper bearing for a speed of 150 r.p.m. under favorable conditions. Life expectancy is 50,000 hr.

4. A throw-out bearing on a cone clutch carries an axial load of 200 lb. and a radial load of 100 lb. The clutch operates at 800 r.p.m. Select a ball bearing for this service.

CHAPTER 14

FLYWHEELS AND HIGH-SPEED ROTORS

226. Flywheel Action. A flywheel is a heavy rotating body which acts as a reservoir for absorbing and redistributing kinetic energy. There are two distinct types of service for which this device is used: first, to store and deliver power in intermittent service; second, to absorb and cushion regularly varying cyclic forces in power production and power consumption.

As an example of the first type, let us consider a machine used for manufacturing purposes, such as a punch press or shear on which work is performed intermittently. During each cycle of operation, energy is consumed at a variable rate. The maximum demand occurs during the work period, while over the remainder of the cycle the only energy required is that which is necessary to overcome friction and other internal losses. The average power consumption is very much less than the maximum; hence, it would be uneconomical to supply a drive large enough to suit the maximum demand. The function of the flywheel in this case is to absorb excess energy during the "idling" period, with a resulting increase in velocity, and then give up this energy during the work period, with an accompanying decrease in velocity. Obviously a change in velocity is necessary if the flywheel is to deliver any energy.

If the weight W of a flywheel is assumed to be concentrated in the rim, and the rim revolves with a velocity of v_1 ft. per sec., then the kinetic energy of the flywheel in ft. lb. is $Wv_1^2/2g$, where g is the acceleration due to gravity = 32.2 ft. per sec. per sec. If the velocity is now reduced to v_2 ft. per sec., the energy E given up by the flywheel is

$$E = \frac{W(v_1^2 - v_2^2)}{2g}.$$

Hence we have

$$W = \frac{2gE}{v_1^2 - v_2^2}. \tag{1}$$

As an example, assume that a small shear is to have sufficient capacity to cut 1 in. square steel bars and that the work required per cut is 25,000 in. lb. The cutting operation is to occupy only one-tenth of a revolution and there are to be 15 cuts per minute. The flywheel has a mean diameter of 3 ft. and is mounted on a shaft running at 120 r.p.m. If the weight of the flywheel is assumed to be concentrated entirely at the rim, determine the weight of the flywheel.

Solution. If there is a cut at every stroke of the shear the cutting action occupies $1/15 \times 1/10 = 1/150$ minute, and the useful work supplied by the drive during that time is $25{,}000/10 = 2{,}500$ in. lb. The flywheel must then provide $25{,}000 - 2{,}500 = 22{,}500$ in. lb. $= 1880$ ft. lb. of energy. The rim speed is $\pi \times 3 \times 120/60 = 19$ ft. per sec. If we assume that the flywheel may be slowed down 10 per cent while delivering its energy, the speed would drop to 17.1 ft. per sec. The required weight is then $W = 2 \times 32.2 \times 1880/(19^2 - 17.1^2) = 1760$ lb.

For this type of service a speed reduction of more than 10 per cent may be permissible with direct or geared motor drives. Such reductions are permissible with belt drives, according to O. S. Beyer,* only if the reductions occur at intervals of four or five seconds. For presses working continuously at speeds of about 90 r.p.m., a reduction of only 5 to 6 per cent is advisable.

As an example of the second type, let us consider a flywheel used in conjunction with a heat engine. In this case the flywheel enables the engine to deliver power at a practically constant rate, while power is being generated by the engine at a variable rate. The flywheel absorbs excess energy during those periods of the cycle in which the power developed in the cylinders exceeds the demand or energy delivered, and it gives up this stored energy when the demand from the load exceeds the power being generated.

It is customary to designate the permissible speed variation for prime movers by a factor called the *coefficient of fluctuation*, which has the value $(v_1 - v_2)/v$, where v is the average speed and is equal to $(v_1 + v_2)/2$. If the coefficient of fluctuation is δ, then $v_1 - v_2 = \delta v$. It is also customary to express the energy to be absorbed, or given up, by the flywheel in terms of the engine output. With a given engine diagram representing the amount of useful work H per revolution, the horsepower hp is $H \times r.p.m./33{,}000$. The excess energy E available during part of the revolution may be expressed as KH, where K is a fractional constant. Therefore

$$E = \frac{K \times hp \times 33{,}000}{r.p.m.}.$$

From equation (1),

$$W = \frac{2gE}{v_1^2 - v_2^2} = \frac{2gE}{(v_1 + v_2)(v_1 - v_2)} = \frac{gE}{\delta v^2},$$

or

$$W = \frac{\text{Constant} \times hp}{\delta v^2 \times r.p.m.}. \tag{2}$$

For steam engines the value of the constant in formula (2) has been determined and is given in various handbooks. For gas engines

* American Machinist, October 17 and 24, 1912.

it can be evaluated into a factor varying only with the compression pressure and the weight of reciprocating parts per square inch of piston area.

The coefficient of fluctuation δ varies from 1/5 for hammering and crushing machinery to 1/300 for three-phase generator drives. For ordinary machine shop drives it may be taken about 1/40. The important thing to note is that the necessary flywheel weight decreases rapidly, not only with increasing r.p.m., but also with increasing diameter. This is due to the fact that the rim speed v is proportional to the diameter.

In many flywheel designs a large part of the weight, perhaps as much as 30 per cent, is concentrated in the arms and the hub. This weight, however, may contribute only 5 to 10 per cent to the flywheel effect. Therefore, the actual rim weight may be made 5 to 10 per cent less than the calculated weight, since the arms and the hub will make up for the difference. The total flywheel weight may then be 20 to 25 per cent greater than the weight required if it were concentrated entirely in the rim.

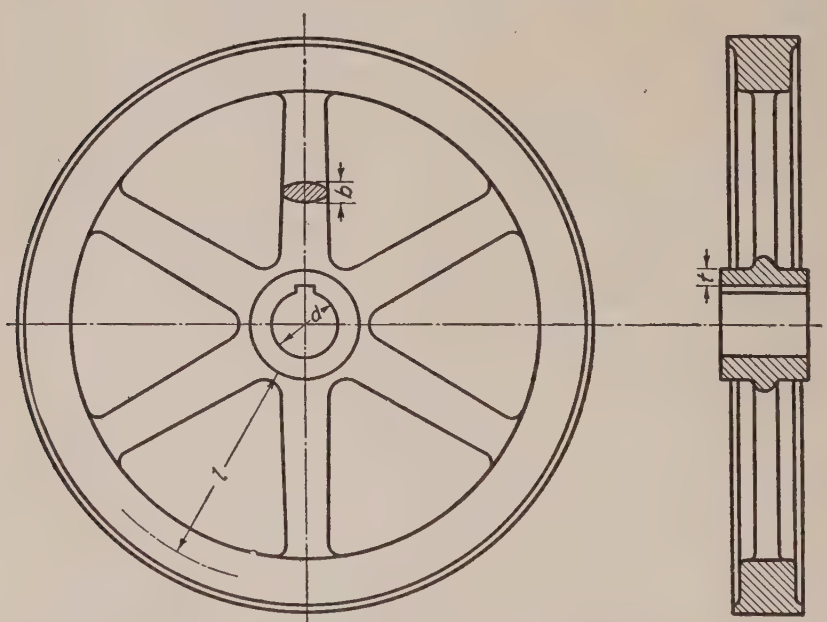


FIG. 268. FLYWHEEL. (SOLID, ONE-PIECE CONSTRUCTION.)

227. General Design of Flywheels. A typical flywheel design is shown in Fig. 268. Various types of arms may be used as shown in

Fig. 268 and Fig. 269. In small wheels the arms may be replaced by a solid web for simplicity of design and greater strength. In large wheels, serious casting stresses may be induced in the arms as a result of the unequal cooling rates of the metal in the rim, arms, and hub.

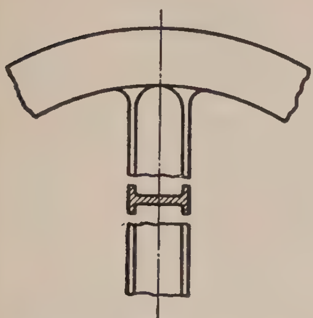


Fig. 269. Section of Flywheel with I-Beam Arm.

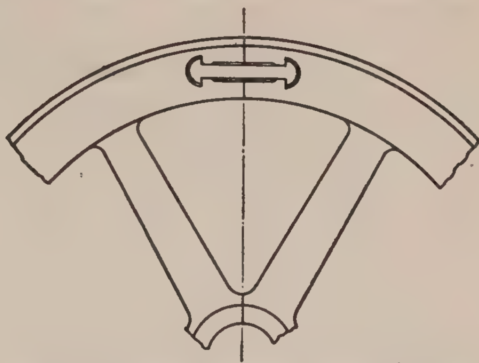


Fig. 270. Split Flywheel with Rim Joint Fastened with Shrink Links.

Such stresses are indeterminate and at times may be of such magnitude as to break the arms. Careful foundry practice is essential in order that the casting may be cooled as uniformly as possible. By splitting the hub through the center (Fig. 270), or by splitting the hub between each pair of arms, the arms are relieved and free to contract while cooling, which tends to reduce the cooling stresses very appreciably.

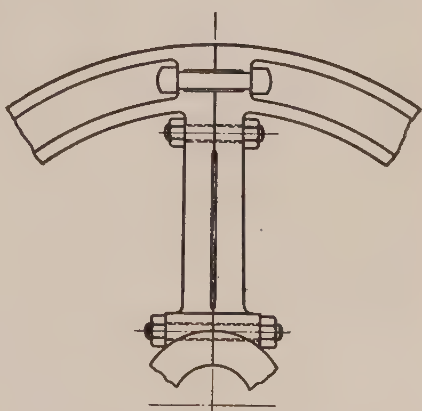


Fig. 271. Split Flywheel with Joint Passing through a Pair of Arms.

For greater ease in transportation and assembly, flywheels may be cast in sections (Fig. 271). Although in practice the joint is frequently placed between the arms, the proper design is to have the wheel split through a pair of arms, as shown in the figure. Since the bending moment on the rim due to the centrifugal force is a maximum at the middle of the section between the arms, a joint at this point would have a serious weakening effect on the wheel. This is true particularly if the construction of the joint is such as to cause a concentration of an additional load at this same point of weakness.

228. Stresses in Flywheels. The maximum permissible speed of a flywheel is limited by the stresses induced in the rim. These stresses, resulting from the effect of the centrifugal force, are a direct "hoop" tension caused by the tendency of the rim to expand like a boiler shell and bending stresses due to the restraining moments of the arms. The portion of the rim between the arms is somewhat similar to a beam fixed at the ends with a uniform loading equal in intensity to the centrifugal force. The derivation of the bending stress by the beam theory is rather inaccurate, however, because the rim section at the arms is not quite unyielding. The arms of a flywheel are not absolutely rigid and under the centrifugal action of the rim they are stretched to some extent. Obviously, if the arms stretched to allow the rim to expand freely as a rotating ring, there would be no bending stress induced. The actual state of the rim is evidently at some intermediate point between that of a free rotating ring and a beam fixed at the ends.* Since the theoretical determination of this stress is rather complex,† its derivation will be omitted.

The bending stress in a flywheel may be reduced by increasing the number of arms, thus reducing the length of the rim section between the arms; and by providing a web construction instead of arms the bending stress is eliminated entirely. Whenever possible, the cross-section of the rim should be proportioned to resist the bending action; that is, the radial depth of the ring should be made equal to, or somewhat greater than, the width.

Even when the bending action is eliminated, the magnitude of the hoop stress may be sufficiently serious to limit the upper speeds of discs or drum rotors, such as are used in steam turbines, centrifugal pumps, and blowers.

229. Determination of the Hoop Stress. Consider a free rotating ring having a mean radius of r in. and cross-sectional area of A sq. in., as shown in Fig. 272. If the mass of the material per cu. in. is m, then the mass of a rim element of length $rd\alpha$ is $mArd\alpha$. If the mean rim velocity is v ft. per sec., the centrifugal force acting on this element is $12mv^2Ard\alpha/r = 12mAv^2d\alpha$. The component of force in the direction of the resultant F is $12mAv^2 \sin \alpha d\alpha$. The total resultant force on

* It has been proposed, for instance, by Professor Lanza that the stress in the ordinary types of flywheels may be assumed to be equal to three-fourths of the hoop stress plus one-fourth of the bending stress. The bending stress in this case is based on a straight beam having a length equal to the arc distance between arms. The beam is considered fixed at both ends and uniformly loaded by centrifugal force. Inasmuch as these conditions are arbitrarily assumed, the resulting stress values are questionable. However, a stress determined by this method will be higher than the hoop stress alone. (Transactions A.S.M.E., vol. 20, 1899, p. 951.

† See Timoshenko, *Strength of Materials*, Part 2, 1930 ed., p. 451.

half the circumference is

$$12mAv^2 \int_0^\pi \sin \alpha d\alpha = 24mAv^2.$$

This force is resisted by the tensile stresses s_t at sections A-A and B-B, each of area A. Therefore

$$s_t = \frac{24mAv^2}{2A} = 12mv^2 = \frac{12wv^2}{g}, \tag{3}$$

where w is the weight of the material per cu. in. and g the acceleration due to gravity in ft. per sec. per sec.

This formula shows that in a free ring the tensile stress in the rim due to the centrifugal force depends only on the rim velocity and the density of the material, but not on the rim dimensions or the diameter, except as the latter is involved in the velocity v as $2\pi r \times$ r.p.m./(12×60). Consequently, it is possible to establish certain maximum rim velocities that should not be exceeded for various materials. Suppose, for instance, that we regard 9000 p.s.i. as the maximum permissible stress in cast iron even under the most favorable loading conditions. The weight of cast iron is 0.255 lb. per cu. in., hence, the permissible rim velocity $v = \sqrt{(9000 \times 32.2)/(12 \times 0.255)} = 307$ ft. per sec. Similarly, if we assume 25,000 p.s.i. as being the maximum permissible stress of low-carbon steel weighing 0.28 lb. per cu. in., the maximum rim velocity for steel rings or drums would be 488 ft. per sec.

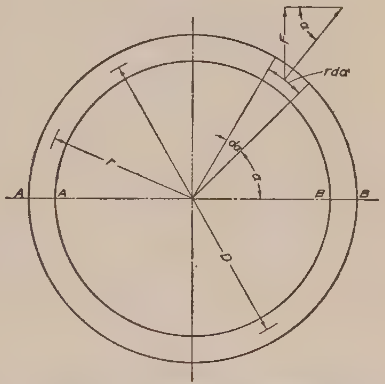


Fig. 272. Force Analysis of the Hoop Stress in a Free Rotating Ring.

Such velocities presuppose not only very smooth and shockless running conditions, but also the absence of any casting stresses or bending stresses due to arms. Where these conditions are not satisfied, much lower velocities must be used. Thus, for ordinary cast-iron flywheels with arms, 6000 ft. per min. is regarded as a practical maximum velocity, although 300 ft. per sec. as found above is equivalent to 18,000 ft. per min. For pulleys, even velocities of 6000 ft. per min. are considered too high, unless the pulleys have been carefully

balanced. Flywheels with built-up rims of laminated steel plates, and cast flywheels having steel spokes cast in place at the rim and hub, somewhat similar to the spokes in a wire wheel, have been run at very high speeds.

230. Hubs, Arms, Etc. If the shaft on which a flywheel is mounted transmits only the torque going to, or coming from, the flywheel, and if in addition the bearings are so close together that the bending stress is negligible, then manifestly, the hub and the arms can be proportioned in some simple relation to the shaft diameter or the bore. If, however, the shaft transmits a torque greater than the flywheel torque, or if it is proportioned to resist bending or excessive deflection, then the arms and the hub should be proportioned to the flywheel torque and not to the shaft diameter.

Usually the hub diameter varies between $2d$ and $2.5d$, where d is the shaft diameter or bore. The former value is used when the shaft size exceeds the size required for the flywheel torque, whereas the latter value applies if the shaft is proportioned to the flywheel torque.

The following formula from Hütte may be used to proportion the hub thickness t outside the key seat, as shown in Fig. 268:

$$(4) \qquad t = \tfrac{1}{5}(d_0 + 0.5d) + \tfrac{3}{8}'' \quad \text{to} \quad \tfrac{1}{4}(d_0 + 0.5d) + \tfrac{3}{8}''.$$

Here $d_0 = \sqrt[3]{16M_t/(\pi s_s)}$, where M_t is the actual torque in in. lb. transmitted to and from the wheel, s_s is the permissible shear stress in p.s.i. in the shaft, and d is again the shaft diameter in in.

In a flywheel with a rim of considerable weight and stiffness, the arms may be regarded as beams, clamped or built in at both ends. On the basis of this assumption, the bending moments at the rim and hub are equal, and the dimensions of the arms should then be the same at both ends. The bending moments in this case are only one half the value of a simple cantilever loaded at the free end. In flywheels with relatively thin, flexible rims, the arms more nearly approximate simple cantilever beams. In such cases the arms should taper toward the rim, since then, theoretically, there is no bending moment at the rim. While some differentiation might be made between these two conditions, it is quite customary to proportion the arm at the hub in either case by formulas applicable to the bending of a simple cantilever. This procedure is justified in view of the fact that in addition to the bending stress, there exists a direct tensile stress due to the centrifugal action of the rim, as well as certain indeterminate casting stresses.

If the maximum torque transmitted to or from the flywheel is M_t and the number of arms is n, each arm is subjected to a bending moment somewhat less than M_t/n, since the length of the arm is less than the radius of the wheel. The bending moment is more nearly equal to Fl, where $F = M_t/nr$, F being the circumferential force at the mean rim diameter, r the mean flywheel radius in in., and l the distance from the rim to the hub in in. See Fig. 268. The arm proportions computed from this moment are the dimensions at the outside of the hub.

The most commonly used shape for the arms is the elliptical section, in which the thickness is made one-half the width. If the width is a and the thickness b, the section modulus is $\pi a^2b/32$. A taper toward the rim of 1/4 in. per ft. is commonly used on the width and about half this amount on the thickness. The width at the rim, however, never should be less than 0.8 times the width at the hub.

231. The Joining of Split Flywheels. According to formula (3), the tensile stress in a freely rotating flywheel rim is $12wv^2/g$. If the cross-sectional area is A, the total force on the rim joint is $12Awv^2/g$. This force must be taken up by bolts, or by bolts and shrink links (Fig. 271), or simply by shrink rings, or by links (Fig. 270). The bolts at the hub are usually made of the same size as those at the rim. Very often, however, the bolts are not proportioned to the actual rim stress but to the maximum rim stress that might reasonably be expected in a flywheel of the type under consideration at the maximum permissible speed.

Example. Let us consider the detail design of the flywheel computed on page 276, and assume it to be split.

The required weight of the flywheel, if concentrated at the rim, was computed to be 1760 lb. Let us assume that the arms and hub supply 5 per cent of the flywheel energy; then the actual rim weight required is $0.95 \times 1760 = 1675$ pounds. If the wheel is made of cast iron, the required volume of the rim is $1675/0.255 = 6570$ cu. in.

With a mean rim diameter of 3 ft., the necessary cross-sectional area is $6570/(\pi \times 3 \times 12) = 58.1$ sq. in. If the depth of the section is made 1.2 times the width, the dimension of the rim will be 7 in. $\times$ 8⅜ in., approximately.

The flywheel must deliver a total of 22,500 in. lb. of energy in 1/150 min. = 2/5 sec. The average mean rim velocity is $(19 + 17.1)/2 = 18.05$ ft. per sec., and the total distance traveled by the rim in the allotted time is $2 \times 18.05/5 = 7.2$ ft. = 86.4 in. The average force exerted in supplying this energy is $22{,}500/86.4 = 260$ lb.

The shaft on which the flywheel is mounted will be subjected to shock loads both in bending and torsion, and will therefore fall into the classification of head shafts. To compute the shaft accurately, a layout would be necessary showing the location of the bearings and the forces acting. As an alternative in this case, we

will determine the shaft size by means of the formula for head shafts given in Table 24, page 196, in which

$$D = \sqrt[3]{\frac{133.7 \times hp}{r.p.m.}}.$$

Then

$$D = \sqrt[3]{\frac{133.7 \times 260 \times \pi \times 3 \times 120}{120 \times 33{,}000}} = 2.15, \text{ say } 2\tfrac{1}{4} \text{ in.}$$

If we assume that the shaft size is to be controlled entirely by the flywheel torque, then the hub diameter of the wheel may be taken as 2.5 times the shaft diameter, or $5\frac{5}{8}$ in. With the key dimensions recommended in Table 26, page 203, the keyseat may project 1/4 in. into the hub. Checking the thickness of the hub over the keyseat by formula (4), $t = 0.2 \text{ to } 0.25 \times (2.25 + 0.5 \times 2.25) + 0.375 = 1.05 \text{ to } 1.22$ say $1\frac{1}{4}$ in. This dimension plus 1/4 in. for the keyseat is within $1\frac{11}{16}$ in., the wall thickness of the hub.

We will assume six arms to reduce bending stress and consider that each arm is subjected to a bending moment of $260(18 - 5.625/2)/6 = 658$ in. lb. It is to be observed that the bending moment is reversed as the flywheel passes from the state of absorbing energy to that of giving up energy. Under such conditions and with suddenly applied loads and impacts as would occur in a machine of this type, a factor of safety of 10 is not too high. If we assume a permissible bending stress of 2500 p.s.i., the required section modulus is then $658/2500 = 0.263$. Assuming an elliptic arm section with a width twice the thickness, and calling the width a, we have $\pi a^3/64 = 0.263$, or $a = 1.75$ in. Such a slender arm joined to a heavy 7 in. $\times$ $8\frac{3}{8}$ in. rim would produce a very unsatisfactory casting.

To form a satisfactory design, this flywheel should have had a diameter of perhaps 5 ft. instead of 3 ft. This proportion would reduce the necessary flywheel weight in the ratio of $(3/5)^2$, since the weight decreases inversely with the square of the rim velocity. It would reduce the dimensions of the rim section to $3\frac{1}{4}$ in. $\times$ $3\frac{7}{8}$ in. Even this lighter section seems to be too heavy to join to arms having a width of only $1\frac{3}{4}$ in. at the hub. In this case it would be desirable to increase the arm width to $2\frac{1}{2}$ or 3 in. in order to produce a satisfactory casting.

Of course a substantial reduction in flywheel weight could be effected if the r.p.m. of the flywheel shaft were increased. If a flywheel diameter of 5 ft. seems too large, such a speeding-up should be undertaken. It would then be necessary to increase correspondingly the gear ratio between the flywheel shaft and the crankshaft of the shear.

The computation of bolts for the rim of a split flywheel will depend on whether the wheel is designed for just one particular speed and load condition, or whether it is designed with a view to the possibility of a variety of service. The extreme and limiting case would be to make the tensile strength of the bolts or other elements holding the rim together as great as that of the rim itself, at least at the point where it is split. Suppose, for instance, we wished to provide bolts for the 5 ft. flywheel computed above in which the rim section measures $3\frac{1}{4}$ in. $\times$ $3\frac{7}{8}$ in. If we considered a bolt steel with a tensile strength 3 times that of cast iron, the combined root area of the bolts at the rim would then have to be $(3\frac{1}{4} \times 3\frac{7}{8})/3 = 4.2$ sq. in. If we resorted to bolts only, evidently there would have to be at least four $1\frac{1}{4}$ in. bolts at each rim joint. Since this construction would be quite heavy, shrink links as shown in Fig. 270 would no doubt be preferred, even though they reduced the net section of the rim considerably. However, if the split is located at the arm, as

Fig. 271 shows, the strength of the arm would compensate for the reduction in section of the rim. In the example under consideration, the velocity of a 5 ft. rim running at 120 r.p.m. is only 31.4 ft. per sec., and the tensile stress due to the centrifugal force would be $12 \times 0.255 \times 31.4^2/32.2 = 94$ p.s.i. This stress is almost negligible, and would necessitate only very small bolts.

It would be more conservative to proportion the bolts for a rim velocity of 6000 ft. per min. or 100 ft. per sec., which usually is stipulated as the maximum permissible speed for ordinary cast-iron flywheels. This velocity would produce a stress of 950 p.s.i. The total load on a section $3\frac{1}{4}$ in. $\times$ $3\frac{7}{8}$ in. would be about 12,000 lb. Two standard coarse-thread bolts of 1 to $1\frac{1}{8}$ in. nominal diameter would readily carry this load.

It will be found instructive to compute the arm section also from the point of view of the maximum speed and maximum energy delivery permitted for this flywheel, say 6000 ft. per min. with a maximum speed reduction of 10 per cent in 1/10 revolution.

In the computation of flywheels for punches, shears, and similar equipment, many arbitrary assumptions must be made, particularly if the machines are used over a wide range of service. Flywheels for prime movers and pumps and compressors have well defined torques to deliver, and the computation of the arms, for instance, need not be fraught with any uncertain estimates.

232. High-Speed Discs. The fact that a free rotating ring or drum cannot be run at a velocity in excess of about 500 ft. per sec., even if made of steel, presents a serious limitation in the design of single-stage steam or gas turbines, as well as of rotary blowers. It is easily possible to provide a nozzle which will discharge steam at a velocity of several thousand feet per second; but to utilize this kinetic energy efficiently, the turbine runner on which it impinges should have a speed of about one-half the steam velocity. At the present time such operating speeds have not been attained.

A great advance beyond the possibilities of the free ring was achieved by De Laval when he introduced his single-stage impulse steam turbine. The rim of the impeller disc was joined to a web of such shape that the stress induced from the centrifugal force was uniform and at the permissible maximum throughout. In the design of this disc, weight was reduced to the utmost, and in consequence, the centrifugal force was reduced to a minimum. With the disc pulling inwardly on the rim with a uniform stress, and thus balancing an equal amount of centrifugal force acting outwardly, speeds have been attained beyond the limitations of a free ring.

The contour of the section of a disc of uniform strength was originally determined by a purely mathematical derivation. It is of interest to note that this application represents one of comparatively few examples of an epoch-making mechanical-engineering invention based on a rational mathematical analysis. The derivation is as follows.

With reference to Fig. 273, consider in a disc of yet unknown shape, the forces acting on a certain element at a distance r from the center of rotation. The volume of this element is $xr\,d\phi\,dr$. If the mass per unit volume is m and the angular velocity ω, the centrifugal force on this element is $m\omega^2r^2x\,d\phi\,dr$.

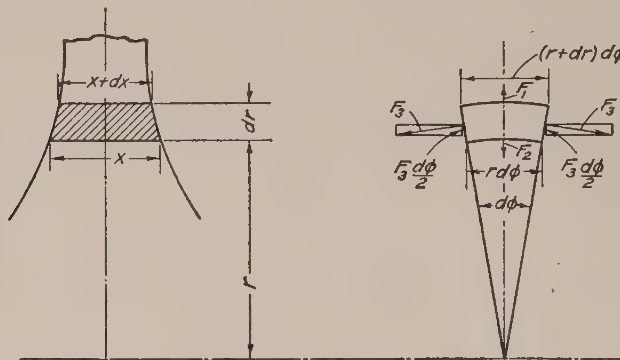


FIG. 273. FORCES ACTING ON AN ELEMENT OF A ROTATING DISC.

Suppose now that we neglect the stresses acting at right angles to the plane of rotation, and assume that a uniform tensile stress s exists throughout the element in all other directions. Then on the outer surface of the element, the adjoining part of the disc exerts an outward pull F_1 acting in the same direction as the centrifugal force and equal to $s(x + dx)(r + dr)d\phi$. On the inner surface, the adjoining disc exerts an inward pull F_2 acting toward the center of rotation and equal to $sxr\,d\phi$. In addition to these forces, there exists the hoop tension F_3 which is equal to $sx\,dr$. If we let $\sin d\phi = d\phi$ for small angles, the radial component of this force acting toward the center is $sx\,dr\,d\phi$.

Cancelling $d\phi$, we have at equilibrium

$$m\omega^2r^2x\,dr + s(x + dx)(r + dr) - sxr - sx\,dr = 0.$$

From this equation, neglecting products of differentials, we have

$$m\omega^2r\,dr = -s\frac{dx}{x},$$

or, integrating,

$$\frac{m\omega^2r^2}{2s} = -\log_e x + \text{constant}. \tag{5}$$

Now if this disc is joined to a rim at the radius r_2 (Fig. 274), the effect of the centrifugal force produces a certain stress on a unit length

of the rim. If the rim is not to be overstressed, the disc must exert an inward pull per unit length of such magnitude that the effect of the centrifugal force is partly balanced, thus leaving the resultant rim stress within the permissible limits. If the thickness of the disc where it joins the rim is x_2, the force F_2 pulling inwardly per unit rim length is $x_2 s$, or $x_2 = F_2/s$. Introducing the values r_2 and x_2 in equation (5), we

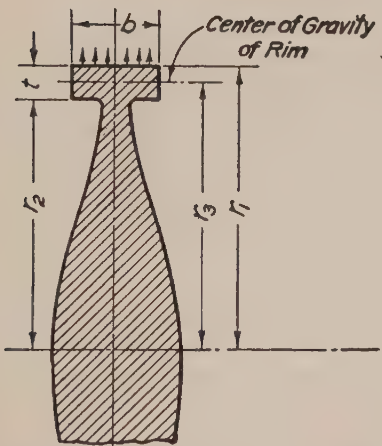


FIG. 274. CROSS SECTION OF HIGH-SPEED DISC.

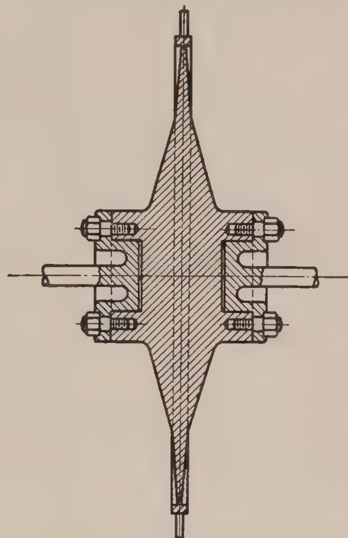


FIG. 275. UNPERFORATED HIGH-SPEED DISC WITH SHAFT CONNECTIONS.

can determine the value of the constant. Thus

$$\log_e \frac{x}{x_2} = \frac{m\omega^2}{2s}(r_2^2 - r^2)$$

or

$$x = x_2 e^{\frac{m\omega^2}{2s}(r_2^2 - r^2)}, \tag{6}$$

where x is the thickness of the disc at radius r.

An examination of formula (6) shows that the disc thickness is a maximum at the center of rotation where $r = 0$. Consequently, a disc of uniform strength cannot be perforated. To provide a means of attaching such a disc to a shaft, De Laval developed the construction shown in Fig. 275. Without the shaft, the disc has the shape shown in Fig. 274.

If it is desired to provide the wheel with a bore, then the disc must be reinforced at the bore with a substantial hub. Both the rim and

the hub must be so computed that their radial extension where they join the disc must be equal to that in the disc under the stress used. While the computation of the extension of a thin rim is not difficult, the formulas for a thick hub are rather complicated and will therefore be omitted.* The engineer who undertakes a difficult and advanced design in a special field must become acquainted with the knowledge and practice in that field. Our object here is simply to introduce the subject and the means available for the solution of various problems.

PROBLEMS

1. It is desired to determine the dimensions of two solid flywheels, built up of plate discs bolted together, which will have the same energy stored as in an automobile weighing 3200 lb. and traveling 80 m.p.h. The tire diameter is 28 in. and the flywheel outside diameter is to be 48 in. If the flywheel is driven from direct peripheral contact with the wheels, determine the necessary moment of inertia of the flywheel and its dimensions. What material is advisable, shock being present?

2. The flywheels above were redesigned to have walls made of solid 1 in. steel plate, 6 in. apart, bolted to a cast hub. The steel rim consisted of an annular ring between the plates and an annular ring 2 in. thick on the outside of each plate, all securely bolted together. If the added rim portion is 10 in. wide, what thickness of rim is necessary? The effect of the hub may be neglected.

3. A gas engine develops 45 hp. at 300 r.p.m. The maximum energy variation per revolution is 30 per cent of the mean energy and the total speed variation is to be 3/4 per cent. Determine the cross-sectional area of cast-iron rim necessary if the mean rim velocity is 4000 ft. per min.

4. The indicated horsepower of a steam engine is 90 at 240 r.p.m. If the maximum variation of energy per revolution is 10 per cent of the mean energy and the allowable speed fluctuation is 1/40, determine the mean radius and cross-section of the rim taking into account the effect of the hub and arms. What would be the approximate total weight of the flywheel?

5. A press is designed to punch a 3/4 in. hole in a 3/4 in. plate. A maximum force of 105,000 lb. and an energy of 55,000 in. lb. is required. The press is operated at 30 r.p.m. through gears having a ratio of 8 to 1. The punching operation takes place over 75 deg. of eccentric travel and the speed fluctuation is to be limited to 15 per cent. The overall mechanical efficiency is 80 per cent. (a) For a radius of 20 in., determine the necessary cross-section of the flywheel rim. (b) What is the total weight of the flywheel? (c) What horsepower motor must be supplied to the press with the flywheel? Without the flywheel?

6. A flywheel rotating at 120 r.p.m. has the following proportions: outside dia., 8 ft., rim section, 12 in. deep by 10 in. wide, bore dia., 6 in., 6 elliptical section arms, 7 in. by 3½ in. (a) Determine the hoop stress in the rim neglecting the restraining effect of the arms. (b) What would be the bending stress in the rim if the arms were rigid? (c) Fix upon the diameter and the length of the hub and the size of key required. (d) If the flywheel is made in two parts and connected by two rectangular links at each joint of the rim what cross-sectional area of the links is necessary to limit the induced stress in the links to 3000 p.s.i.? (e) The

* See Stodola, *Steam and Gas Turbines*, translated by Loewenstein and published by McGraw-Hill Co.

links are shrunk in place to develop a stress of 35,000 p.s.i., close to the yield point. If the machined length of the slots in the flywheel is 8 in. what should be the machined length of the links, between the heads, if the deflection of the rim is neglected?

7. A flywheel for a punch press has a mean diameter of 36 in. It must be capable of supplying 2000 ft. lb. of energy during the $\frac{1}{4}$ revolution while the hole is being punched. The normal speed of the wheel is 200 r.p.m. and is decreased 10 per cent during the cutting stroke. (a) Determine the weight of the flywheel rim. (b) Determine the dimensions of the arms at the center of the hub for an allowable bending stress of 2000 p.s.i. There are six arms of elliptical cross-section with a major axis twice the minor axis.

8. A flywheel having a mean rim diameter of 5 ft. rotates at 300 r.p.m. The cross-section of the rim is 8 in. by 8 in. While delivering energy over 1/4 revolution the flywheel speed drops 10 per cent. There are six arms of elliptical cross-section with a major axis twice the minor axis. (a) Determine the dimensions of the arms at the center of the hub for an allowable stress of 3000 p.s.i. (b) Neglecting the restraining effect of the arms determine the hoop stress in the rim caused by centrifugal force. (c) If the rim is considered as restrained by rigid arms, determine the bending stress induced in the rim by centrifugal force.

9. A high-speed disc of uniform strength has an outside diameter of 3 ft. and rotates at 4000 r.p.m. (a) If the thickness at the periphery is 1/2 in., what is the thickness at the center? at 1/2 ft. radius? at 1 ft. radius? (b) What would be the thickness at the center at 8000 r.p.m.? Allowable stress is 25,000 p.s.i.

CHAPTER 15

BELTS AND PULLEYS

233. Introduction. Belts and ropes are flexible connectors transmitting force by friction only. Schematically, a belt or rope drive is shown in Fig. 276. Each side of the belt is in tension; the tension T_1 tends to turn the driven pulley counter-clockwise, the tension T_2 to turn it clockwise. If T_1 is greater than T_2, the resulting turning moment is $T_1R - T_2R = (T_1 - T_2)R$, where R is the radius of the pulley. T_1 is called the *tight-side* tension, T_2 the *slack-side* tension, and $T_1 - T_2$ the *effective pull.* If the belt speed is V in ft. per min., the horsepower transmitted is $(T_1 - T_2)V/33{,}000$. For drives with the belt approximately horizontal, the slack side of the belt should be on the top of the pulleys, as this arrangement increases the *arc of contact* beyond the theoretical amount obtained with no slack (Fig. 276).

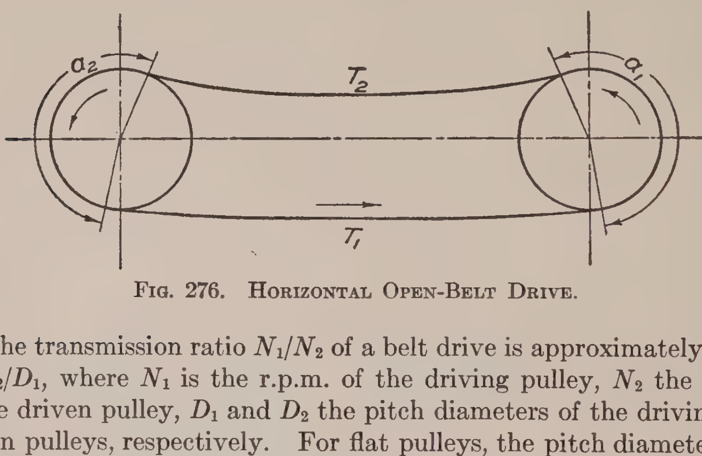


FIG. 276. HORIZONTAL OPEN-BELT DRIVE.

The transmission ratio N_1/N_2 of a belt drive is approximately equal to D_2/D_1, where N_1 is the r.p.m. of the driving pulley, N_2 the r.p.m. of the driven pulley, D_1 and D_2 the pitch diameters of the driving and driven pulleys, respectively. For flat pulleys, the pitch diameter theoretically extends to the neutral fiber in the belt, so that for a pulley diameter of D, and a belt thickness t, the pitch diameter is $D + t$. Pulleys are, however, often *crowned* (see page 305, Fig. 289b); in this case, it is not possible to determine accurately at what point on the sloping sides the diameters should be measured, nor is the belt thickness or the location of the neutral fiber in the belt section accurately known. With grooved pulleys the belt sinks to a somewhat uncertain depth into the groove, and again the location of the neutral fiber in the belt is but roughly determinable. In the case of grooved

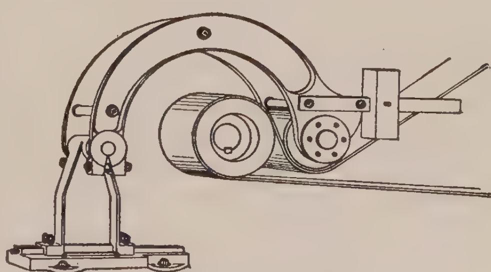
pulleys, the diameters as given in manufacturers' catalogs are sufficiently accurate for computing transmission ratios. In the case of flat pulleys, the actual pulley diameters are generally used.

234. Creep and Slip. When power is being delivered by a belt the transmission ratio does not hold exactly, because an unavoidable speed loss varying from less than 1 per cent to 4 per cent occurs, due to the *creep* and *slip* of the belt.

Creep is always present in belt drives and results from the elasticity of the material. As the belt passes over the driven pulley it stretches from the length at tension T_2 to the greater length at the higher tension T_1. Thus a greater length of belt runs on to the driving pulley than runs off and this can occur only if the belt creeps backward over the surface of the driving pulley. Up to certain limiting loads, experiments show that no slip occurs between the belt and pulley at the running-on point. At the rated load, creep may be about 1 per cent.

Beyond a limiting pull, *slip* begins at the running-on points. This condition results in many belts overheating and slipping off the pulley, particularly belts having a dry surface, as have most textile belts. In the case of belts containing a greasy or oily filler, such as do leather belts, a reasonable amount of slip will cause some of this filler to melt out over the pulley surface; the intimacy of contact is intensified, a sort of semi-viscous friction or drag is produced, and the transmissive power is increased. A slip of 2 or 3 per cent may then be permissible.

235. Belt Drives. On horizontal or slightly inclined drives having considerable distance between shaft centers, the weight of the belt hanging between the pulleys maintains the tensions, even if the belt is quite slack. For dependable and elastic drives of this kind, a considerable center distance is desirable, preferably 15 to 25 feet. For steeply inclined drives and for short center distances, the tension is not satisfactorily maintained by belt weight alone. In such cases, to maintain the tension needed for friction, a swinging and weighted idler is often used, as shown in the Lenix drive (Fig. 277). Recently, pivoted motor suspensions have been introduced (Fig. 278) in which the weight of the motor itself produces the tension in the belt.



F. L. Smidth & Co., New York, N. Y.
FIG. 277. LENIX DRIVE.

Manila or cotton ropes running on grooved pulleys or sheaves were commonly used in the past to transmit up to several thousand horsepower. A long center distance was regularly provided to maintain the tension automatically, unless a special tensioning device was used. Now, however, electric power transmission has replaced rope drives almost entirely, and the increasing use of individual motor drives on machine tools and other production machinery has led many engineers to believe that belt driving was doomed to disappear. This supposition is not true, however, and the belt remains an extremely simple and efficient form of power transmission. In many cases it is desirable because of its ability to slip in case of overload, thus providing a natural safety device against breakdowns of more expensive and vital machine parts.

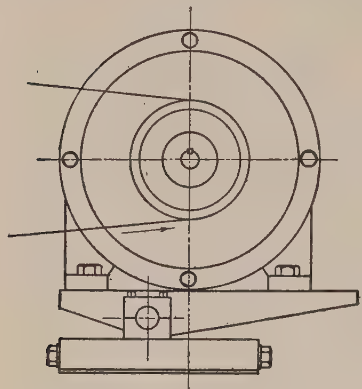


Fig. 278. Pivoted Motor Suspension.

Moreover, in view of the fact that electric motors are essentially high-speed devices, it is desirable to have a cheap, simple, and efficient means of reducing the motor speed to the much lower speeds commonly required in production machines. The belt is an excellent device for this purpose, if the need for a long center distance or a special tensioning device is eliminated. With the ordinary flat belt, the swinging motor mounting (Fig. 278) is a solution; perhaps even a simpler solution is the rubberized V-belt running on grooved sheaves. The transmissive power of a flat belt, which is dependent on surface conditions, may be reduced by wear, whereas the transmissive power of the V-belt is largely dependent on the wedging action in the grooves. This characteristic of the V-belt is a simple mechanical advantage which, as investigations show, leaves the transmissive power of the belt

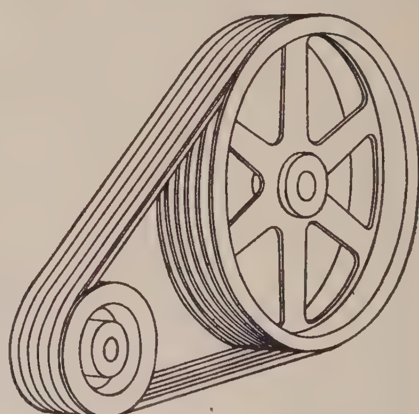


Fig. 279. Multiple V-Belt Drive.

unchanged even after years of service. With this wedging action, a rubberized fabric surface acquires as high a transmissive power as that of an excellent oil or grease filled leather belt. Because of the wedging action, V-belts may be run quite slack and hence require but little care and adjustment.

V-belts can be run singly or in multiple (Fig. 279). A great many such drives have been installed in the last ten or fifteen years, not only for speed reduction, but also for speed increase.

236. Types of Flat Belts. The leather belt is the oldest type of flat belt and at present generally consists of a single or double thickness of hide. In the past, triple thicknesses were at times installed on large drives.

In this country, single belts are run with the smooth, or hair, side against the pulley. Double belts are made with the flesh sides glued together. The *oak-tanned* belt has been the standard for many years, but excellent *mineral-tanned* belts have been developed in the last few decades. An oak-tanned belt must be run for some time before it acquires its full transmissive power, and should be dressed occasionally with an oil or grease preparation to retain its flexibility and adhesion. Mineral-tanned belts are naturally flexible and are so treated that they have their full transmissive power at the outset. They also, however, require dressing to maintain their properties. Leather is an expensive material and the best quality belts are made from the narrow portion of the hide adjacent to the backbone. Leather belts must be scarfed and joined together from pieces only 4 or 5 ft. long, which is not a most desirable method of manufacture.

Woven or twisted textile belts are cheaper and easier to produce in long lengths than leather belts. The most commonly used textile belt today is the *rubber belt*, built up of layers of cotton fabric held together by intervening layers of vulcanized rubber. Such belts do not adhere to the pulley as well as leather belts, but they have the advantage of being water proof and cheaper. They are much used where operating conditions are unfavorable for leather belts. At very low temperatures *balata* gum is a better material than rubber for cementing the layers of fabric together, but the gum gets sticky at temperatures not much over 100° Fahr.

Impregnated or unimpregnated cotton belts stitched together are cheap, but the stitching interferes with the contact on the pulley and may wear through. This type may be desirable, however, for small belts performing intermittent or temporary service under conditions which would soon wear out any kind of a belt. It is possible to weave

a fabric to the full thickness of single and double leather belts, and some very high-grade impregnated *solid woven* fabric belts are manufactured. Belts of this type are suitable for severe service, and tests * have shown excellent adhesion to the pulley, at least when they are new.

Belts made of band steel similar to that used in band saws, with the ends brazed to special fasteners, have been used fairly extensively abroad and transmit large amounts of power for the section used. Steel belts require pulleys covered with canvas, cork, or rubber, as well as very accurate shaft alignment. They have not gained acceptance in this country.

237. V-Belts. The tensile strength of V-belts is secured by the number and size of the cotton cords used in their structure. To secure lateral stiffness in the groove, the cords are embedded in rubber and fabric as shown in Fig. 280. This construction gives considerable bulk to the belt for its cord strength, and reduces its flexibility. For high-speed drives, it is desirable to have a very light belt, which is less affected by centrifugal force, and flexes readily over small pulleys. Flat rubberized cord belts have been produced for these conditions; such a belt is shown in section in Fig. 281. These belts are very strong and light, but adhesion to the pulley is usually not in proportion to their strength. They must be put on very tight in order to prevent slipping, and this condition somewhat limits their usefulness.

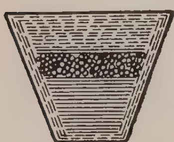


FIG. 280. SECTION OF RUBBERIZED V-BELT.

FIG. 281. SECTION OF RUBBERIZED CORD BELT.

V-belts made of fabric instead of cords were introduced in Germany and are now used, experimentally at least, for certain types of drives in this country. Railway lighting-set drives are the leading example. Belts of this type may be joined at the ends by means of a modified type of hinged joint.

238. Belt Fasteners. V-belts are usually manufactured endless, and flat cord belts regularly so. A satisfactory fastener has been developed for V-belts made of vulcanized layers of fabric. Leather belts may be glued or cemented together (Fig. 282a), laced as in Fig. 282b, or joined by belt hooks (Fig. 282c), or by plate fasteners as in Fig. 282d. Rubber belts may be joined together by overlapping the layers of the splice and vulcanizing them together. The use of the

* Tests conducted by C. A. Norman and G. N. Moffat. See Bulletin No. 41, Engineering Expt. Sta., The Ohio State University, Columbus, Ohio.

belt hook is steadily increasing for transmission belts both of leather and fabric. The joint in this case is formed by a hinge pin of rawhide, metal, or bakelite.

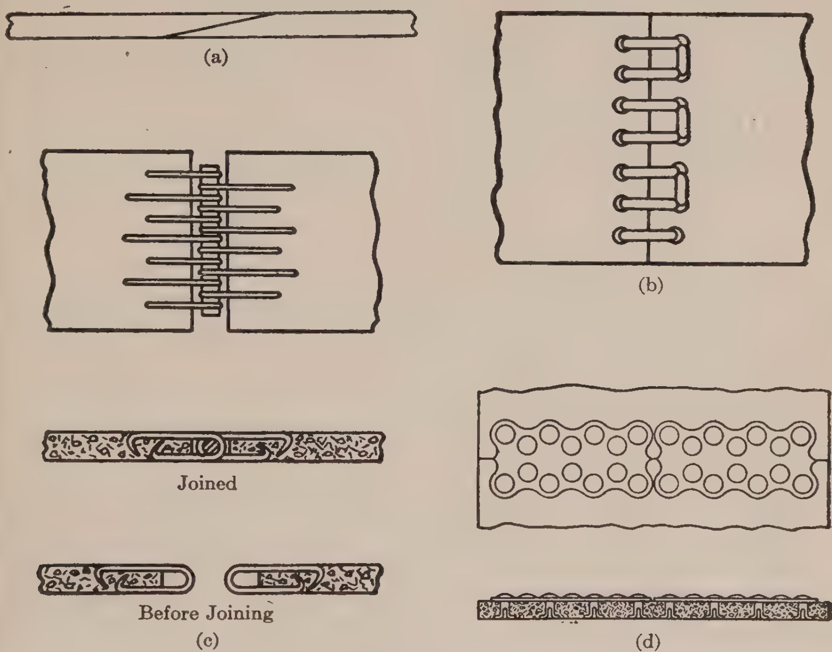


FIG. 282. TYPES OF BELT JOINTS.
(a) SPLICED. (b) LACED.
(c) WIRE HOOKS. (d) PLATE FASTENERS.

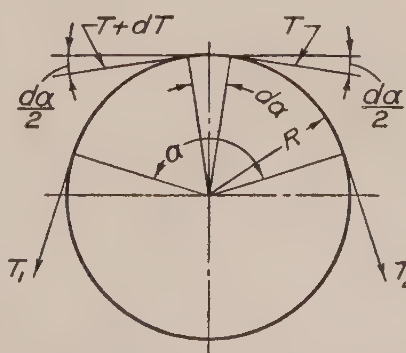


FIG. 283. FORCES ACTING ON FLAT BELT.

239. Tension Ratio and Centrifugal Force. If we assume that the slipping of the belt over the pulley face is prevented by ordinary solid friction, the relation between tension, contact angle, and centrifugal force may be developed as follows.

With reference to Fig. 283, assume $d\alpha$ to be a small angle through which the tension grows from T to $T + dT$. The tension difference dT tends to slip the belt over the pulley. The tensions T and $T + dT$, being practically equal, both have components $T \sin d\alpha/2$ pressing

the belt against the pulley. Setting the small angle equal to the sine, the sum of these components is $T d\alpha$. If the weight * of the belt per ft. of length is w and the belt speed in ft. per sec. is v, the centrifugal force of the belt element of length $R\,d\alpha$ is equal to $wv^2R\,d\alpha/Rg = wv^2\,d\alpha/g$, where R is the pitch radius of the belt in ft. and g the acceleration of gravity. The net force pressing the belt against the pulley is then $(T - wv^2/g)d\alpha$. If the coefficient of friction is f, the force which prevents the belt from slipping is $f(T - wv^2/g)d\alpha$.

If slip is to be prevented, we must have $dT \leqq f(T - wv^2/g)d\alpha$, or

$$\frac{dT}{T - wv^2/g} \leqq f d\alpha.$$

Since $d(T - wv^2/g)$ is equal to dT, integrating between the limits T_1 and T_2, the tight and slack-side tensions, respectively, and 0 and α, the angle of contact in radians, we find

$$\text{(1)} \qquad \log_e \frac{T_1 - wv^2/g}{T_2 - wv^2/g} = f\alpha, \qquad \text{or} \qquad \frac{T_1 - wv^2/g}{T_2 - wv^2/g} = e^{f\alpha}.$$

With a given friction coefficient and a given contact angle, the right side of equation (1) is a constant. Hence, if $T_2 - wv^2/g$ becomes zero, $T_1 - wv^2/g$ will become zero also. According to this formula, if the centrifugal tension equals the slack-side tension, the drive can no longer transmit force. For a complete understanding of the situation it should be noted, however, that the tension wv^2/g is dependent only on belt weight per unit length and belt speed, and not on the radius to which the belt is bent.† Hence, this tension is set up also in the arc through which the belt passes between the pulleys. This increase in tension exactly balances the reduction in the force by which the belt is pressed against the pulley, so that if the belt did not stretch, centrifugal force would have no effect whatever on transmissive power or pull on the shaft. Owing to the fact that the belt stretches, there *is* a reduction in shaft pull and pressure on the pulley, but the equation for this reduction is complicated and cannot be given here. It should be stated, however, that the correction for centrifugal force as given by formula (1) is on the safe side and, in many cases, not much in error.

* The weights per cu. in. of leather and textile-rubber belts may be taken at 0.035 lb. and 0.045 lb., respectively. Haven and Swett, in their *Treatise on Leather Belting*, classify leather belts as follows: (Thicknesses are approximate) Light Single—1/8 in., Medium Single—5/32 in., Heavy Single—3/16 in., Light Double—1/4 in., Medium Double—5/16 in., Heavy Double—3/8 in., Medium Three Ply—1/2 in., Heavy Three Ply—9/16 in.

† The complete analysis is set forth mathematically and experimentally in a bulletin on high-speed belt drives published by C. A. Norman as Engineering Experiment Station Bulletin No. 83 of The Ohio State University.

These conditions have been known to belt engineers in a general way for some time. Thus belt and rope drives *have* been run at speeds at which the centrifugal force should very nearly cancel the slack-side tension. Automobile fan belts are required to transmit their maximum torque at maximum speed and are often run so slack that they would not be able to do this, if centrifugal force were as serious as formula (1) would indicate. It is the opinion of some engineers, as the result of direct test measurements, that the actual effect of centrifugal force may be only a certain percentage of the theoretical force. It is only recently, at least in this country, that the quantitative relations have been properly analyzed and set forth in mathematical form.

240. Tension Ratios. The tension ratio T_1/T_2 (corrected or uncorrected for centrifugal force as the speed may require) is an important indicator of the transmissive capacity of the belt. It is a more practical and a more rational indicator than the coefficient of friction, which can be computed only from formula (1), a formula that does not really apply if the friction, as in leather belts, is of semi-viscous character. The tension T_2 exerts a turning moment in the wrong direction and is necessary only to obtain normal pressure between belt and pulley. If it is high, the pull on the shaft and the loads on bearings will be high, and consequently tension T_1 must be high if it is to produce the required effective pull. This high tension has a very serious effect on the life of the belt, which, according to tests by the Goodyear Tire and Rubber Company, and one of the authors,* decreases with the fourth to the sixth power of the tension. Moreover, a high tension causes stretch and necessitates frequent tightening of the belt. Everything considered, the most economical and dependable drive is obtained if the slack tension can be kept very low and hence the ratio T_1/T_2 high.

For most textile belts on steel or cast iron pulleys, slip begins at tension ratios † of from 3 to 5; for cord belts at rated load perhaps at 2. Well run-in oak-tanned leather belts on steel or cast-iron pulleys may permit tension ratios of 7 to 10, but, unfortunately, new belts have tension ratios as low as 2. New high-grade mineral-tanned belts may have ratios up to 20 and 30, but in the course of a few years,

* See the *Handbook on Belting* by the Goodyear Tire and Rubber Company for flat belts, and articles by C. A. Norman in Product Engineering of June, 1932, p. 248, or American Machinist of May 12, 1932, p. 605, for V-belts.

† On the basis of these ratios obtained by experiment, the coefficients of friction would vary from about 0.25 to values close to 1.0. A safe value for ordinary conditions would be 0.3 for both new oak-tanned leather belts and rubber belts on either cast iron or steel pulleys.

unless the belt is treated with impregnations, the value may drop below 10. V-belts maintain the tension ratio much better. For steadiness of running and dependable service, it may be well to put these belts on with tension ratios no higher than 5 or 7, but tension ratios may be carried as high as 14 or 15, even for old belts, without slip occurring. (The preceding statements refer to contact angles of 180 deg.)

241. Effect of Contact Angle. The tension ratio $(T_1/T_2)_1$ at a contact angle α_1 may be obtained from the tension ratio $(T_1/T_2)_2$ at an angle α_2 by the formula

$$\text{(2)} \qquad \log\,(T_1/T_2)_1 = \frac{\alpha_1}{\alpha_2}\log\,(T_1/T_2)_2,$$

which is easily derived from formula (1). Experiments by Norman indicate that this formula gives somewhat lower values than may actually be obtained in practice.

The torque and horsepower transmissible are proportional to $T_1 - T_2$. The fraction of the maximum tension converted into effective pull is

$$\text{(3)} \qquad \frac{T_1 - T_2}{T_1} = \frac{e^{f\alpha} - 1}{e^{f\alpha}}.$$

If the tension ratio in one case is a and in another b, and if the maximum tension remains unchanged, then the ratio of the powers transmitted is

$$\text{(4)} \qquad \frac{T_1 - T_{2b}}{T_1 - T_{2a}} = \frac{(b - 1)a}{(a - 1)b}.$$

In laying out new belt drives it is always necessary to assume the most unfavorable conditions that may occur. Consequently, shafts and bearings for flat belt drives may have to be computed for tension ratios as low as 2, as required by new leather belts and cord belts, although it is not expected that such low tension ratios will prevail in regular operation.

242. Transmissive Capacity of Belts. In view of the fact that continuous progress is being made in belt design and manufacture, it is well to consult manufacturers' catalogs for transmissive capacity, especially in the case of new developments like rubberized V-belts and flat cord belts.

For first estimates with speeds of 3000 ft. per min. and less, and with pulleys of sufficient size, the effective pull per in. of width for ordinary leather belts may be taken equal to 40 lb. for single belts

and 60 lb. for double belts, and for belts of highest quality, 50 lb. for single belts and 80 lb. for double belts.

For ordinary rubber and other textile belts made up of plies of fabric, the effective pull may be taken at 10 lb. per in. of width per ply and for high-grade belts at 15 lb. If the pulley diameter is small, the bending stress in the belt is high and the effective pull must be reduced. For four-ply rubber belts, the figures given may apply on 10 in. pulleys, but may be only half as high on 4 in. pulleys. An old rule used for computing rubber belts to operate at maximum effective pull was to allow 4 in. of pulley diameter for every ply of belt thickness. At the present time, 3 in. might be considered sufficient. Sometimes, however, as little as 1 in. per ply is allowed. The power transmitted should then be considerably reduced.

In Table 39, the recommendations of a prominent German belt firm are given relative to the effective pulls suitable for leather belts operating at various speeds and with different pulley diameters. The reason why the effective pull increases with the speed, within certain limits, is no doubt the fact that the adhesive power of leather belts increases with some slip. At low belt speeds, a slip of sufficient magnitude to produce a noticeable effect might constitute an excessive percentage of speed loss, while at high speed it would not be objectionable.

Incidentally, this table shows that the centrifugal force is not so serious as it has commonly been believed. It has often been said that centrifugal force reduces the tensions so seriously at speeds over 3000 ft. per min., that at 4000 ft. there is no further gain in horsepower from increased speed. According to the table, the effective pull is well maintained at speeds much higher than this, particularly for large pulleys.

For rubber belting, the *Handbook of Belting* of the Goodyear Tire and Rubber Company contains some valuable horsepower tables. In it ratings are given for belt speeds of 10,000 ft. per min. for flat cord belts, and on large pulleys the horsepower increases with the speed, up to 7500 ft. per min. For fabric belts, the ratings are carried to a maximum speed of 4000 ft. per min. for some belts, and to 6000 ft. per min. for others, dependent, no doubt, on the relation of the weight of duck to the belt strength. For comparatively large pulleys, the horsepower increases up to the maximum speed, but for small pulleys, the increase ceases at speeds varying from 3500 to 5000 ft. per min.

The Goodyear Company computes the horsepower by the following formula:

$$hp = \frac{bnv(T_1' - w'v^2/g)}{550}\left(1 - \frac{1}{e^{f\alpha}}\right), \tag{5}$$

TABLE 39

EFFECTIVE PULL PER INCH WIDTH FOR HIGH-GRADE, SINGLE AND DOUBLE LEATHER BELTS

SPEED, FEET PER MINUTE *

Diameter of Pulley, Inches	500		1000		2000		3000		4000		5000		6000		7000		8000		9000		10,000	
	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.
4	10		14		17		18		19		20		20		20		19		18		16	
8	16		23		28		31		33		35		36		37		37		37		36	
12	22	26	28	34	34	39	38	45	42	50	45	53	48	56	49	55	50	56	51	56	50	56
16	27	33	33	45	39	50	45	56	50	61	53	65	56	67	58	69	60	70	61	70	61	70
20	33	42	39	53	45	62	51	68	56	73	59	75	61	76	63	77	65	78	66	79	66	79
24	38	50	44	61	50	70	56	77	62	84	67	87	70	90	72	92	74	94	75	94	75	95
30	43	60	50	70	56	81	62	90	67	98	71	102	73	105	75	107	76	109	77	111	77	112
40	49	70	56	84	61	96	68	107	73	117	76	122	78	125	80	128	82	130	83	132	83	133
59	55	80	61	95	67	107	72	118	76	129	79	139	81	145	83	149	84	152	85	154	86	156
79	60	90	67	106	73	118	76	130	79	140	82	150	84	156	86	160	87	163	89	166	90	168

s. = single. d. = double.

* After recommendations of G. Otto Gehrckens, Hamburg.

where b is the belt width in in., n the number of plies, v the velocity in ft. per sec., T_1' the tight side tension in lb. per in. width and ply, w' the weight of belt in lb. per in. width and ply per ft. of length, and g the acceleration of gravity. This formula is based on the consideration that the pressure on the pulley is relieved by the full amount of the centrifugal force.

The effective pull for the standard sizes of rubberized V-belts now on the market may be found from Table 40, which is based on horsepower data for the Allis Chalmers so-called Texrope drives.

TABLE 40

EFFECTIVE PULL ON STANDARD TEXROPES (LB.)

SPEED, FT./MIN.	SIZE IN IN. (WIDTH × THICKNESS)				
	1/2 × 11/32	21/32 × 7/16	7/8 × 5/8	1¼ × 3/4	1½ × 1
1000	30	40	100	184	250
2000	28	38	91	165	230
3000	26.5	35	82	160	218
4000	23.0	34	74	144	193
5000	18.4	27.5	59	116	155

RECOMMENDED PULLEY DIAMETERS (IN.) FOR ABOVE BELTS

Lowest recommended	4	6	9	13	22
Absolute minimum	3½	5¼	8½	12	20

243. Effect of Tension and Pulley Size on Service Life. The serious effect of high tension and small pulley diameter on belt life is very well shown by the following formula given in the *Handbook of Belting* of the Goodyear Tire and Rubber Company:

$$\text{(6)} \qquad \text{Length of service in flexing} = \frac{\text{constant} \times D^{5.35}}{V^{0.5} \times n^{6.27} \times T^{4.12}},$$

where D is the pulley diameter in in., n the number of plies, V the belt speed in ft. per min., and T the tension in lb. per ply. In accelerated life tests * on rubberized V-belts, one of the authors found that the life decreases with the 5.85 power of the pull and inversely with a power of the pulley diameter at least equal to 5.05. In other words, a 50 per cent overload will reduce the belt life at least to one-fifth, and reducing the pulley diameter to two-thirds will have an even more serious effect. Naturally this statement is of importance particularly for small diameters, where a change in diameter of an inch or two means a large percentage change.

* See, for instance, article by C. A. Norman in Product Engineering, June, 1932, p. 248.

The advantages of light belt loads were first emphasized by F. W. Taylor.* He recorded, for about nine years, the performance of countershaft and machine belts on the same machine tool drives and found that it was good economy to reduce the effective pull on double leather belts from the commonly used 60 to only 35 lb. per in. of width. This reduction increased the life of the belts from 6 or 7 years to perhaps 20 or 30 years, and the time between take-ups was very much lengthened.

244. Formula for the Influence of Wedge Grooves. In the preceding discussion, the extent to which wedge grooves may be expected to improve the tension ratio was briefly explained. For hoisting purposes and belt applications, an analysis of the effect of the wedge action is of importance.

Referring to Fig. 284, assume that F_n is the normal force acting between the rope, or belt, and the side of the groove. Neglecting friction, the component $F_n \sin(\theta/2)$ of this force will prevent the rope from sinking deeper into the groove, if θ is the wedge angle. If the force pulling the rope into the groove is F, we have at equilibrium $F = 2F_n \sin(\theta/2)$, since there are two components of F_n of the same magnitude. Thus $2F_n = F/\sin(\theta/2)$. The friction force preventing slip is $2fF_n = fF/\sin(\theta/2)$, where f is the coefficient of friction.

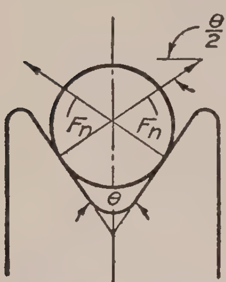


FIG. 284. FORCE ANALYSIS OF ROPE, OR BELT IN WEDGE GROOVE.

Now if the pulley is flat, the angle $\theta/2$ is 90 deg., and the force preventing slip is simply fF. We notice then that the wedge action has produced the same effect as if the coefficient of friction were increased from f to $f/\sin(\theta/2)$. Neglecting centrifugal force, the formula for the tension ratio of ropes or belts in wedge grooves then becomes

$$\frac{T_1}{T_2} = e^{f\alpha/\sin(\theta/2)}. \tag{7}$$

The criticism may be made that the derivation of formula (7) does not take into account the fact that friction also prevents the belt from sinking into the groove. The effect of this force would reduce the amount of side pressure F_n required, and consequently would also reduce the available frictional transmissive force $2fF_n$. Experience, however, seems to justify the use of the formula as de-

* For the report on Taylor's tests, see Transactions A.S.M.E., 1893–4, vol. 15, p. 204.

rived, since it is in general use for transmission belts as well as elevator cables.

245. Pulleys and Their Characteristics. Cast iron is extensively used for pulleys (Fig. 285), and has the advantage of cheapness and

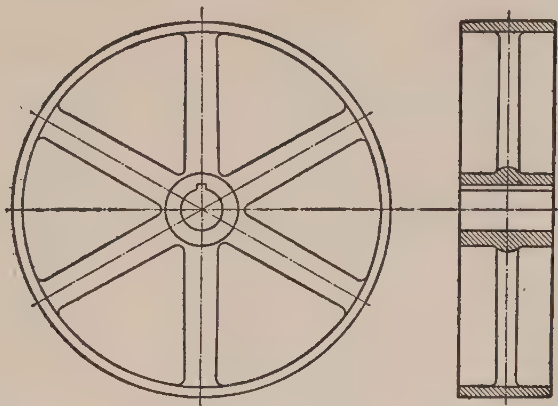
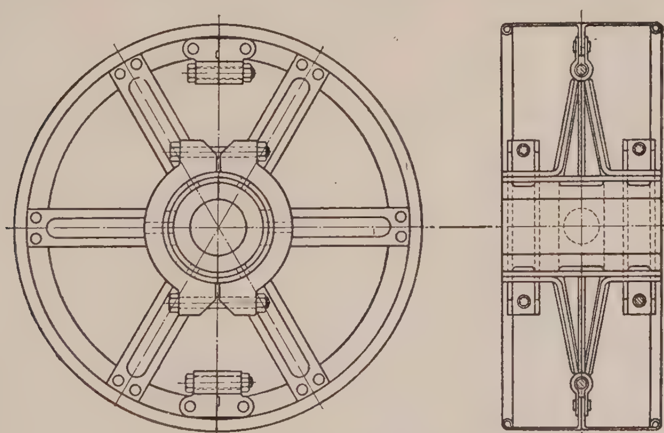


FIG. 285. CAST-IRON PULLEY.

good transmissive properties with leather belts. It has the disadvantage of being heavy and brittle, and does not give very good adhesion

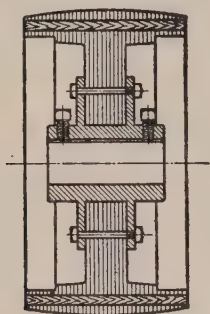


American Pulley Co., Philadelphia, Pa.

FIG. 286. PRESSED STEEL PULLEY.

with rubber belts. Pressed steel pulleys (Fig. 286) are lighter and stronger and show about the same transmissive characteristics as cast-iron pulleys.

Wood has a greater strength per unit of weight than cast iron; consequently, wood pulleys may be considered in place of cast-iron pulleys for speeds at which rupture might occur from centrifugal force. Wood pulleys show good friction qualities at low speeds, where the creep velocity is small, but the adhesion is not so good at speeds where slip velocities become considerable. Wood also is a poor conductor of heat, and the heat developed from slip and friction may result in overheating and injury to the belt. Finally, wood pulleys will split and warp. Considering all factors, wood pulleys are not so extensively used for modern high-speed drives as they were in the past when drives were operated at lower speeds.



Rockwood Mfg. Co., Indianapolis, Ind.

Fig. 287. Paper Pulley.

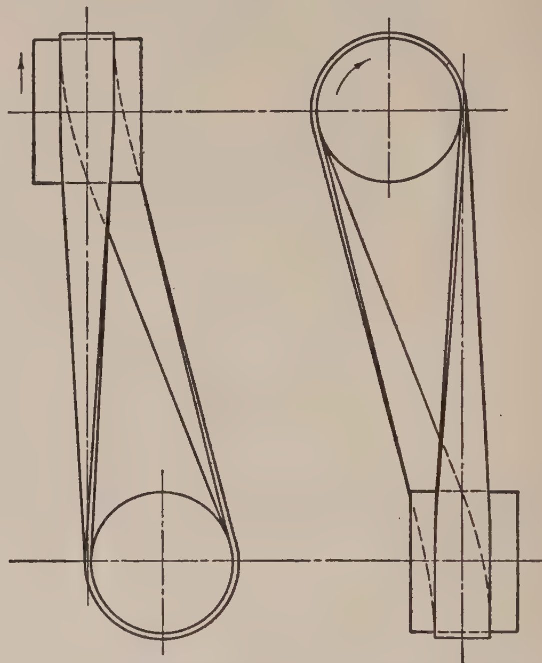


Fig. 288. Non-Reversible Quarter-Turn Drive.

Pulleys of pressed paper (Fig. 287) improve the adhesive power considerably for both leather and rubber belts, and may be of value in raising the tension ratio for rubber belts. They have found considerable application, for instance, as motor pulleys. A paper pulley may glaze over after a period of time and the only means of restoring its frictional properties is to turn down the pulley diameter about an eighth of an inch. In such cases, it should be ascertained whether the belt had been unduly slack or unduly overloaded, because either condition would result in excessive slipping and the formation of a glazed surface.

246. Relative Position of Belt and Pulley. In order that a belt may stay on a pulley, the center line of the belt, as it runs toward the pulley, must be in the central plane of that pulley (Fig. 288). Even so, if the alignment is not perfect, the belt may have a tendency to run off a flat-faced pulley. It is customary to have the pulley about one-half inch wider than the belt, although Carl G. Barth favors a greater allowance.

247. Flanged and Crowned Pulleys. Narrow belts may be run on *flanged* pulleys (Fig. 289a) but the rubbing action of the flanges has a

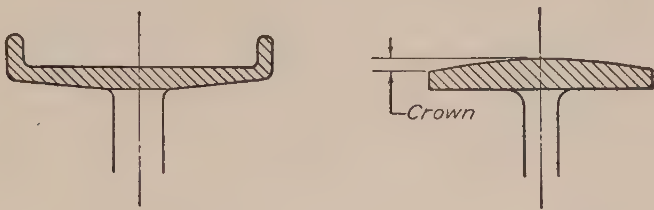


FIG. 289. (a) FLANGED PULLEY. (b) CROWNED PULLEY.

tendency to fray the edges of the belt. For belts 3 in. wide or wider, *crowning* (Fig. 289b) is an effective way to keep the belt on the pulley. The crown causes the belt to ride towards the ridge, since on a conical pulley the lateral stiffness tends to make a running-on belt travel toward the larger diameter (Fig. 290).

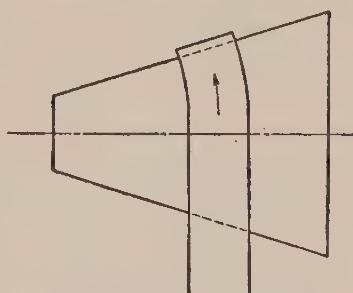


FIG. 290. ACTION OF BELT RUNNING ON A CONICAL PULLEY.

Crowning is by no means universal. It cannot be applied, for instance, to quarter-turn belts, or to belts to be shifted over the pulley face. When the drive permits the use of crowned pulleys, it is advisable to crown the smaller pulley. The height of the crown may be made about 1/8 in. per ft. of pulley width, but a smaller allowance is permissible on wide pulleys.

248. Angular Belt Drives. Quarter-turn drives are not reversible, since the belt can run on in the central plane of the pulley in one direction of running only. If the drive must be reversible, an idler pulley is employed to guide the belt. Such pulleys are usually mounted on adjustable stands and can be arranged to guide the belt onto pulleys in any relative position. Complicated drives should

always be avoided if possible, but there may be cases where drives like those shown in Figs. 288 and 291 are found convenient.

249. General Remarks on Pulley Applications. Pulleys for all ordinary applications may be purchased as standard commercial items

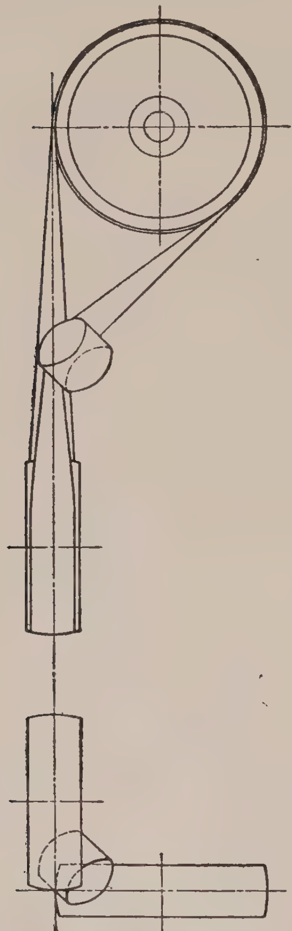


FIG. 291. REVERSIBLE QUARTER-TURN DRIVE WITH GUIDE PULLEY.

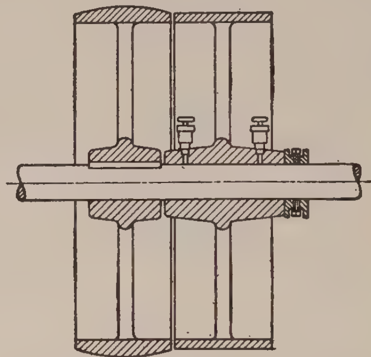


FIG. 292. TIGHT AND LOOSE PULLEYS.

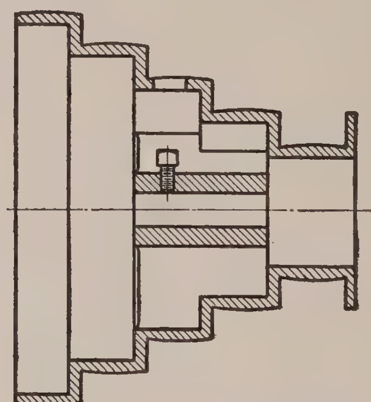


FIG. 293. STEP CONE PULLEY.

and are regularly carried in stock. Tight and loose pulleys are frequently employed on machines which are operated intermittently. A disadvantage is that in order to start the machine the belt must be running before it can be shifted. The oiling of an idler, or loose pulley, is not very convenient. A simple grease cup is often used, but it is

not dependable (Fig. 292). A ball or roller bearing application (Fig. 257) is a much better construction, but is also more expensive. Friction-clutch pulleys on the driving shaft obviate the necessity of having the belt running before the machine is started and are preferable to tight and loose pulleys. Friction clutches of this type will be dealt with in the chapter on clutches.

Cone pulleys and step pulleys (Fig. 293) are speed-changing devices which are being replaced by gearing, variable-speed belt drives, and variable-speed motors. Yet, for a perfectly continuous, stepless speed change, the cone pulley was long considered the best available mechanical device, at least when powers of some magnitude were involved. Nevertheless, cone-pulley operation is unsatisfactory, since the belt tends to ride toward the larger diameter and must be held in position by guide rollers. Rollers should also be applied to belt shifters (Fig. 294) to reduce fraying of the belt edges. The shifter should have a parallel movement, as shown, and the shifting arm should play freely between pins, so that there is no tendency for the arm to shift the belt by its own weight.

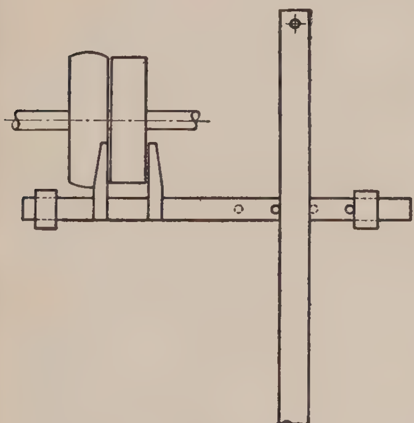
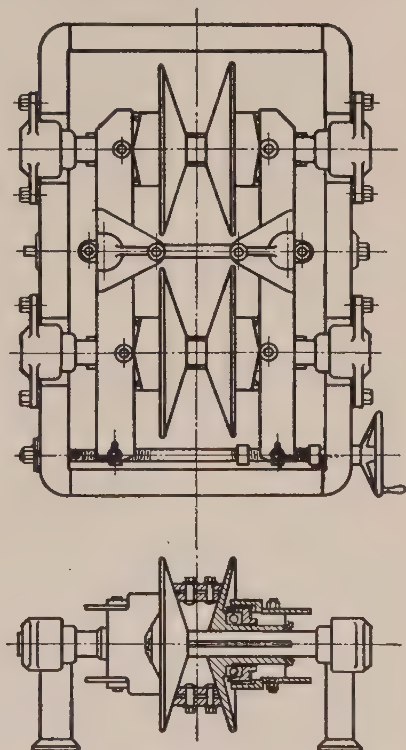


FIG. 294. BELT SHIFTER.

Note: Stops are shifted from position shown in full lines to the position shown in dotted lines.

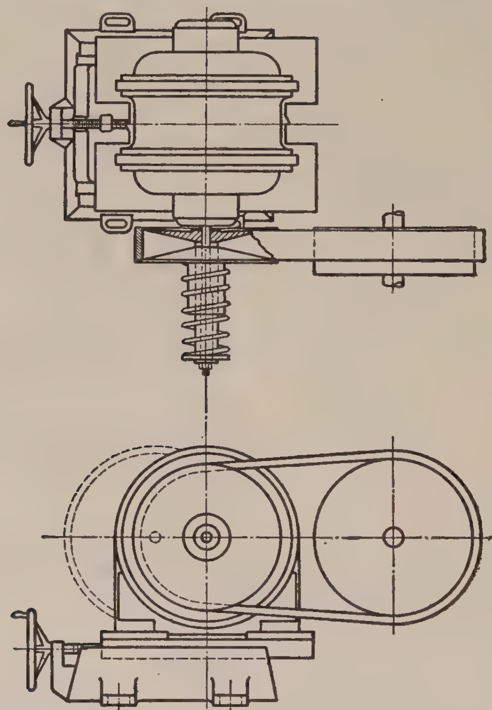


Reeves Pulley Co., Columbus, Ind.

FIG. 295. SPEED VARIATOR.

A truly stepless speed change is accomplished by the variable speed transmission shown in Fig. 295. The conical discs are moved toward

or away from each other permitting the belt to drop toward the center of rotation or forcing it toward the outer circumference. The device is now often built with the motor as an integral part of the unit. A similar method of changing speeds, although through a more limited range, is accomplished by the device shown in Fig. 296.



Reeves Pulley Co., Columbus, Ind.

FIG. 296. AUTOMATIC SPEED VARIATOR.

250. Pulley Speeds. A peripheral speed of 4000 ft. per min. is usually considered the permissible maximum for ordinary cast-iron pulleys with arms. If carefully balanced, 6000 ft. per min. may be considered permissible (see Chapter on Flywheels and High Speed Rotors, page 281). Webbed cast-iron pulleys may be run up to speeds of 9000 to 10,000 ft. per min. under favorable conditions, although manufacturers may refuse to guarantee them for such speeds.

Pressed-steel pulleys, whether welded or riveted, are usually not guaranteed for speeds higher than those recommended for cast-iron pulleys, on account of lighter construction, weakness of joints, or imperfect balance. Wood pulleys, however, permit higher speeds than cast-iron pulleys, since wood is lighter for its strength than cast iron.

251. Examples of Belt Design. Determine the size of belt required for a 60 hp. main drive running from a motor operating at a speed of 1750 r.p.m. to a line shaft revolving at 450 r.p.m.

Design Based on Leather Belts. Very often a pulley is furnished with the driven machine, and its size then fixes the size of the driving pulley. In the present case, we shall assume that the designer is at liberty to select all pulley sizes. A preliminary computation with a belt speed of 3000 ft. per min., a commonly recommended speed, indicates that the pulleys would be too small, that is, give undesirable belt widths with this speed. A belt speed of 4500 ft. per min. will give better proportions with a pulley of approximately 10 in. diameter on the motor. If we select a 10 in. pulley, the belt speed is then $10 \times \pi \times 1750/12 = 4580$ ft. per min.

We have $T_1 - T_2 = 60 \times 33{,}000/4580 = 434$ lb. If we wish to use leather belts, a double belt is advisable on a main drive to obtain evenness of running and good life. A 10 in. diameter pulley is smaller than the smallest one recommended for double belts in Table 39, but light extra flexible double belts are now available. Considering the values given in the table, we may assume that such a belt would be capable of transmitting an effective pull of 50 lb. per in. of width at 4580 ft. per min. This value gives a belt width of $434/50 = 8.7$ in. A width of 9 in. would doubtless be used.

If a single leather belt were to be considered, it could be loaded to an effective pull of 40 lb. per in. of width, which would mean a belt width of 11 in. Ordinarily, such a belt would run unevenly and with flapping.

Design Based on Textile Belts. If we wished to use a four-ply rubber belt on this drive, it would be advisable to load the belt to only 12 lb. per in. of width and ply, since there would not be quite 3 in. of pulley diameter per ply of fabric. The effective pull per in. of width would be 48 lb. and the width required would be $434/48 = 9$ in. The Goodyear *Handbook of Belting* gives almost exactly this width for their highest grade of fabric belt. A single endless-cord belt would, however, need to be only 6 in. wide.

Design Based on V-Belt. If the center distance of the drive is short, V-belts might be considered. A 10 in. pulley would be suitable for 7/8 in. $\times$ 5/8 in. belts. According to Table 40 a single V-belt would transmit an effective pull of about 66 lb. The number of belts would then be $434/66 = 6.6$, say 7.

These computations were based on arcs of contact of 180 deg. If the arc is less, the belt must be put on somewhat tighter, and if the maximum tension is to remain the same, the power transmitted is reduced. If it is desired to operate with the same maximum tension per in. of width, the effective pull transmitted per in. of width would then be reduced according to formula (2). The actual arc of contact must be found by layout or computation. The arc of contact for an open belt (see Fig. 322) on the smaller pulley may be computed by means of the following equation:

$$\cos\left(\frac{\text{angle of contact}}{2}\right) = \frac{R - r}{C}, \tag{8}$$

where R is the radius of the larger pulley, r the radius of the smaller pulley, and C the distance between pulley centers.

The angle of contact for a crossed belt is always greater than 180 deg. (see Fig. 297). For this reason a crossed belt can often be made to pull on a short center drive where the pulley diameters are unequal, when an open drive would be inadequate.

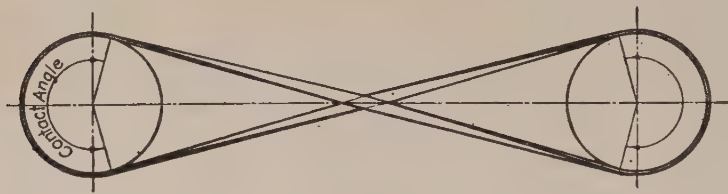


FIG. 297. CROSSED-BELT DRIVE.

252. Shaft and Bearing Loads. A belt produces a cross-bending load on the shaft equal to $T_1 + T_2$. With $T_1/T_2 = 2$, this force will be equal to $3(T_1 - T_2)$. With flat belts, it is necessary to compute the size of shaft and bearings to withstand this force, plus the force resulting from the weight of the shaft and pulleys. With V-belts a higher T_1/T_2 value may be assumed. With $T_1/T_2 = 3$, $T_1 + T_2 = 2(T_1 - T_2)$.

PROBLEMS

1. A single ply oak-tanned leather belt on cast-iron pulleys, 4 in. and 20 in. in diameter, is to transmit 2.5 hp. at 1200 r.p.m. of the smaller pulley. What is a reasonable ratio of net tensions? What width of belt is necessary? What is the resultant bearing force?

2. A rubber belt of four plies is to transmit 18 hp. at a belt speed of 2500 ft. per min. Select the diameters of pulleys necessary for a velocity ratio of 7 to 1 with a slip of 1½ per cent, and determine the width of belt necessary.

3. A high-grade mineral-tanned belt, installed for long service, is to transmit 40 hp. at 1300 r.p.m. of the driving pulley. What ply and width of belt are required? What size pulley may be used?

4. A leather belt transmits 25 hp. from a pulley 75 in. in diameter rotating at 150 r.p.m. The contact angle on the large pulley is 240 deg. and the coefficient of friction is 0.3. The contact angle on the small 10 in. pulley is 300 deg. and the coefficient of friction is 0.25. Calculate the necessary width of a belt 3/8 in. thick if the allowable stress is 300 p.s.i. and starting torque is twice running torque.

5. An 80 hp. Corliss engine running at 75 r.p.m. and having an 8 ft. flywheel is connected to a 42 in. pulley by means of a leather belt 1/2 in. thick. There is no idler pulley in the system. The contact angle of the large pulley may be taken as 200 deg. Allowable tensile stress in the belt is 200 p.s.i. Coefficient of friction on the flywheel is 0.45, on the 42 in. pulley 0.30. For these assumed conditions calculate the width of belt.

6. A leather belt 6 in. wide and 1/4 in. thick runs at 4000 ft. per min. and connects pulleys 12 in. and 60 in. in diameter. For the small pulley the contact angle is 270 deg. and the coefficient of friction 0.3; for the large pulley the corresponding values are 240 deg. and 0.4. If the weight of the belt is 0.035 lb. per cu. in. and the

allowable tension is 100 lb. per in., calculate the maximum hp. that can be transmitted. Consider the effect of centrifugal force if you deem it necessary.

7. A wood pulley 6 in. in diameter on a motor developing 25 hp. at 1200 r.p.m. drives a cast-iron pulley 36 in. in diameter. Determine the necessary width of a medium two-ply mineral-tanned leather belt, if the contact angle on the small pulley is 230 deg. and on the large pulley is 200 deg. Coefficient of friction is 0.3 and allowable belt stress is 300 p.s.i.

8. An electric motor operating at 1100 r.p.m. drives a punch-press drive shaft at 200 r.p.m. through a belt 5 in. wide and 1/4 in. thick. When the clutch is engaged the belt slips. It is proposed to correct this condition by increasing the contact angle with an idler pulley, and using the same belt with the same pulleys. If an increase in transmission capacity of 20 per cent will prevent slipping, determine the contact angle necessary. The original contact angle on the 8 in. motor pulley is 160 deg., the original tension ratio is 2.3, and the net tension 60 lb. per in. of width.

9. A variable-speed belt drive uses a special belt made by riveting trapezoidal wooden blocks to the outside surface of a leather belt. The tapered ends of the wooden blocks are leather surfaced and bear against two beveled discs attached to the motor shaft (similar to Fig. 296). The inside of the belt contacts a flat pulley. Determine the horsepower that can be transmitted if conditions are as follows:

Effective disc diameter	= 3¼ in.
Pulley diameter	= 10 in.
Disc r.p.m.	= 1160
Coef. of friction for disc	= 0.20
Coef. of friction for pulley	= 0.30
Belt section	= 4 in. × 1/4 in.
Effective wt. of belt	= 0.1 lb. per cu. in.
Allowable unit tension	= 250 p.s.i.
Contact angle for disc	= 120 deg.
Contact angle for pulley	= 240 deg.
Included angle between discs	= 40 deg.

10. A rubberized V-belt drive consists of 4 strands 7/8 in. × 5/8 in. on a driving pulley of 9 in. diameter. What horsepower would be transmitted at 920 r.p.m. of the driver?

11. Select a single rubberized V-belt to transmit 15 hp. at 3000 ft. per min. What size pulley should be used?

12. A flat rubber belt, 8 in. wide and six plies thick, operates at 4500 ft. per min. from a shaft rotating at 1560 r.p.m. Consider the belt 3/8 in. thick. What transmitted horsepower would be expected? Is this value near the maximum obtainable for the dimensions and belt given? If the speed were increased to 5000 ft. per min., what would be the percentage reduction in flexing life?

CHAPTER 16

BRAKES AND CLUTCHES

253. Friction. Friction is employed in brakes and friction clutches to regulate speed or to transmit torque. Brakes convert the energy stored in a rotating shaft into frictional energy and dissipate it as heat. Friction clutches provide a means of gradually applying a torque from a driving shaft to a stationary shaft until both are engaged and rotate as a unit. Without some such flexible coupling device, an automobile engine, for instance, could not be engaged and disengaged smoothly with the driving mechanism of the car.

254. Block Brakes. The simplest form of brake consists of a block pressed against the rim of a wheel, as illustrated in Fig. 298. If the pressure force between block and wheel is F_n and the coefficient of friction is f, the brake force is fF_n and the brake torque is fF_nr, where r is the radius of the wheel or drum. For the location of the pivot and the direction of rotation as shown, the tangential friction force tends to apply the brake. Thus, if distance c were sufficiently large compared to the distance a, the value of F might be zero, and the brake would yet be applied. When the friction force tends to draw the block into closer contact the brake is said to be *self-energizing*.

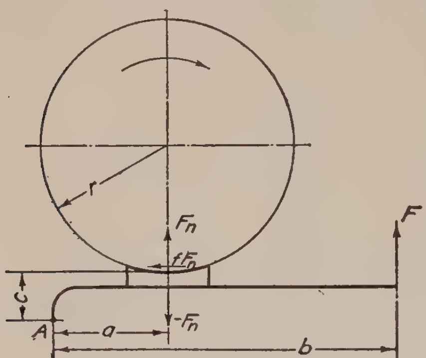


FIG. 298. BLOCK BRAKE.

The relationship between the forces on the brake may be obtained by considering the block and lever as a free body in equilibrium under all the forces acting upon it. Considering the weight of the brake arm as W and concentrated at the same point as the force F_n, by taking the summation of moments about the pivot A as equal to zero, we have

$$Fb + fF_nc - (F_n + W)a = 0,$$

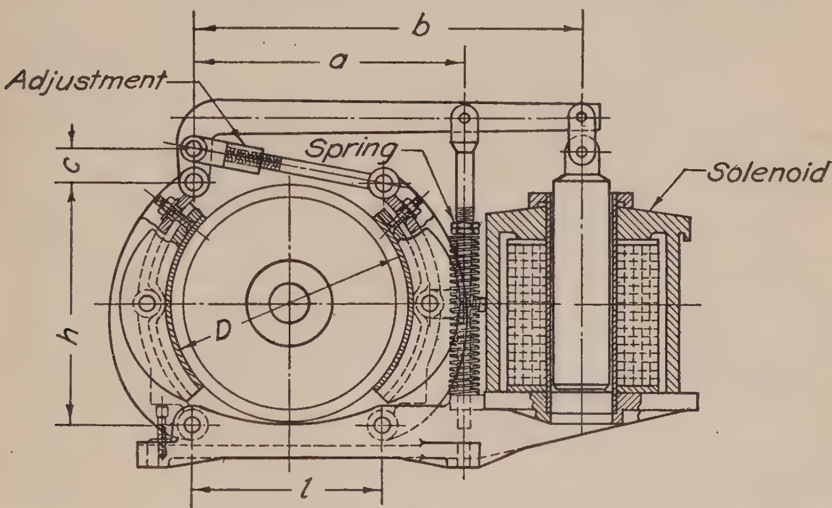
or

$$F = \frac{(F_n + W)a - fF_nc}{b} = \frac{F_n(a - fc) + Wa}{b}. \tag{1}$$

If the brake arm is nearly vertical and the torque Wa exerted by the weight W is negligible, it will be seen that F becomes zero even with a finite value for F_n, when $a = fc$, or $a/c = f$. In other words, no outside force is then necessary to produce a brake torque and the brake is self-energizing. For a common value of f equal to 0.3, the brake becomes self-energizing when a is equal to $0.3c$.

The use of one friction block results in high pressures on the drum-shaft bearings; consequently block brakes are generally made with blocks diametrically opposite to one another in order to equalize more nearly the bearing forces. This type of brake is frequently used in hoisting machinery and block brakes with either one or two shoes are universally used in railway applications.

A crane brake actuated by a solenoid is illustrated in Fig. 299. The spring setting is adjusted to apply the shoes when the solenoid



The Cleveland Crane and Engineering Co., Cleveland, Ohio

FIG. 299. CRANE BRAKE ACTUATED BY SOLENOID.

is not energized. When current is sent to the hoisting motor of the crane, the solenoid is energized and exerts a force on the lever sufficient to overcome the effect of the spring force and allow rotation of the drum. Failure of the electric current makes the solenoid inoperative and the brake is applied by the spring. A force analysis of this type of brake consists in treating each member of the linkage as an isolated body in equilibrium under the forces acting upon it. This procedure applies regardless of the peculiarities of the linkage.

255. Band Brakes. A stationary belt, or band, wrapped around a pulley acts as an effective brake. The law of belt friction applies (of course, without any allowance for centrifugal force in the band) and $T_1/T_2 = e^{f\alpha}$, where T_1 and T_2 are the tensions on the tight and loose sides of the band, $e = 2.718$, f is the coefficient of friction, and α the angle of contact in radians. The brake torque is

$$(T_1 - T_2)r = T_2(e^{f\alpha} - 1)r,$$

where r is the drum radius.

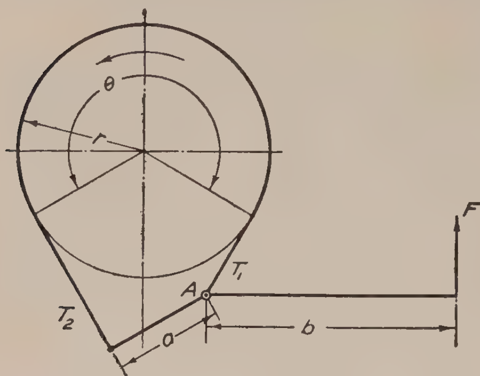


FIG. 300. BAND BRAKE.

Where the direction of rotation of the drum is always the same, the tight side of the band may be fastened to the pivot point, as shown in Fig. 300. This arrangement is desirable since only the smaller tension T_2 is applied on the brake arm. In Fig. 300 the relationship between F and T_2 is simply $T_2a = Fb$, or $T_2 = Fb/a$.

With an angle of contact equal to three-fourths of the circumference and a coefficient of friction equal to 0.3, as for wood or asbestos band lining, it will be found that $e^{f\alpha} = 4$, approximately.

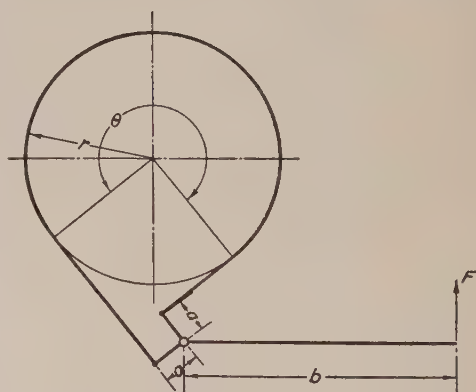


FIG. 301. TWO-WAY BAND BRAKE.

With these values the band torque is $3T_2r = 3Frb/a$. The ratio of b/a may be made as large as 10 for band brakes, while about 5 is the practical limit for block brakes. Thus the band brake shown in Fig. 300 may have a brake torque of $30Fr$, while the maximum for the block brake would be only $1.5Fr$, or one-twentieth as great. It is this feature of very high holding power that makes the band brake popular in spite of definite disadvantages. The band brake is difficult to set evenly, or to clear the drum dependably, and is very ineffective in dissipating the friction heat.

If the drum is subject to rotation in both directions, the arrangement shown in Fig. 300 is not satisfactory, since then the greater band pull T_1 would have to be sustained at times by the lever. For the ratio of $T_1/T_2 = 4$, this band brake is only one-fourth as effective in one direction as it is in the other. Figure 301 shows a lever arrangement which makes the brake equally effective in both directions of rotation. The ends of the bands are attached to two lever arms at equal distances from the pivot point. For the same operating force F this brake is only one-fifth as effective as when the tight side is attached to the frame.

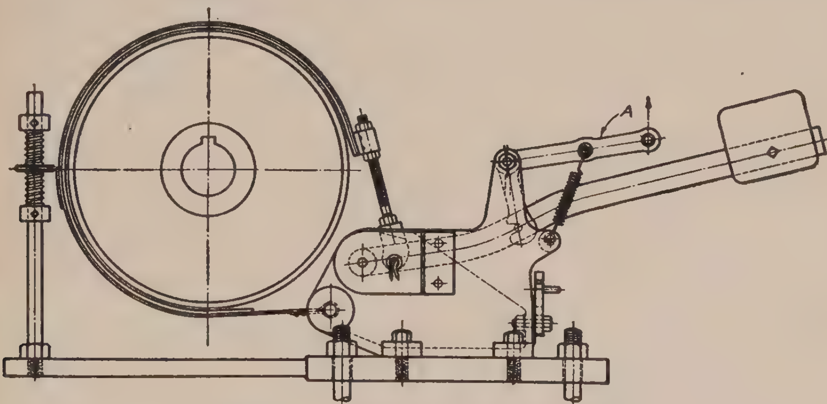


Fig. 302. Industrial Type Band Brake.

In Fig. 302 an industrial type of safety band brake is shown. By releasing latch A the brake is applied by means of the weighted lever. The latch-release mechanism is connected to a circuit-breaker so that the motor is disengaged when the brake is applied.

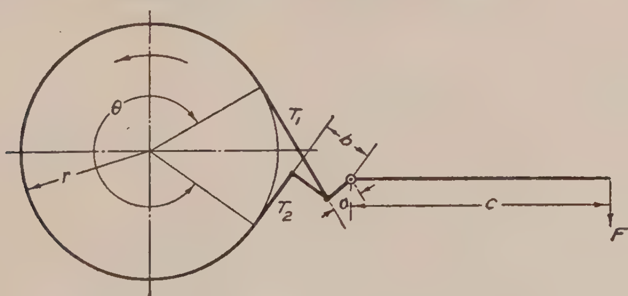


Fig. 303. Differential Band Brake.

256. Differential Band Brakes. The band brake may be made partially or wholly self-actuating by properly proportioning the lengths of the two arms in the arrangement of Fig. 303, and attaching the

tight side of the band to the lever so as to aid the actuating force F while the loose side opposes it. This brake, known as a differential band brake, sets automatically and allows rotation in only one direction. It is frequently used instead of a pawl and ratchet on cranes to prevent the load from descending under its own weight.

For the differential band brake shown in Fig. 303, rotating as shown, we have for equilibrium

$$Fc = T_2b - T_1a = T_2(b - ae^{f\alpha}).$$

For F to be zero or negative (automatically setting),

$$(2) \qquad b \leqq ae^{f\alpha}, \qquad \text{or} \qquad \frac{b}{a} \leqq e^{f\alpha}.$$

For operation in the opposite direction the brake releases automatically.

Example. The hand brake illustrated in Fig. 304 has an angle of contact of 280 deg. and is to sustain a torque of 3500 in. lb. The band has a compressed woven lining and bears against a cast-iron drum of 14 in. diameter. Determine the necessary operating force F. Assume the coefficient of friction f to be equal to 0.3. Then

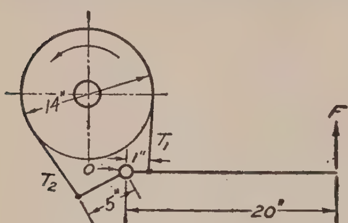


Fig. 304. Differential Band Brake.

$$\frac{T_1}{T_2} = e^{f\alpha} = e^{0.3 \times 280 \times (\pi/180)} = e^{1.47} = 4.35,$$

and

$$T_1 - T_2 = \frac{3500 \times 2}{14} = 500 \text{ lb.}$$

Combining these two equations, we have

$$T_1 - \frac{T_1}{4.35} = 500 \text{ lb.},$$

$$T_1 = \frac{500}{1 - \frac{1}{4.35}} = 650 \text{ lb.},$$

$$T_2 = 650 - 500 = 150 \text{ lb.}$$

To determine F, take $\Sigma M_0 = 0$; then $20F + T_1 = 5T_2$, and

$$F = \frac{5 \times 150 - 650}{20} = 5 \text{ lb.}$$

257. Internal Brakes. Where protection from dirt is necessary, as well as for other design reasons, internal brakes are used. They may be of the following types: (a) expanding-band, (b) two-shoe, (c) tandem-shoe. The expanding-band type is used on machinery for light loads and was used at one time, along with the contracting external band, almost universally on automobiles. It consists of a flexible band pivoted at some point and expanded so as to make con-

tact with a surrounding drum. The expanding force F required at the band for operation is the sum of force F_1 necessary to bring the band into contact with its drum, and force F_2 necessary to press the band against the drum to develop sufficient friction. The thickness of the band is small compared to its diameter, but because it is pivoted at only one point it is not free to expand and make uniform contact around the internal circumference of the drum. The relationship between the radius of curvature ρ to which a beam is bent by a bending moment M, the modulus of elasticity E, and the moment of inertia I of the band section, is

$$\frac{M}{EI} = \frac{1}{\rho}, \tag{3}$$

and if the radius of curvature is changed from ρ_a to ρ_b,

$$\frac{M_b - M_a}{EI} = \frac{1}{\rho_a} - \frac{1}{\rho_b}.$$

If ρ_a is the radius in the unstressed state, $M_a = 0$; therefore

$$\frac{M_b}{EI} = \frac{1}{\rho_a} - \frac{1}{\rho_b}, \quad \text{or} \quad M_b = M = EI\left(\frac{1}{\rho_a} - \frac{1}{\rho_b}\right). \tag{4}$$

The difference in diameter between the band and drum may be about 1/32 in. or slightly more. For the condition in Fig. 305, $M_1 = F_1D$ and $M_2 = F_2D$, where D is the diameter of the band. If we assume the unit contact pressure p between the band and drum to be uniform, we have

$$F_2 = \frac{pbD}{2} \quad \text{and} \quad M_2 = \frac{pbD^2}{2}, \tag{5}$$

where b is the width of the band. The total moment $M = M_1 + M_2$ produces maximum bending at the pivot point A, if it is placed as in Fig. 305. (The pivot point could also be placed at one end of the ring.)

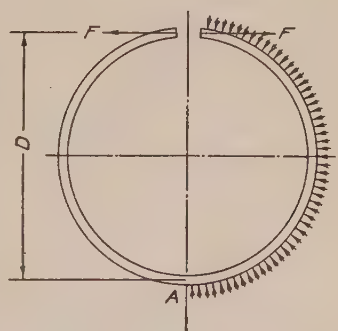


Fig. 305. Expanding-Band Brake.

In calculating the stress in the band, we may disregard the direct stress F/A_b, where A_b is the cross-sectional area of the band, and the effect of curvature, since the uncertainty of the distribution of contact pressure renders the formulas but rough approximations in any case. With stress values proven in practice, however, the formulas may be used as the basis for design.

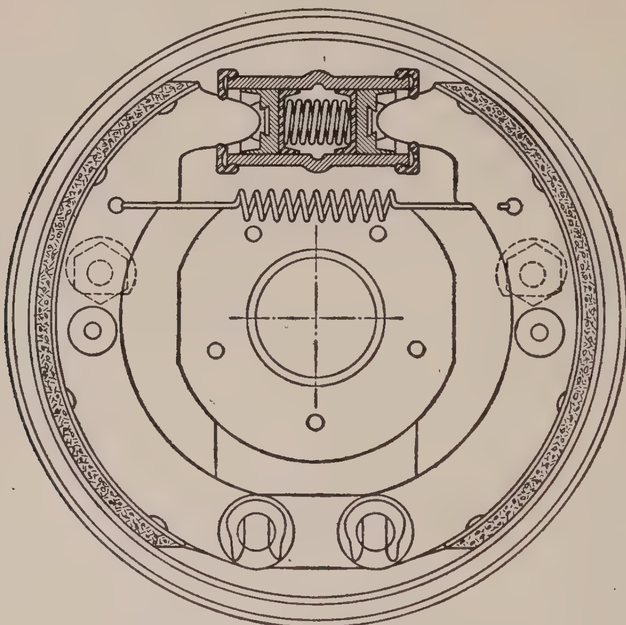
The torque that may be sustained by the brake is

$$M_t = \frac{\pi D^2 pbf}{2} = \pi M_2 f, \tag{6}$$

and the pressure producing force is

$$F_2 = \frac{M_t}{\pi f D}. \tag{7}$$

The two-shoe type of brake is used in American automotive practice with the shoes pivoted at a common point or at separate points. The shoes are much more rigid than a band but, of course, suffer some distortion along with the drum. The total arc of contact of each shoe is about 120 deg. This type of brake is self-acting. A modern automotive brake is illustrated in Fig. 306.



The Chrysler Corp., Detroit, Mich.

FIG. 306. HYDRAULIC TYPE AUTOMOBILE BRAKE.

258. Brake Capacity. The capacity of a brake is dependent, in addition to strength, upon its ability to dissipate heat and to withstand repeated applications without excessive wear. For continuous dragging of the brake, the work of friction is

$$\text{Work} = \frac{fpA_c\pi DN}{12} \text{ ft. lb. per min.,} \tag{8}$$

where A_c is area of contact in sq. in., N the number of revolutions per minute, and all other symbols as in the preceding equation. The coefficient of friction f varies not only with the materials, but also with the condition of the surfaces, so that the values selected in design must allow for glazing of the contact surfaces. The General Electric Company uses a value of 0.3 for asbestos-lined block brakes with a brake pressure of 15 to 20 p.s.i. Coefficients may be assumed as follows: asbestos 0.3 to 0.5, wood 0.3, greasy linings 0.2, unlubricated cast-iron shoes 0.15, lubricated cast-iron shoes 0.07.

For a certain relative speed and pressure, the friction heat generated is proportional to the contact area of the shoes times the drum diameter D; consequently it is proportional to bD^2, where b is the width of the shoes. The heat dissipating capacity is not quite proportional * to bD; hence large brakes do not dissipate the generated heat so well as small brakes, and should be designed for a lower unit pressure than small brakes. If fan blades are attached to the wheels, the heat dissipating capacity may be increased about one-third. A convenient indicator for brake capacity is the horsepower per square inch of projected contact surface, $hp/(bD)$. As a result of experiences with excavating machinery, Rasmussen † gives for hoisting brakes, $hp/(bD)$ ratios of 0.20 to 0.30 for open exposed band brakes and 0.25 to 0.40 for open exposed band and cone clutches; these values are rather high. The General Electric Company gives values of about 0.05 for a motor-shaft speed of 900 r.p.m.; this factor should not be exceeded for conservative design and severe conditions.

The allowable average unit pressure p in p.s.i. is dependent upon the rubbing velocity V, in ft. per min. Hütte gives the following limiting values:

pV = 55,000, for intermittent operation of short duration (stopping brakes) and poor heat dissipation (wooden blocks),

pV = 28,000, for continuous operation (lowering brakes) and poor heat dissipation (wooden blocks),

pV = 83,000, for continuous operation (lowering brakes) and good heat dissipation (oil bath).

Rasmussen gives as maximum average pressures:

Molded compressed friction blocks	100 p.s.i.
Molded band friction lining	75 p.s.i.
Woven and compressed friction lining	50 p.s.i.

and twice these values for construction machinery where space is limited.

* See Norman, *Principles of Machine Design*, p. 651.

† A. C. Rasmussen, *Heat-radiating capacity of clutches and brakes*, Product Engineering, December, 1931, vol. 2, pp. 529–532.

259. Clutches. The simplest form of clutch is the jaw clutch (Fig. 307), in which torque is transmitted by the positive engagement of interlocking teeth or jaws. This type of clutch is used when a positive drive is required and when a gradual engagement between driving and driven members is not a definite requirement. The fixed half of the clutch is tightly keyed to its shaft, whereas the movable half is free to slide axially to engagement, but is prevented from turning relatively to its shaft by splines or one or more feather keys. Two feather keys promote free sliding. It is evident that this type of clutch should be engaged when the connecting shafts are at about the same speed; otherwise severe impact occurs due to inertia loads. In the spiral jaw clutch (Fig. 308) the teeth are shaped to drive in one direction only. With this type, engagement can be made at moderate speeds.

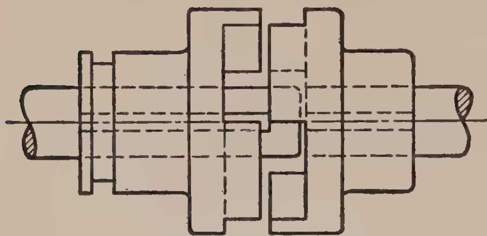


Fig. 307. Jaw Clutch.

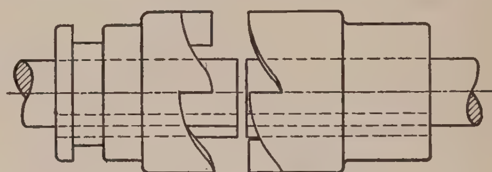


Fig. 308. Spiral Jaw Clutch.

In automotive design, direct drive from the engine to the propeller shaft is made through a positive clutch consisting of an external gear meshing with an internal gear of the same pitch and tooth number. In automotive transmissions in which helical gears in constant mesh are used, one of the gears is engaged by means of a jaw clutch having a limited number of teeth. The speed of the shaft and the gear is synchronized, however, by a cone friction clutch which comes into engagement before the jaw clutch.

The development of the power transmission clutch as used in the automobile progressed from the cone clutch to the multiple-disc metal-to-metal clutch operating in oil, and then to the single-plate dry clutch in general use today. Performance, transmissive capacity, and heat dissipation, as well as space requirements and cost of manufacture, are the considerations which have prompted this development.

260. Cone Clutch. The cone clutch is a very simple device for coupling two shafts by friction without the use of excessive axial

pressure. The slope of the cone face is made small enough to give a high normal force (hence high friction force) and yet large enough to permit of easy disengagement. The slope of the face angle α is made from 8 to 15 deg., depending upon the coefficient of friction of the surface materials. The contacting surfaces may be metal to metal, leather, asbestos, or tarred fiber. The clutch parts may be held together by springs producing the required axial force for driving and are then released by a force applied against the springs.

It is evident from the forces acting on the cone (Fig. 309) that the engaging force F is opposed by the horizontal components of the normal force F_n, and the friction force fF_n, both of which are actually distributed around the circumference of the cone and not concentrated at two points, as shown in the diagram. Assuming that the surfaces of the mating parts slide along each other while being engaged, which may be true for yielding materials, and in consequence produce frictional resisting forces, we have

(9) $$F = F_n(\sin \alpha + f \cos \alpha).$$

If the friction force along the cone elements is zero, as it well might be with relatively rigid materials, or after engagement is completed, we have

(10) $$F = F_n \sin \alpha.$$

The torque transmitted in either case is $fF_nD_m/2$, where D_m is the mean diameter of the cone. The value of F_n is equal to the product of the unit pressure p and the surface area of the cone in contact, or $F_n = b\pi D_m p$, where b is the width of contact face along the cone element.

Fig. 309. Forces Acting on Cone Clutch.

Allowable unit pressures depend upon the materials and the frequency of use. Rockwood Manufacturing Company uses 100 p.s.i. for clutches operated frequently and 200 p.s.i. for clutches operated infrequently. These values apply to fiber cones operating in cast iron cups. Much lower values are recommended by others. Spooner, for instance, gives 7 to 8 p.s.i. for leather-cone clutches on iron.

Example. A cone clutch with leather-to-iron contact is to transmit 30 hp. at 900 r.p.m. The mean diameter is limited to 12 in., and the slope of the face is to be 12 deg. Determine the necessary axial force to engage the clutch and the width

of face if the coefficient of friction is 0.25. From formula (1), page 193,

$$M_t = \frac{30 \times 63{,}024}{900} = 2100 \text{ in. lb.}$$

$$M_t = fF_n\frac{D_m}{2}, \qquad \text{or} \qquad F_n = \frac{2M_t}{fD_m},$$

so that

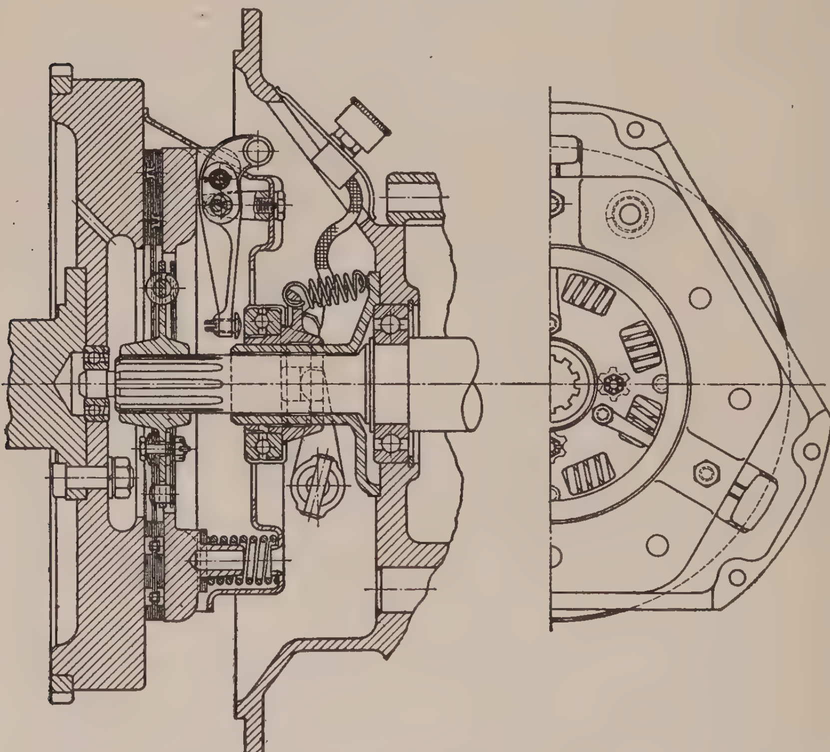
$$F_n = \frac{2 \times 2100}{0.25 \times 12} = 1400 \text{ lb.}$$

Since $F = F_n(\sin \alpha + f \cos \alpha)$ for a soft surface, we find

$$F = 1400(0.2079 + 0.25 \times 0.9781) = 635 \text{ lb.}$$

Assuming an allowable contact pressure of 10 p.s.i., we have

$$b = \frac{F_n}{p \times \pi D_m} = \frac{1400}{10 \times \pi \times 12} = 3.72 \text{ in., say } 3\tfrac{3}{4} \text{ in.}$$

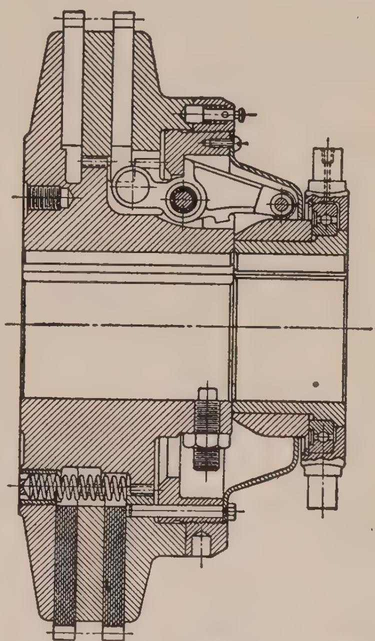


Borg-Warner Corp., Long Mfg. Division, Detroit, Mich.

FIG. 310. AUTOMOBILE SINGLE-PLATE CLUTCH.

261. Disc Clutches. The single-plate disc clutch is best exemplified by the automotive clutch, as illustrated in Fig. 310. This type of clutch has been highly developed to perform its function and to

withstand severe usage. Many design features of the clutch illustrated are worthy of notice. The drive is from the surface of the flywheel to the friction-lined flexible disc through coil shock-absorbing springs and a friction vibration-dampening element, and then to the splined hub which rides on the driven shaft. The friction disc is pressed against the flywheel by a pressure plate actuated by helical compression springs parallel to the shaft axis, and is released by a shifting collar pressing on three forged-steel release levers. The release levers, which compress the springs, are supported by a rolling pin connection to the pressed-steel housing. The pressure plate is operated from the release levers through pins in needle bearings. The release levers are so proportioned that, as the speed of rotation of the clutch increases, centrifugal force causes the plate pressure to be increased, thus adding to the transmissive capacity of the clutch. The purpose of this feature is to allow the transmission of a larger horsepower without undue increase of the spring pressure which would make manual disengagement more difficult. The driven member assembly is made light to minimize inertia, and all rotating parts are carefully balanced.

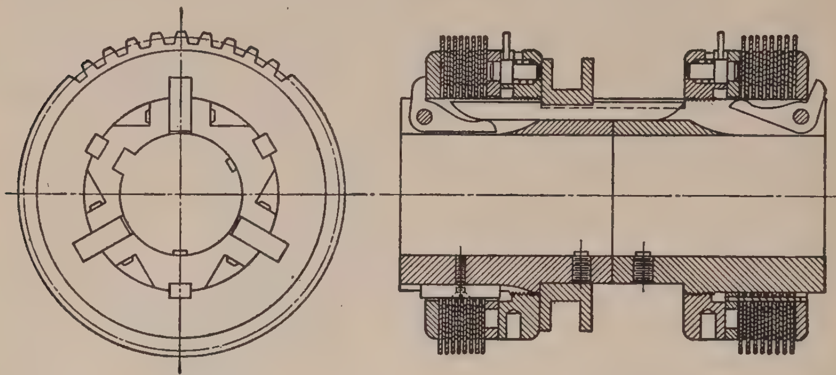


Twin-Disc Clutch Co., Racine, Wis.

Fig. 311. Two-Plate Dry-Disc Clutch.

When the horsepower becomes large, as in trucks, and the clutch diameter is limited, it becomes necessary to use more than one disc. In this manner the tangential force may be increased for the same normal pressure and axial force. Heat dissipation is more difficult, however. Figure 311 shows a two-plate dry clutch for industrial use. The pressure between the plates is provided by an axially operated cam acting through a lever. The outside edges of the discs are toothed and fit into an internal gear housing of the same pitch diameter. A popular type of multiple-disc clutch, designed to run in oil, is shown in Fig. 312. This type of duplex clutch, which functions as two clutches operated by a single cam, is widely used in machine tools. In automatic lathes it may operate 50 times per minute. Rapid adjustment

is possible by turning the screw collar after pulling out the small locking pin shown at the top.



Twin-Disc Clutch Co., Racine, Wis.

FIG. 312. DUPLEX MULTIPLE-DISC CLUTCH (OIL TYPE).

To determine the transmissive capacity of disc clutches, consider the discs to have an inner radius of r_i, an outer radius of r_o, and n pairs of surfaces of contact. If the average unit contact pressure is p_m, the axial force is

$$F = p_m \pi (r_o^2 - r_i^2). \tag{11}$$

If the pressure per square inch on a ring-shaped area of width dr and radius r is p and the coefficient of friction is f, the torque for this area and n pairs of surfaces is

$$dM_t = frpn2\pi r dr = 2\pi fpnr^2 dr,$$

and the torque for all the surfaces is

$$M_t = 2\pi fn \int_{r_i}^{r_o} pr^2 dr.$$

It is probably reasonable to assume that because of greater wear at the outer circumference, the pressure is less at that point. Taking $p = C/r$, the axial force is

$$F = 2\pi \int_{r_i}^{r_o} r \left(\frac{C}{r}\right) dr = 2\pi C(r_o - r_i),$$

and

$$M_t = 2\pi fn \int_{r_i}^{r_o} r \left(\frac{C}{r}\right) rdr = \pi fnC(r_o^2 - r_i^2).$$

Eliminating C, we have

(12) $$M_t = \frac{fnFD_m}{2},$$

where D_m is the mean diameter of the disc. This formula may be used also for uniformly distributed pressure as the error is slight and on the safe side.*

A value of 0.2 for the coefficient of friction is satisfactory for dry-fabric design and 0.1 for operation in oil or grease, with fabric or metal contact. The average unit pressure for single-plate automotive clutches is about 50 p.s.i.

EXAMPLE. A small multiple-disc clutch is to be made up of 6 steel and 5 bronze discs, with an inner diameter of 1 in. What outer diameter and axial pressure are necessary to transmit a torque of 175 in. lb.?

On the basis of the data presented, $n = 10$, $f = 0.1$, and p_m is taken as 50 p.s.i.; then

$$F = p_m\pi(r_o^2 - r_i^2) = 50\pi\frac{(D_o^2 - D_i^2)}{4} = 39.4(D_o^2 - D_i^2).$$

Since $M_t = fnFD_m/2$, we have

$$175 = \frac{0.1 \times 10 \times 39.4(D_o^2 - D_i^2)(D_o + D_i)}{2 \times 2},$$

so that

$$(D_o^2 - D_i^2)(D_o + D_i) = 17.8.$$

Substituting for D_i, we have $(D_o^2 - 1)(D_o + 1) = 17.8$. Inspection shows that D_o is between 2 and 3; try $D_o = 2\frac{1}{2}$. Then $(6.25 - 1)(2.5 + 1) = 18.4$, which is safe. Next try $D_o = 2\frac{3}{8}$; then $(5.65 - 1)(2.375 + 1) = 15.6$. This value is too low, and therefore D_o should be $2\frac{1}{2}$ in., and $F = 39.4(6.25 - 1) = 207$ lb.

PROBLEMS

1. A simple block brake, such as shown in Fig. 298, has the following proportions: $a = 14$ in.; $b = 36$ in.; $c = 1\frac{1}{2}$ in. The block is faced with woven friction lining and the drum is cast iron giving an assumed coefficient of friction of 0.4. If the drum diameter is 28 in., what force, F, is required to sustain a torque of 2000 in. lb. if the direction of rotation is clockwise? Counterclockwise? What direction and value of c would make the brake self-locking?

2. What horsepower can be absorbed by a simple block brake, similar to Fig. 298, with the following conditions: $a = 6$ in.; $b = 18$ in.; $c = -2$ in.; $F = 10$ lb.; diameter of cast-iron drum = 12 in.; r.p.m. of drum = 1200, clockwise. The block is made of wood and is 3 in. wide and 8 in. long. Is the hp. per sq. in. projected contact surface satisfactory?

3. A double-block brake, similar to the one shown in Fig. 299, has the following dimensions: $D = 22$ in., face 6 in., $a = 2$ ft. $2\frac{3}{4}$ in.; $b = 2$ ft. $10\frac{3}{4}$ in.; $c = 2\frac{1}{2}$ in.; $h = 22$ in., $l = 19$ in. The approximate weight of the mechanism referred to the end of the lever is 84 lb. The compression spring has an outside diameter of 3 in., 32 coils of 3/8 in. wire, square ends, and a free height of 24 in. What spring force is necessary to give a rating of 140 hp. at 870 r.p.m.? Assume that the coefficient of friction is 0.35.

* Norman, *Principles of Machine Design*, p. 638.

4. A simple band brake, arranged as in Fig. 300, is to hold a torque of 1500 in. lb. on a drum 18 in. in diameter, rotating at 900 r.p.m. counterclockwise. The contact angle is 280 deg., the coefficient of friction is 0.2, and the force on the end of the lever, F, is 40 lb. Fix upon reasonable values of a and b.

5. A two-way band brake, as in Fig. 301, has a drum diameter = 16 in., $a = 3$ in., $b = 25$ in., the coefficient of friction = 0.25, and the contact angle is 260 deg. What torque will a force, F, of 75 lb. sustain? If the band width is 1 in., what thickness is necessary to limit the tensile stress to 10,000 p.s.i.? Will this brake have sufficient capacity to dissipate heat if operated at 200 r.p.m.?

6. A differential band brake, such as shown in Fig. 303, is to absorb 40 hp. at 1200 r.p.m. of the drum with an applied force, F, of 12.5 lb. If the drum diameter is 30 in., the contact angle is 320 deg., $a = 1$ in., and $b = 5$ in., what is the necessary value of c for a coefficient of friction of 0.2? Brake band width is 4 in. What is the $hp/(bD)$ ratio?

7. A cone clutch, unlubricated cast iron on cast iron, has an outside diameter of 16 in., a face of 4 in., and an included cone angle of 20 deg. What horsepower can be transmitted at 800 r.p.m. if the axial force is 150 lb.? What force would be necessary to engage the clutch to transmit this horsepower? What is the unit normal contact pressure?

8. A cone clutch, asbestos lined, is to transmit 40 hp. at 2500 r.p.m. If the maximum axial force available is 200 lb., what mean radius is necessary? What is the required width of face? Included cone angle should be about 24 deg. and normal unit pressure about 8 p.s.i.

9. A single-disc clutch with two contact faces (cast iron and cast iron) has an inside diameter of 6 in. and an outside diameter of 9 in. What axial pressure is necessary to give a capacity of 7 hp. at 300 r.p.m.? Is the unit normal pressure satisfactory? Clutch operates in oil.

10. A multiple-disc clutch is composed of 7 steel and 6 leather-lined discs and is to transmit a torque of 125 in. lb. If the inner diameter is 2 in., determine the necessary outer diameter and the axial force. Average unit pressure is about 50 p.s.i. and clutch operates in oil.

11. A multiple-disc clutch, steel on bronze, is to transmit 10 hp. at 1500 r.p.m. If the inner radius is 1½ in. and the outer radius 2¾ in., how many discs are required? What is the axial force and the unit normal pressure? Clutch operates in oil.

12. An internal expanding clutch has a cast-iron drum 4 in. in diameter with a 1 in. face. Thickness of band is to be 3/16 in. before expansion. The ring diameter is 0.05 in. smaller than the drum diameter. Clutch is to operate in oil. If the clutch is to have a torque capacity of 120 in. lb., what force is required to spread the bronze ring? What stress is induced in the ring?

13. A friction clutch engaging with the rim of a pulley is to be designed with 4 pairs of fabric-lined shoes, half the shoes in contact with the inner surface of the rim and half in contact with the outer surface. The mean rim diameter of the pulley is 20 in. and 30 hp. is to be transmitted at 200 r.p.m. If the coefficient of friction is assumed to be 0.25 and the permissible bearing pressure 30 p.s.i., determine the area required for each shoe.

CHAPTER 17

CHAINS

262. Open Link Chains. Link chains (Fig. 313) are of great importance for anchoring vessels, for hauling, for tying and slinging loads in hoisting, etc. They are also used for turning the wheels of hand-operated hoists, overhead window shutters, and other similar purposes. For such applications, the wheels are provided with pocketed grooves to receive the chain, as shown in Fig. 314. Chains may still be used for small hoists of various kinds, but for important hoisting purposes, wire rope is now used almost exclusively.

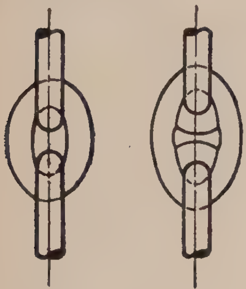


Fig. 313. Open Link Chains.

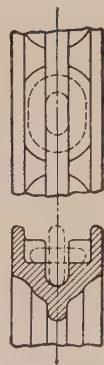


Fig. 314. Pocket-Grooved Wheel for Open Link Chains.

263. Chains for Power Transmission. The types of chains used for power transmission are: *block* (Fig. 315), *roller* (Fig. 316), and *silent* chains (Fig. 317). Each type possesses the advantage of giving a positive transmission ratio. Chains permit a longer center-to-center distance than gears without excessive wheel diameters. On the other hand, they permit shorter shaft distances than flat belts and require no slack side tension to operate. Consequently, chains produce smaller bending loads on the shaft and lower bearing pressures than do belts.

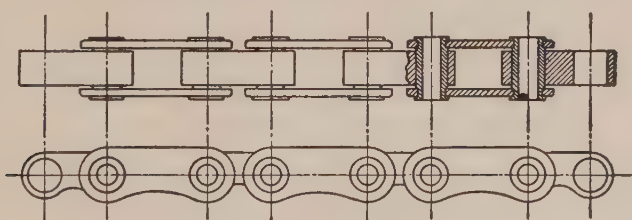


Fig. 315. Block Chain.

The advent of the V-belt has somewhat minimized these advantages, because the V-belt can run with a comparatively low slack-side tension. Chains do not slip, and hence afford no automatic safety

against overload. They do not give quite the same cushioning effect under impact, nor quite the same silence as do belts. Chain drives are rather complicated; to give satisfactory service, they must be lubricated. Furthermore, a chain drive is usually more expensive than a V-belt drive. For these reasons, the latter has found increasing use where a fixed transmission ratio is not required.

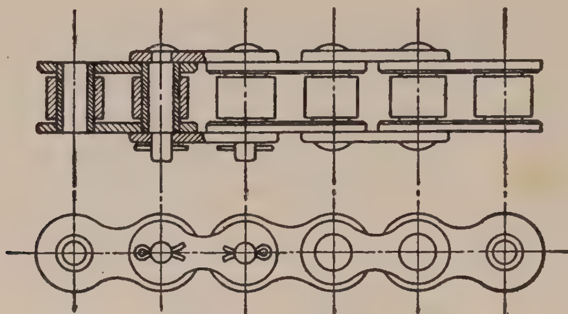


Fig. 316. Roller Chain.

In chain parlance, the toothed wheel through which the power is transmitted is known as a *sprocket*. The distance from chain joint to chain joint is known as *pitch*. The earliest chains were of roller or block type with the chain touching at the bottom of the tooth space. All chains stretch from wear in the joints and from elongation of the links. To allow for this stretching, clearance was provided behind all links except one, which thus transferred the whole load to a single

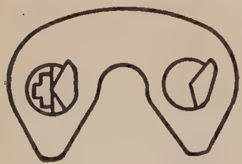
Link-Belt Co., Chicago, Ill.

Fig. 317. Silent Chain.

sprocket tooth. The transfer of the load from tooth to tooth was jerky, and wherever the fit was imperfect impact and noise occurred. To remedy this condition, Hans Renold in England introduced a chain link consisting of stampings with a straight bearing edge which engaged with a straight-sided sprocket tooth, as shown in Fig. 317. This type of chain engages and drives on several teeth simultaneously. If the links wear or stretch, the bearing faces simply make contact on a

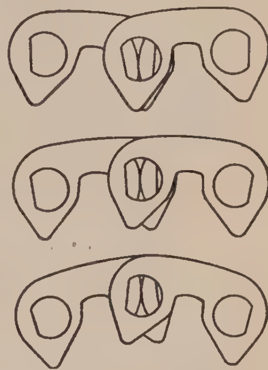
larger diameter, where the pitch distance between successive sprocket teeth is greater. These chains were called *silent* chains because of the smoothness of action which this design afforded.

The silent chain found ready acceptance; was introduced into this country by the Link-Belt Company, with improved joint design, and was later taken up by other concerns, usually with somewhat modified joints. The rocker joint, introduced by the Morse Chain Company, is shown in Fig. 318. The Ramsey Chain Company's joint is illustrated in Fig. 319. Chains were used for a very wide range of power, from the very smallest to several thousand horsepower. For the larger powers the chains were made very wide.



Morse Chain Co., Ithaca, N. Y.

FIG. 318. SILENT-CHAIN LINK SHOWING ROCKER JOINT.



Ramsey Chain Co., Inc., Albany, N. Y.

FIG. 319. SILENT-CHAIN LINKS SHOWING ROCKER JOINT ACTION.

Later on, largely through the efforts of the Diamond Chain and Manufacturing Company, the principle of letting the chain engage on larger diameters as wear occurs was applied also to roller chains, and led to development of the *standard American sprocket tooth form* now recommended by several engineering organizations. In Fig. 320 this tooth is reproduced from the standards of the Society of Automotive Engineers, which are identical with those of the American Gear Manufacturers' Association.

264. Power Computations for Roller Chains. All chain manufacturers' catalogs contain directions for the arrangement of drives and tables covering powers, speeds, etc. It is always recommended that in extreme cases manufacturers' recommendations be sought before the final selection is made, and no material is guaranteed unless their directions are followed. Nevertheless, standardization has now progressed to a point where capacity tables and ratings are also furnished by the American Gear Manufacturers' Association, and since the formulas on which these tables are based are also listed, they will be given here.

The formula for the permissible working load is as follows:

$$F = \frac{2{,}600{,}000A}{V + 600} - \frac{wV^2}{115{,}900}, \tag{1}$$

where F = working load in lb., A = projected pin-bearing area, that is, diameter of pin times length of bushing in in., V = chain velocity

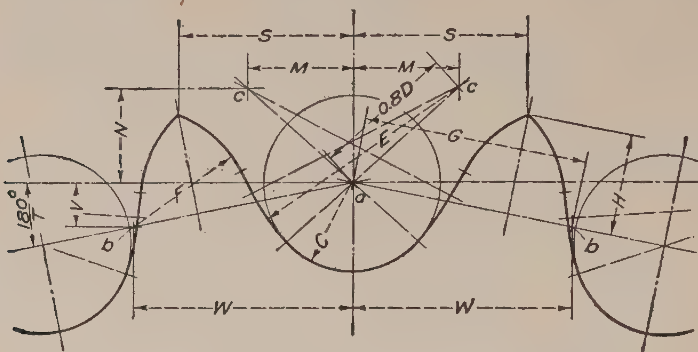


FIG. 320. STANDARD TOOTH FORM FOR ROLLER CHAIN.

P = Pitch of Chain.
D = Nominal Roller Diameter.
T = Number of Teeth.

$$C = \frac{1.005\ D + 0.003 \text{ in.}}{2}.$$

$$N = 0.8\ D \sin\left(35° + \frac{60°}{T}\right).$$

$$M = 0.8\ D \cos\left(35° + \frac{60°}{T}\right).$$

$$E = 0.8\ D + C.$$

$$V = 1.24\ D \sin\frac{180°}{T}.$$

$$W = 1.24\ D \cos\frac{180°}{T}.$$

$$S = \frac{P}{2}\cos\frac{180°}{T} + F \sin\frac{180°}{T}.$$

$$F = 0.8\ D \cos\left(18° - \frac{56°}{T}\right) + 1.24\ D \cos\left(17° - \frac{64°}{T}\right) - E.$$

$$G = 1.24\ D.$$

$$H = \sqrt{\left[F^2 - \left(G - \frac{P}{2}\right)^2\right]}. \quad \text{When } \frac{P}{2} > G, \text{ then } H = F.$$

$$O.D. = \text{Outside Diameter} = P \cot\frac{180°}{T} + 2\ H.$$

Connect the two arcs E and F by a common tangent.

in ft. per min., and w = weight of chain in lb. per ft. The second term allows for impact due to centrifugal force and should be introduced only if the speed exceeds 800 ft. per min. below which speed the effect is negligible.

F may be increased 25 per cent, if the design of the drive, installation, and lubrication are correct. On the other hand, F should be reduced to 50 per cent of the value in the formula, if the lubrication is

poor, if the load is applied suddenly, if the service is quite continuous, if unusually long life is required, or if there is misalignment.

To avoid excessive impact between chain and sprocket, the maximum r.p.m. is limited by the formula

$$\text{(2)} \qquad \text{Max. r.p.m.} = \frac{1920}{P}\sqrt{\frac{A}{wP}},^{*}$$

where P is the pitch in in.

For a first estimate the following formulas of the Diamond Chain and Manufacturing Company are doubtless sufficient and are less complicated:

$$\text{(3)} \qquad \text{Max. allowable pitch} = \left(\frac{900}{\text{r.p.m.}}\right)^{2/3}.$$

$$\text{(4)} \qquad \text{Max. r.p.m. of sprocket} = \frac{900}{\sqrt{P^3}}.$$

In Table 41 are given the necessary data for standard roller chains, as found in the list of Recommended Practices of the American Gear

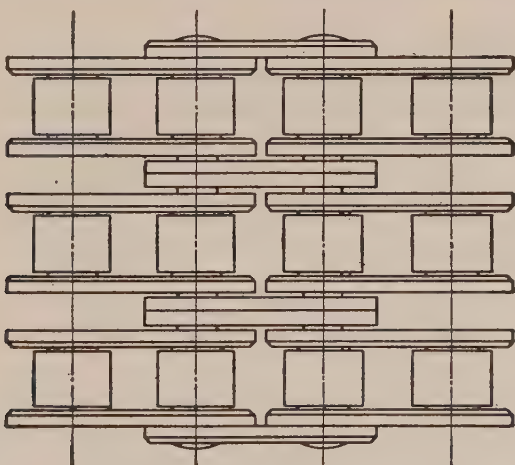


Fig. 321. Triple Roller Chain.

Manufacturers' Association. The weight is taken from the tabulations of the Whitney Mfg. Co. Chains are furnished in single, double, triple, and even greater widths. For approximate values, the weights and widths of the double and triple chains (Fig. 321) may be taken

* This is only one of three alternative formulas listed in standards issued by the A.G.M.A. Space limitations prevent full reproduction of the standards.

as twice and three times, respectively, the values given for the single chain.

TABLE 41

General Dimensions of Standard Roller Transmission Chains

Pitch in.	Roller Dia. in.	Bushing Length in.	Width of Roller in.	Pin Dia. in.	Test Load lb.	Weight lb. per ft.
3/8	0.200	0.288	3/16	0.141	555	0.23
1/2	5/16	0.433	5/16	0.156	740	0.41
5/8	0.400	0.535	3/8	0.200	1400	0.69
3/4	15/32	0.688	1/2	0.234	2018	1.04
1	5/8	0.875	5/8	0.312	3820	1.77
1 1/4	3/4	1.062	3/4	0.375	5626	2.59
1 1/2	7/8	1.374	1	0.437	7760	4.05
1 3/4	1	1.438	1	0.500	10220	5.10
2	1 1/8	1.750	1 1/4	0.562	13008	6.85
2 1/2	1 9/16	2.124	1 1/2	0.781	25340	10.20
3	1.90	2.625	1 7/8	0.937	36634	

265. General Arrangement. The following rules are taken from manufacturers' catalogs.

1. Use the smallest pitch that the load will permit. This practice will insure a quieter drive. If the computed pitch somewhat exceeds a standard pitch, select a slightly larger sprocket diameter, or use a multiple chain, rather than use the next larger pitch.

2. Do not use less than fifteen teeth in any sprocket, unless the speed is very low. Seventeen teeth are often preferred. Frequently the size of the shaft for the sprocket is such that the required diameter of the sprocket will result in a greater number of teeth.

3. Select a wide chain in preference to a narrow one, except in cases where the chain is crossed or where the sprockets must run considerably out of alignment.

4. Select a light chain in preference to a heavy one, if strength and rivet area are adequate.

5. The angle included between the two strands of the chain should not be greater than 45 deg. When the speed ratio does not exceed 2½ to 1, the minimum distance can be one-half the sum of the sprocket diameters plus necessary tooth clearance; when the speed ratio is greater than 2½ to 1, the center distance should not be less than the sum of the sprocket diameters. Smoothness of drive and efficiency are increased, if the span of the chain on the tight side is an exact multiple of the pitch. The center distance should not be more than sixty times the pitch.

6. The center distance should be adjustable. Vertical or sharply inclined drives are permissible if the center distance is kept carefully

adjusted, but are not advisable otherwise. Idlers should be used for tightening the chain only as a last resort. They should be sprockets of sufficient size and tooth number, placed preferably between the two strands of the chain.

266. Sample Computation. Select a suitable roller chain to transmit 30 hp. from a motor running at 1750 r.p.m. to a line shaft running at 300 r.p.m.

For a first approximation to determine the maximum pitch, we will use formula (3):

$$P(\text{max}) = \left(\frac{900}{1750}\right)^{2/3} = 0.641 \text{ in.}$$

Let us select a 5/8 in. pitch and check the maximum permissible speed by formula (2). The weight per ft. for a single chain = 0.69 lb. The projected bearing area = 0.200 × 0.535 = 0.107 sq. in.

$$\text{Max. r.p.m.} = \frac{1920}{0.625}\sqrt{\frac{0.107}{0.69 \times 0.625}} = 1540 \text{ r.p.m.}$$

This speed is a little less than the motor speed of 1750. Actually, the maximum speed listed in the Handbook of the American Gear Manufacturers' Association is 1927 r.p.m., which is 25 per cent higher than the speed computed. The only reservation made is that if actual speed exceeds 80 per cent of the listed maximum speed, that is, if it exceeds the speed computed by the formula, the manufacturer should be consulted. We may therefore assume that we can proceed with the pitch selected.

With 15 teeth in the sprocket, we have $V = 15 \times 0.625 \times 1750/12 = 1360$ ft. per min. This speed exceeds 1000 ft. per min.; hence, according to A.G.M.A. directions, consultation with manufacturers is required. Assuming such consultation, we have, from formula (1),

$$F = \frac{2{,}600{,}000 \times 0.107}{1360 + 600} - \frac{0.69 \times 1360^2}{115{,}900} = 130 \text{ lb.}$$

The chain pull required is 30 × 33,000/1360 = 726 lb. or 5.6 times the pull computed for a single roller chain.

In this case, it would appear that if the widest available chain is only a triple roller chain, we would be required to use two triple chains side by side, or desist from using a roller chain altogether. It is apparent that the horsepower to be transmitted and the speed in this problem are not ideally suited for a roller-chain drive. It will be instructive to compute this same problem using a silent chain.

267. Computation of Silent Chain. As a guide to the computation of silent chains, the following tables were selected from a catalog of the Ramsey Chain Company, because, as compiled, they give a clear

understanding of the flexible and tentative nature of the values involved in chain design.

TABLE 42

SPEED AND HP. LIMITS FOR DESIGNING SILENT-CHAIN DRIVES †

A PITCH	B RANGE OF SPEEDS, R.P.M. DRIVER (SEE NOTE)	C RANGE OF VELOCITY FT. PER MIN. (SEE NOTE)	D RANGE OF HP.	E RANGE OF TENSION PER INCH WIDTH (LB.) (SEE NOTE)	F PINIONS, RANGE OF TOOTH NUMBERS	G SPROCKETS, RANGE OF TOOTH NUMBERS	H RANGE OF MINIMUM DESIRABLE CENTER DISTANCE
				* **			
3/8″	1150–3600	500–2400	0–15.5	48–100	15–29	30–210	Should
1/2″	850–3000	500–2600	3/4–28 3/4	63–135	15–29	30–204	equal
5/8″	650–2600	500–2800	2 1/2–50 3/4	80–165	15–29	30–192	sum of the
3/4″	585–2200	500–3000	3 1/4–80	90–225	15–29	30–186	diameters
1″	400–1800	500–3200	8–148	106–270	15–29	30–171	of both
1 1/2″	150–900	500–3200	22–420	175–485	15–29	30–171	sprockets

NOTE. Tension per inch width * indicates tension for very high foot speed travel of chain. Tension per inch width ** indicates tension for very low foot speed travel of chain.

Chains can be operated at speeds in excess of above table and for all such cases consult manufacturers before designing drive.

Range of hp., teeth, and minimum center distance can vary from above list, but should be approved by manufacturers before proceeding with design.

TABLE 43

STANDARD CHAIN WIDTHS ‡

PITCH	NOMINAL WIDTH OF CHAIN IN INCHES																								
3/8″	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	..	.	..	.	.	.	.	..	..	..	..	..	..	..	..
1/2″	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	5 1/2	6	.	.	.	..	..	..	..	..	..	..	..
5/8″	..	..	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	5 1/2	6	7	8	.	..	..	..	..	..	..	..	..
3/4″	..	..	1	..	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	5 1/2	6	7	8	9	10	..	..	..	..	..	..	..
1″	..	..	.	..	..	2	2 1/2	3	3 1/2	4	4 1/2	5	5 1/2	6	7	8	9	10	11	12	13	14	..	..	..
1 1/2″	..	..	.	..	..	.	..	3	..	4	..	5	..	6	7	8	9	10	..	12	..	14	16	18	20

It should be observed that for speed-up drives, the smaller sprocket should preferably have not less than 23 teeth and certainly not less than 21. For smooth driving, the span of the chain on the tight side should be an exact multiple of the pitch.

Returning now to our 30 hp. drive at 1750 r.p.m. motor speed, we notice in Table 42 that this motor speed is very nearly the normal for 5/8 in. pitch. Since the linear speed of this chain may run as high as 2800 ft. per min., we may assume as many as 21 teeth in the pinion. With a driven sprocket speed of 300 r.p.m. we would have $1750 \times 21/300 = 123$ teeth in the larger sprocket, which is within the permis-

† From catalog of Ramsey Chain Co., Albany, N. Y.
‡ *Ibid.*

sible range. It should be ascertained, however, whether or not the diameter of this sprocket is too large for the space allowable. The diameter is $123 \times 0.625/\pi = 24.42$ in., which does not seem prohibitive. The linear speed is $21 \times 0.625 \times 1750/12 = 1910$ ft. per min. The chain pull is $30 \times 33{,}000/1910 = 518$ lb. Selecting as permissible a pull per in. of width of about 120 lb., we find that the chain width is $518/120 = 4.3$ in. A 4½ in. chain would be applicable. It will be seen that a silent chain drive is very well suited for this speed and power.

268. Length of Chain. With reference to Fig. 322, the exact geometric length of a belt or chain stretched tightly over two pulleys

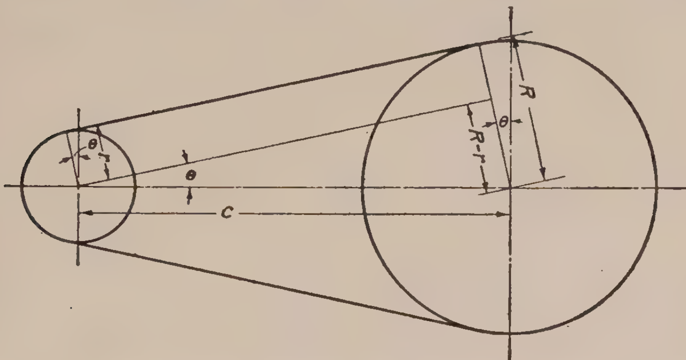


FIG. 322. LAYOUT FOR COMPUTING THE LENGTH OF ROPE OR CHAIN.

or sprockets of radii R and r and center distance C may be obtained by first computing the value of the angle θ from the relation

$$\sin \theta = (R - r)/C,$$

and then the belt length L from the expression

$$(5) \quad L = 2C \cos \theta + \pi(R + r) + 2\theta'(R - r),$$

in which θ' is merely θ expressed in radians, that is, it is $2\pi\theta/360$, where θ is in deg. It is customary in the computation of chain lengths to use the following approximate formula, which in general gives sufficiently accurate results:

$$(6) \quad L = 2C + \pi(R + r) + (R - r)^2/C.$$

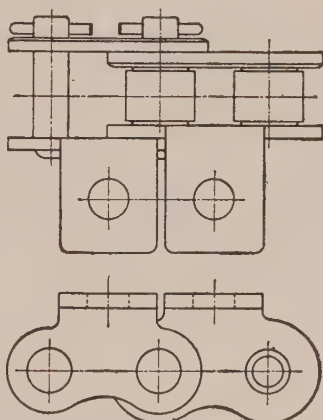


FIG. 323. ATTACHMENT FOR ROLLER CHAIN.

If the chain length is desired in terms of the pitch, the tooth numbers of the sprockets are used instead of the diameters. If the number of teeth on the larger

sprocket is N and on the smaller sprocket n, the formula becomes

$$L = 2C + P\left[\frac{N + n}{2} + \frac{0.0257(N - n)^2}{C}\right]. \tag{7}$$

In the preceding example of the silent chain, the length computed by the approximate formula (6) for a center distance of 53 in. is found to be 153 in., whereas formula (5) gives 152.8 in.

269. Conveyor Chains. The use of chains for conveyors is of great importance. For this purpose the chain links may be provided with lug extensions, as shown in Figs. 323 and 324. Buckets, scrapers,

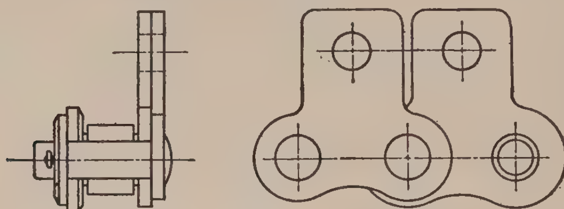


FIG. 324. ATTACHMENT FOR ROLLER CHAIN.

or other carrying or conveying devices may be attached to these extensions.

PROBLEMS

1. Select a roller chain to transmit 50 hp. at 1200 r.p.m. to a shaft at 275 r.p.m. Transmission is smooth and operation continuous. Determine the pitch, width, length, and number of teeth on the sprockets.

2. A roller chain is to transmit 15 hp. at 800 r.p.m. under conditions of intermittent operation with mild shock. Speed of driven shaft is 325 r.p.m. Select a chain and give necessary pitch, width, length, and number of teeth on sprockets.

3. A silent chain is used on the drive of a machine tool. The maximum horsepower is 15 at 1200 r.p.m. and is delivered to a shaft operating at 700 r.p.m. Select a suitable chain, giving all specifications.

4. Select a silent chain to transmit 120 hp. at 3000 r.p.m. under favorable conditions. Give necessary specifications.

CHAPTER 18

SPUR GEARS

270. Gearing. The word *gearing* may be used as a general term for all machine elements used in transmitting motion. If a constant velocity ratio and a positive drive are not essential, belts, ropes, or friction rolls may be used to transmit motion from one shaft to another. When a positive drive and a constant velocity ratio are required, chains or toothed wheels are usually employed. Often the word *gearing* is taken to cover transmission only by such wheels. In this chapter we shall deal only with gearing in which parallel shafts are connected by wheels having teeth parallel to the axes of the shafts. This type of gearing is known as *spur gearing.*

The essential purpose of gearing is to transmit angular motion in the same ratio as it would be transmitted by two rotating bodies maintaining contact with one another *without slipping.* The sole function of the teeth is to prevent slipping.

If the teeth were infinitesimally small, we would have friction gearing, the bodies transmitting motion from one to another simply by friction at the line of contact. The angular-velocity ratio of the friction wheels would be the same as that of the gears.

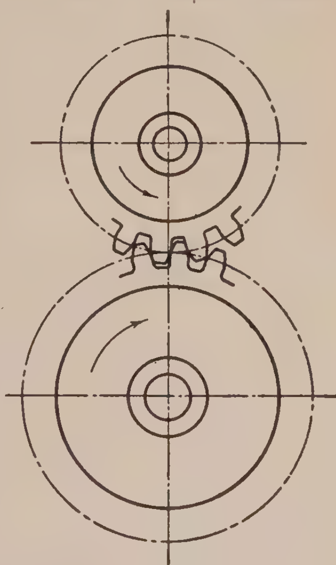


FIG. 325. GEAR-TOOTH ACTION.

271. Action of Gear Teeth. The action of gear teeth is usually dealt with in textbooks on kinematics, while the strength of the teeth and the power transmissible are dealt with in textbooks on machine design. Actually, gear-tooth shape, tooth action, and power transmissible are so closely related that it is desirable to consider all the phases of gear action as a unit.

The problem of gear-tooth action is so to shape a pair of tooth profiles (Fig. 325) that they will transmit motion as if by two cylinders rolling in contact without slipping (Fig. 326), that is, with the same circumferential velocity.

Let us consider the action of two bodies A and B rotating about fixed centers o_1 and o_2, in contact at c (Fig. 327). If A is turned counterclockwise, it will drive B clockwise. Through c, draw N_1N_2, the common normal to the two curved surfaces, which intersects the line of centers at p. Draw o_1a and o_2b perpendicular to this normal. Since the direction of motion of a point moving about a fixed center is perpendicular to its radius, the direction of motion of the point a about o_1 and b about o_2 is along the line N_1N_2. It is apparent that if the two engaging surfaces are to remain in contact, the linear velocities of a and b must be equal; otherwise the surfaces would separate, or penetrate each other. Therefore

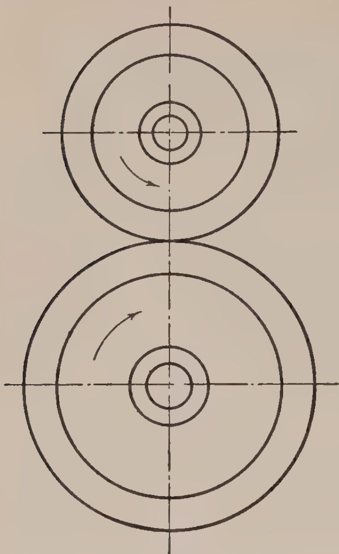


FIG. 326. ROLLING CYLINDERS.

$$\frac{\text{ang. vel. of } A}{\text{ang. vel. of } B} = \frac{\dfrac{\text{lin. vel. } a}{o_1a}}{\dfrac{\text{lin. vel. } b}{o_2b}} = \frac{o_2b}{o_1a}.$$

Also, by construction, triangles o_1ap and o_2bp are similar, from which

$$\frac{o_2b}{o_1a} = \frac{o_2p}{o_1p}.$$

Then

$$\frac{\text{ang. vel. of } A}{\text{ang. vel. of } B} = \frac{o_2p}{o_1p}.$$

Through p, with centers at o_1 and o_2, draw two circles A_1 and B_1. If these circles are in rolling contact, we have

$$\frac{\text{ang. vel. of } A_1}{\text{ang. vel. of } B_1} = \frac{o_2p}{o_1p}.$$

Therefore, the two bodies A and B with their surfaces engaged in sliding contact at c have the same angular-velocity ratio at this instant as the two circles A_1 and B_1 in rolling contact at p.

If two gear teeth are to transmit motion in the same ratio as two bodies rotating in contact without slipping, the common normal to the tooth profiles at their point of contact must pass through the point of tangency of the rotating bodies. If there is no slipping, the point of tangency,

which is called the *pitch point*, is always located on the line joining the centers of rotation of the two bodies.

The point of tangency is a fixed point if the velocity ratio is constant. The rotating bodies are then circular in section, and the circles determining the transmission ratio are called *pitch circles*. If the bodies are not circular in section, the pitch point moves along the line of centers. It is always located on this line, however, unless there is slipping. A pair of teeth engaging to give correct gear action are called *conjugate* teeth.

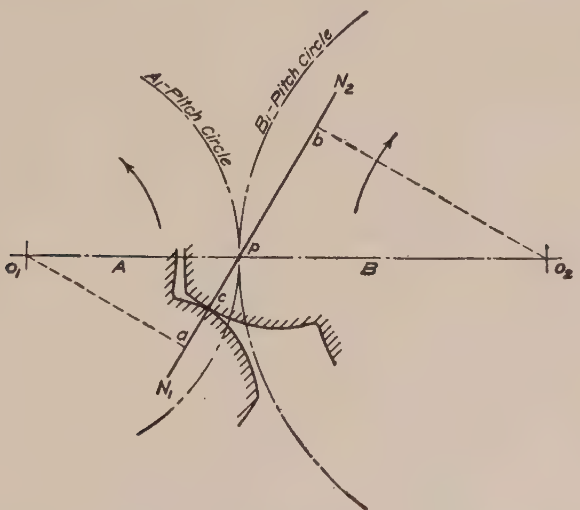


FIG. 327. TWO BODIES IN CONTACT ROTATING ABOUT FIXED CENTERS.

272. Gear-Tooth Forms. With certain limitations, the tooth profile of one gear can be chosen arbitrarily, and by a graphical construction, the profile of the conjugate tooth of the mating gear found. During the action of engaging tooth profiles, the point of contact describes a line in space called the *line of action* or *path of contact* (Fig. 328). A definite relation exists between a gear-tooth profile and its path of contact. If either one is given, then the other is fixed.

From a theoretical point of view, and with certain restrictions, any path of contact can be chosen. However, only certain tooth profiles have desirable characteristics and are practicable. For instance, if the angle between the normal at the point of contact of the tooth curves and the line of centers is comparatively small (Fig. 327), the tangential component or effective driving force will be small, while the component of force along the line of centers, tending to spread the gears apart, will be large. Another consideration which limits the choice of

profiles is the shape and strength of the gear teeth. If motion is to be transmitted in both directions, the teeth must be formed with the

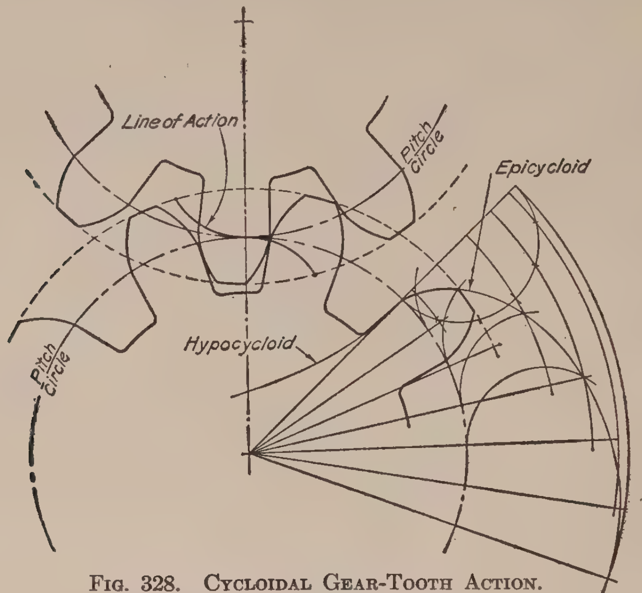


FIG. 328. CYCLOIDAL GEAR-TOOTH ACTION.

proper curves on both sides, and yet the section at the root of the tooth must not be unduly weak.

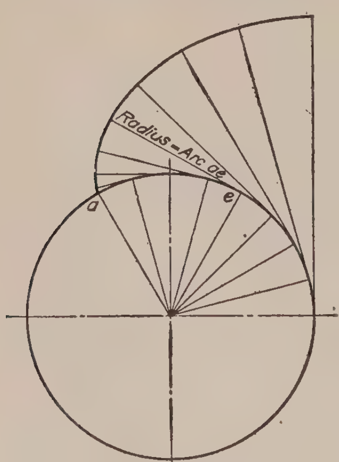


FIG. 329. GENERATION OF INVOLUTE CURVE.

In practice, only two systems of gear-tooth forms have been generally used, namely, the *cycloidal* and the *involute*.

273. Cycloidal Teeth. The cycloidal tooth form was one of the first regular curves used in gearing. It gives a comparatively wide distribution of tooth contact, which reduces wear, and it can be used without interference for a gear having a comparatively small number of teeth.

The line of action, as shown in Fig. 328, consists of two circular arcs tangent to each other at the pitch point of the gears. The tooth profiles are composed of the cycloidal curves, described by a tracing point on a circle rolling on the outside and on the inside of the pitch circles of the gears.

On account of the difficulty and cost of producing accurately formed teeth, purely cycloidal teeth are now practically obsolete in commercial gearing. A combination of the cycloidal curve and the involute curve, however, forms what is known as the *14½ deg. composite system* of gearing. This tooth form will be discussed under the involute system.

Impellers of cycloidal profile are extensively used in pumps and blowers. Their action of engagement is the same as conjugate gear-tooth action.

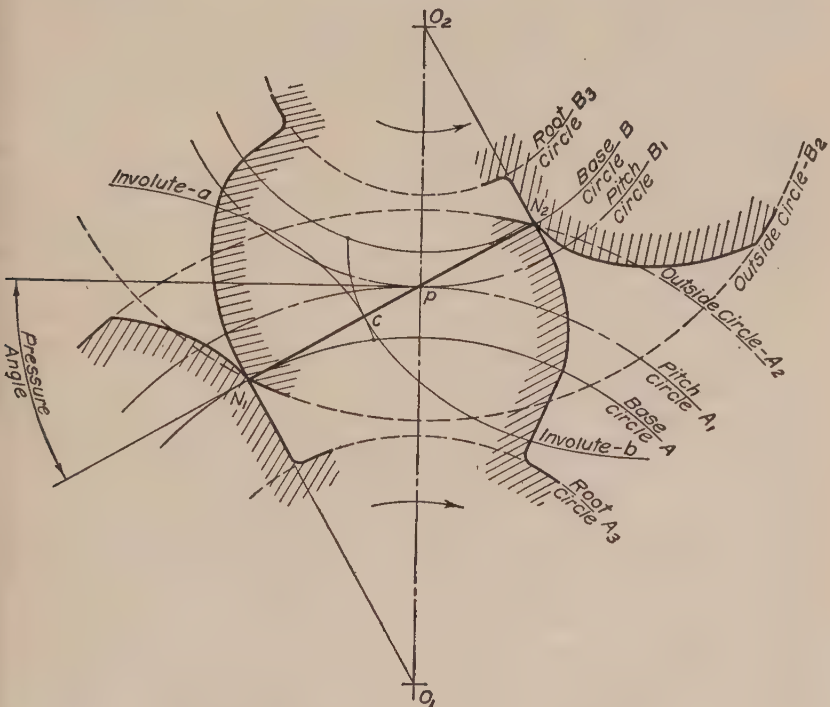


FIG. 330. INVOLUTE GEAR-TOOTH ACTION.

274. Involute Gear-Tooth Action. The curve used almost exclusively for gear-tooth outlines today is the *involute*. It has desirable properties and characteristics, particularly from the manufacturing point of view.

The *involute* is a curve traced by a point on a string as it is unwound from the circumference of a circle, commonly known as the *base circle* (Fig. 329). In its application to gear-tooth profiles, let us consider two base circles A and B with centers at o_1 and o_2 (Fig. 330). A common tangent N_1N_2 connecting these circles will intersect the line

of centers at p. Suppose this tangent to be a string wrapped around and fastened to each circle. With the string in tension, if circle B is turned in the direction of the arrow, the circle A will revolve as shown.

Now if the string is cut at any point c on the line N_1N_2, an involute a will be traced by c as the string is unwound from circle A, and involute b will be traced by c as the string is unwound from circle B.

275. Line of Action. These two involutes just discussed will be in contact with each other at point c and N_1N_2 will be their common normal. During rotation, the point c moves along N_1N_2; hence it is evident that both involutes traced by c will be in continuous contact from N_1 to N_2. This straight line is therefore the *line of action* and *path of contact* of the involute curves.

The rectilinear path of contact is one of the unique properties of the involute in its application to gear-tooth forms. The *circles* A_2 and B_2, drawn through the points of tangency N_1 and N_2 of the line of action with the base circles, will be the top circles for the teeth if the full length of the line of involute action is utilized.

With tooth contact extending over the full length of the line of action, as shown in the figure, the circle B_2 extends some distance within base circle A, and A_2 extends within base circle B. Since involute action cannot extend below the base circles of the involutes, the profiles of the teeth below these circles must be so formed that they clear each other during rotation. This part of the profiles is usually formed by straight radial lines ending with fillets for added strength, but other forms, which will not cause interference between mating teeth, may be used. The bottoms of the tooth spaces form the root circles A_3 and B_3.

276. Angular-Velocity Ratio. Since N_1N_2, the common normal to the tooth curves, always passes through the point p, circles A_1 and B_1 drawn through this point will be the *pitch circles* of the gears, and point p the *pitch point.* From the similarity of triangles o_1pN_1 and o_2pN_2,

$$\frac{o_1p}{o_2p} = \frac{o_1N_1}{o_2N_2}.$$

The angular-velocity ratio of the *pitch circles* is therefore equal to the angular-velocity ratio of the *base circles.* The power transmission of involute gearing proceeds as if the base circles were a pair of pulleys with the line of action corresponding to a portion of a crossed belt transmitting motion from one pulley to the other.

277. Pressure Angle. The angle between the line of action N_1N_2 and a common tangent to the pitch circles is called the *pressure angle*

(Fig. 330). If we change the center distance between the base circles, the pressure angle will change, but the form of the curves will remain the same. The form of an involute depends only upon the size of its base circle; hence the angular-velocity ratio will remain constant. This characteristic is an important property of the involute curve. If, through faulty workmanship or wear of journal bearings, the correct center distance is not maintained, the gear teeth will nevertheless transmit uniform motion with the required angular-velocity ratio.

278. Angle of Action. In order to transmit motion uniformly and smoothly, the involute profiles must be so selected that there is an overlapping of tooth action, that is, one tooth should not pass out of contact with its mating tooth until another pair of teeth are properly engaged.

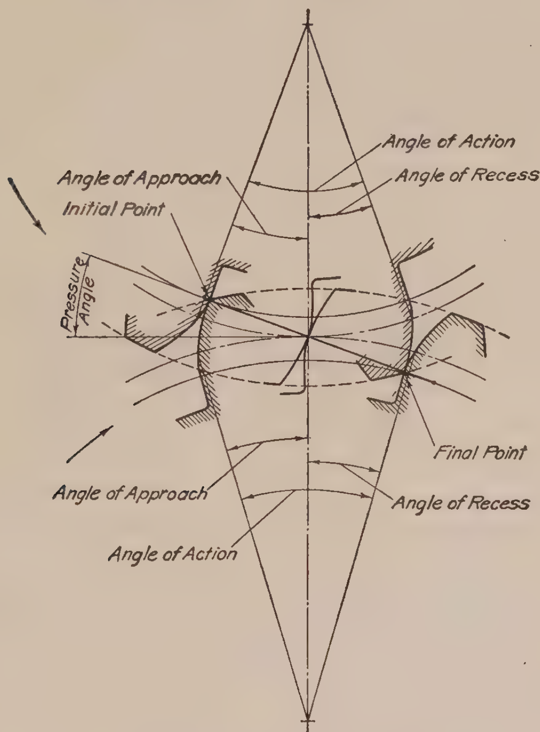


FIG. 331. ANGLE OF ACTION.

The angle through which one tooth travels from the time it first makes contact with a mating tooth until contact ceases is called the *angle of action* (Fig. 331). It is divided into the angle of approach and the angle of recess. The *angle of approach* is the angle through

which a tooth moves from the time it first makes contact with a mating tooth until it is in contact at the pitch point. The *angle of recess* is the angle through which a tooth moves from the time it is in contact with a mating tooth at the pitch point until contact ceases. The number of teeth in contact is found by dividing the angle of action by the angle between two successive teeth.

According to Earle Buckingham,* a contact as low as 1.20 tooth intervals is used in extreme cases, but such short contact requires extreme accuracy in the gears for smooth and quiet running. For any appreciable amount of power, 1.40 tooth intervals or more should be secured, if possible.

279. Sliding Action of Gear Teeth. The action between meshing gear teeth consists of a combination of rolling and sliding. The *rate of sliding* is greatest when contact starts, decreasing gradually until the pitch point is reached, where it ceases for an instant, then changes direction, gradually increasing again until contact ends. This variation in the rate of sliding is due to the fact that unequal parts of the tooth profiles come into contact for equal angles of rotation.

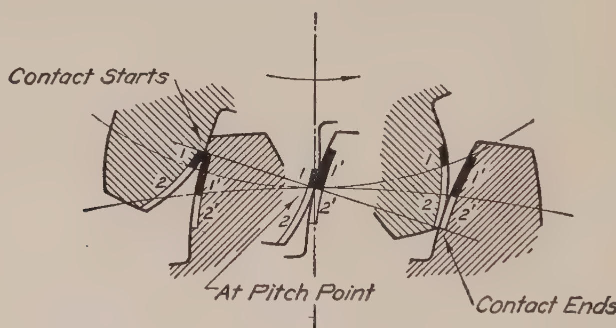


FIG. 332. SLIDING ACTION BETWEEN GEAR TEETH.

The action between two teeth is illustrated in Fig. 332. The portions of the profiles which work together are similarly numbered, such as 1 and 1^1, 2 and 2^1. There would be no sliding and pure rolling motion would exist, if these portions were of equal length. The difference in length between 1 and 1^1, 2 and 2^1 represents the amount of sliding. Sliding leads to friction, wear, and noise.

280. Definitions of Gear-Tooth Parts. A knowledge of the terms commonly applied to gear-tooth parts is necessary for a discussion of standard gear-tooth systems. These terms are illustrated in Fig. 333.

* *Spur Gears*, McGraw-Hill Book Co., 1928, p. 113.

Addendum is the radial distance between the pitch circle and the top of the tooth.

Dedendum is the radial distance between the pitch circle and the bottom of the tooth space.

Clearance is the radial distance between the top of a tooth and the bottom of the mating tooth space.

Whole depth is the radial distance from the top of the tooth to the bottom of the tooth space.

Working depth is the greatest depth to which a tooth of one gear extends into the tooth space of a mating gear.

Tooth face is the surface between the pitch circle and the top of the tooth.

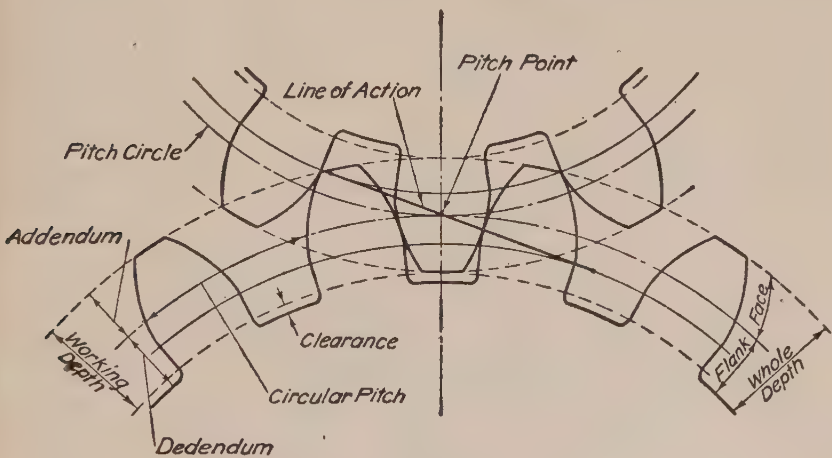


FIG. 333. GEAR-TOOTH PARTS.

Tooth flank is the surface between the pitch circle and the bottom of the tooth space. It includes the fillet.

Circular pitch is the distance on the circumference of the pitch circle between corresponding points of adjacent teeth. It is equal to the circumference of the pitch circle divided by the number of teeth. If the pitch diameter is D and the number of teeth N, the circular pitch $p_c = \pi D/N$.

Diametral pitch is the number of teeth per inch of pitch diameter. With the notation just used, the diametral pitch $p_d = N/D$. Hence we have $p_c = \pi/p_d$, or $p_d = \pi/p_c$.

281. Involute Rack. Of two gears mating together, the smaller is called the *pinion*, and the larger, the *gear*. A *rack*, theoretically, is a segment of a gear of infinite diameter (Fig. 334). Obviously the

pitch and base circles then become straight lines. The profile of a rack tooth is an involute generated from a base circle of infinite radius and becomes a straight line perpendicular to the line of action. This property of the involute is of great importance in connection with various methods used in cutting gear teeth and will be discussed later in more detail.

282. Interference. Interference is contact between mating teeth at some point other than along the line of correct tooth action. Interference never occurs as long as conjugate involute curves are in contact.

It was shown in Fig. 330 that true involute gear-tooth action extends along the line of contact only to the points of tangency of the line with the base circles.

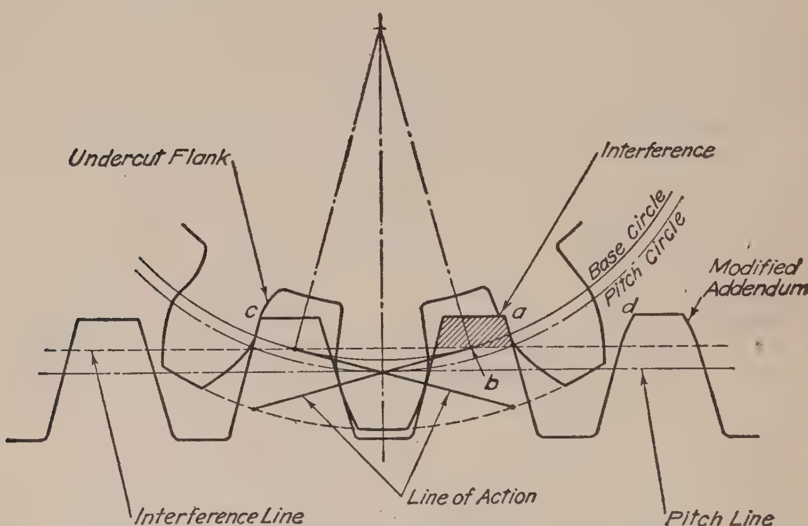


FIG. 334. INTERFERENCE BETWEEN PINION AND RACK.

In Fig. 334, a pinion of 12 teeth is shown in mesh with a rack. Involute action cannot extend inside the base circle of the pinion, and with the tooth height shown, the rack tooth will interfere with the radial flank of the pinion tooth, as shown at a. If the addendum of the rack tooth did not extend beyond b, that is, if the cross-sectioned portion were removed, this interference would not occur. In view of the fact that interference begins at the points of tangency of the line of action with the base circles, these points are called *interference points*.

Instead of shortening the tooth, interference may be avoided by undercutting and relieving the flanks of the pinion tooth, as shown

at *c*. This method, however, is less advisable because it weakens the tooth at the base where the bending stress is greatest.

Since it is customary to make the flanks of gear teeth radial below the base circle, interference could also be avoided by trimming off both sides of the rack tooth at the corners, from the point of interference to the top of the tooth, as shown at *d*. This is the method generally adopted in practice; it will be explained later in more detail.

283. Standardization of Gear Teeth. It is often difficult or impossible to design gear teeth so that all elements—the pressure angle, the angle of action, freedom from sliding and interference, ease of manufacture, etc.—will be attained to the full measure desired. The tooth forms found in practice are therefore, as a rule, compromises.

In the earlier days, gears were generally cast and had teeth of the cycloidal form. With the subsequent development of the cut-tooth gear, the involute system gained preference, and our present standardized gear-tooth forms are in the main based on the involute curve. In some cases, however, this curve is modified to avoid interference.

There are in use today a number of standard spur gear-tooth systems. In most of these systems, it is required that all gears must be capable of meshing with a standard rack, and the system is characterized by the shape of the basic rack-tooth form. In 1932 the American Standards Association, sponsored by the American Gear Manufacturers' Association and the American Society of Mechanical Engineers, approved the forms described below.

284. 14½ Deg. Composite System. This system was introduced in the early days when gear teeth were cut principally by means of formed milling cutters.* It has been widely known as the "Standard 14½ Deg. Involute System," yet the tooth profile is actually a combination of cycloidal curves and the involute curve. The basic rack for the 14½ deg. composite system is shown in Fig. 335.

All gears in this system are interchangeable; that is, they will run with one another if they are of the same pitch and of standard proportions. The system is based on a 12-tooth pinion as the smallest that will give satisfactory tooth action.

It has already been shown in the article on *interference* that a 12-tooth pinion in a system with full involute-tooth action will have the flanks of its teeth badly undercut. This is especially the case with a small pressure angle of 14½ degrees. For this reason, the involute is modified in the composite system. Only the parts adjacent to the

* When gears are cut with formed milling cutters, a set of eight cutters is required for each pitch. These cutters are designed to cut tooth forms approximately correct from a pinion of 12 teeth to a rack.

pitch circle are of involute shape. The extreme parts both of the face (addendum) and of the flank (dedendum) theoretically are given a cycloidal shape.

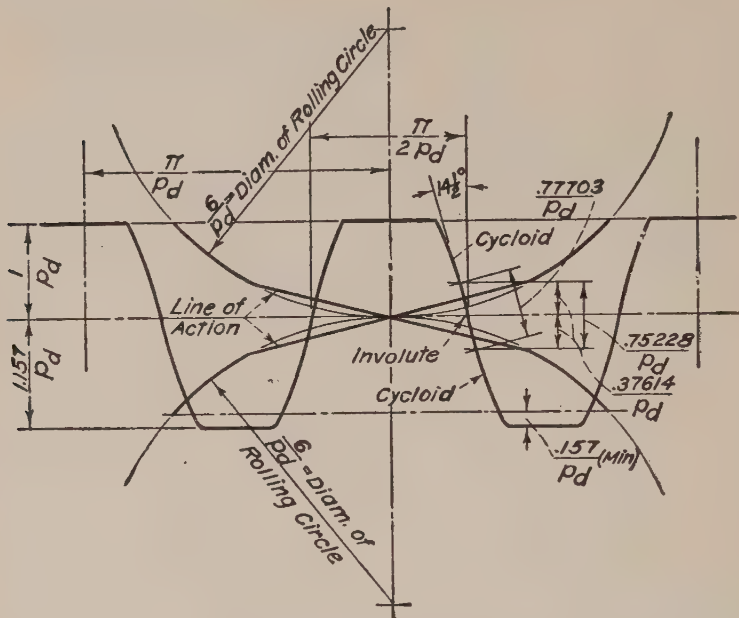


FIG. 335. BASIC RACK OUTLINE OF 14½ DEG. COMPOSITE SYSTEM, INVOLUTE GEAR-TOOTH FORM.

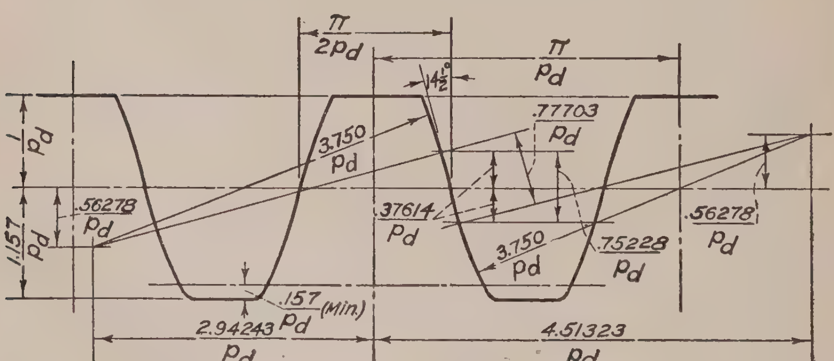


FIG. 336. APPROXIMATE METHOD OF DETERMINING THE BASIC RACK OUTLINE OF 14½ DEG. COMPOSITE SYSTEM BY MEANS OF CIRCULAR ARCS.

Since the cycloidal form is difficult to produce accurately, it is customary in practice to approximate it on the basic rack by the use

of circular arcs, as shown in Fig. 336. The tooth form has the following proportions:

TABLE 44
14½ Deg. Composite System

	In Terms of Diametral Pitch (in.)	In Terms of Circular Pitch (in.)
Addendum	$\frac{1}{p_d}$	$0.3183 \times p_c$
Minimum dedendum	$\frac{1.157}{p_d}$	$0.3683 \times p_c$
Working depth	$\frac{2}{p_d}$	$0.6366 \times p_c$
Minimum total depth	$\frac{2.157}{p_d}$	$0.6866 \times p_c$
Minimum clearance	$\frac{0.157}{p_d}$	$0.05 \times p_c$
Pitch diam.	$\frac{N}{p_d}$	$0.3183 \times N \times p_c$
Outside diam.	$\frac{N + 2}{p_d}$	$0.3183 \times (N + 2) \times p_c$

N = number of teeth. p_d = diametral pitch. p_c = circular pitch.

285. 14½ Deg. Full-Depth Involute System. Several new gear-tooth systems were established with the introduction of the generating or molding processes of forming gear teeth. These processes will be described in some detail later. In any of these methods, a gear tooth

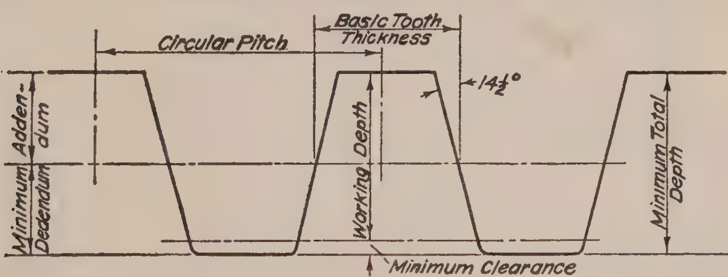


Fig. 337. Basic Rack Outline of 14½ Deg. Full-Depth Involute System.

is generated by a cutter of basic rack form, or else the cutter is generated from the basic rack and this generated cutter in turn generates the tooth. The simplicity of the straight-sided involute rack and the accuracy attainable in reproducing this form on cutters led to the adoption of the 14½ deg. full involute system, the basic rack of which is shown in Fig. 337.

The tooth proportions given in Table 44 apply also to full depth 14½ deg. teeth. However, the gears of both systems are not interchangeable. In the 14½ deg. full involute system, gears with small tooth numbers are excessively undercut. When there are enough teeth in the gears to avoid this condition, very satisfactory results are obtained. With standard proportions, 32 teeth is the smallest number for full involute action with a rack.

To obtain full involute action when a pinion with less than 32 teeth contacts with a 14½ deg. basic rack, the center distance and the tooth height of the pinion must be increased, or else the height of the rack tooth decreased.* The same condition is true, to a smaller degree, if gears are substituted for the rack.

For drives involving large tooth numbers, gears with 40 teeth or more, the full-depth tooth with low pressure angle is known to be quite satisfactory.

286. 20 Deg. Full-Depth Involute System. This system has the same proportions as the 14½ deg. involute, except that the pressure

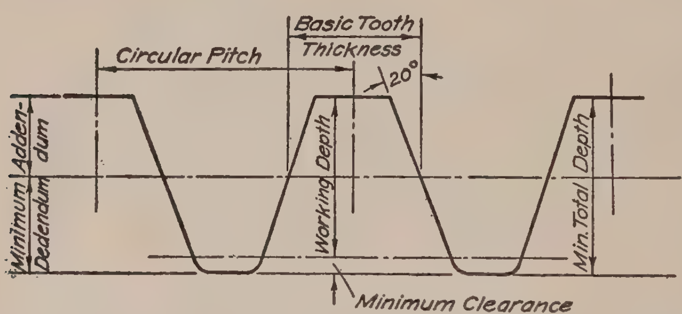


Fig. 338. Basic Rack Outline of 20 Deg. Full-Depth Involute System.

angle has been increased to 20 deg. (Fig. 338). With a larger pressure angle, there is less interference and undercutting. In this system, an 18-tooth pinion is the smallest for full involute action with a rack. The 20 deg. tooth is thicker at the base than the 14½ deg. and is therefore stronger.

287. 20 Deg. Stub Involute System. The basic rack of the 20 deg. stub tooth approved by the American Standards Association is shown in Fig. 339. The tooth form has the following proportions:

* See pamphlet of American Standards Association on *Spur Gear Tooth Form*, B6.1, 1932.

TABLE 45

20 Deg. Stub Tooth

	In Terms of Diametral Pitch (in.)	In Terms of Circular Pitch (in.)
Addendum	$\frac{0.8}{p_d}$	$0.2546 \times p_c$
Minimum dedendum	$\frac{1}{p_d}$	$0.3183 \times p_c$
Working depth	$\frac{1.6}{p_d}$	$0.5092 \times p_c$
Minimum total depth	$\frac{1.8}{p_d}$	$0.5729 \times p_c$
Minimum clearance	$\frac{0.2}{p_d}$	$0.0637 \times p_c$
Pitch diam.	$\frac{N}{p_d}$	$0.3183 \times N \times p_c$
Outside diam.	$\frac{N + 1.6}{p_d}$	D + (2 addendums)

N = number of teeth. p_d = diametral pitch. D = pitch diameter. p_c = circular pitch.

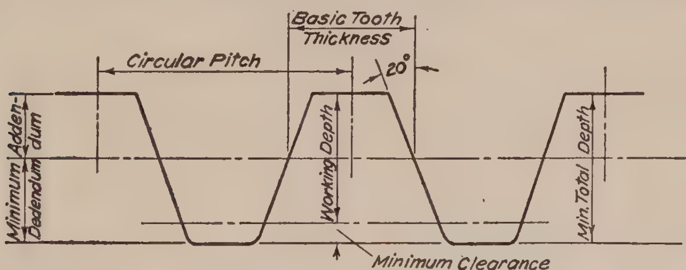


Fig. 339. Basic Rack Outline of 20 Deg. Stub Involute System.

The stub-tooth system was developed to meet the need for a more satisfactory gear of small tooth numbers, one free from undercut and of greater strength. In this system, the pressure angle is increased from 14½ to 20 deg. and the tooth is made shorter than the full-depth tooth. Pinions of 12 and 13 teeth are slightly undercut, although not enough to have any appreciable effect on their tooth action.

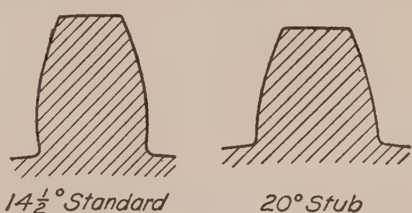


Fig. 340. Comparison between 14½ Deg. Composite Involute Tooth and 20 Deg. Stub Involute Tooth.

The effect of these changes on the tooth form is readily appreciated when a 20 deg. stub tooth is compared with a 14½ deg. standard tooth of the same pitch, as in Fig. 340. The 20 deg. pressure angle pro-

duces a tooth which is strong at the base; and, since the reduced tooth height decreases the sliding action between the teeth, there is less wear. The angle of action, however, is less; and, with fewer teeth in contact, it is necessary to cut the gears accurately for quiet operation.

The stub-tooth system has been extensively used in the automotive industry where strength and gears with small tooth numbers are essential. Commercially, it is a cheaper tooth to cut than a full height tooth because there is less metal to remove. It is also used for industrial purposes, especially for gear reducers, where the increased strength of the stub tooth is of considerable importance.

288. Other Gear-Tooth Forms. In addition to the recently approved standardized gear-tooth forms already described, various manufacturers in the early development of gear-cutting practice established their own tooth proportions.

The Fellows Gear Shaper Company, in 1899, developed the original stub-tooth gear and their system is still extensively used, especially in the automobile industry. It is based on a pressure angle of 20 deg., and the tooth proportions are determined from two diametral pitches. The pitch of the teeth is designated as 4/5, 6/8, etc. The numerator of the fraction indicates the diametral pitch used in determining the thickness of tooth, the number of teeth, and the pitch diameter; whereas the denominator indicates the dimensions for the addendum and dedendum. That is, a 4/5 pitch tooth has a diametral pitch of 4, and an addendum height equal to 1/5 in., while a full-depth tooth would have an addendum height of 1/4 in.

The R. D. Nuttall Company originated a 20 deg. stub-tooth gear system in which the tooth dimensions are based upon the circular pitch. The addendum is made 0.250 $\times$ the circular pitch, and the dedendum, 0.300 $\times$ the circular pitch.

There is correct tooth action between gears cut to these systems and those cut to the A.S.A. 20 deg. stub system, the only dimensions affected being the clearance.

289. Standard Pitches. In Fig. 341 gear teeth of different diametral pitches are shown drawn to scale. It is customary and convenient to specify the size of the teeth of cut gears by the diametral pitch. However, for large cut teeth of 3 in. circular pitch or more, the circular pitch is usually designated. The standard diametral pitches commonly used in practice are as follows: 1 to 2 varying by 1/4 pitch, 2 to 4 by 1/2 pitch, 4 to 10 by 1 pitch, 10 to 20 by 2 pitch, and 20 to 40 by 4 pitch. Other cutters may be obtained, but they should be avoided unless they are absolutely necessary.

Cast tooth gears are generally based on the circular pitch as a convenience to the patternmaker. The standard circular pitches commonly used are: 1/2 in. to 1½ in. varying by 1/8 in., 1½ in. to 3 in. by 1/4 in., and 3 in. to 4 in. by 1/2 in.

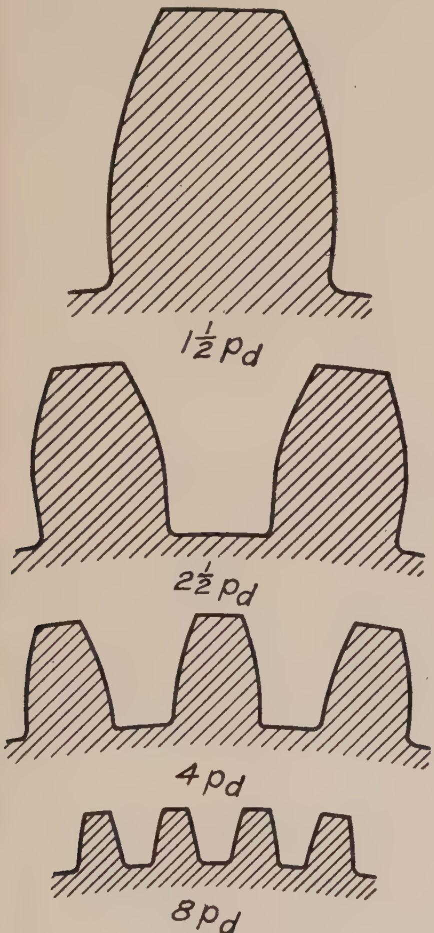


Fig. 341. Comparison of Standard Diametral Pitches Drawn to Scale.

290. Unequal Addendum Teeth. By reference to Fig. 334, it can be seen that with equal addendum on a regular involute rack and pinion, the tip of the rack tooth may foul the flank of the pinion, while the tip of the pinion does not interfere with the flank of the rack tooth. To avoid interference, therefore, we may reduce the addendum of the rack (or larger gear) and still maintain or even extend the

addendum of the pinion. Incidentally, this has the effect of decreasing the angle of approach, in which sliding is greater, and of increasing the angle of recess, where sliding is less.

Maag gears,* developed in Switzerland, are designed on the basis of this principle. They are true involute gears in which the relative addendum heights, as well as the pressure angle, vary so as to obtain the best possible combination for every pair of gears. There is no interference or undercutting and the teeth are very strong at the base. In Fig. 342, a comparison is made between a pair of standard 14½ deg.

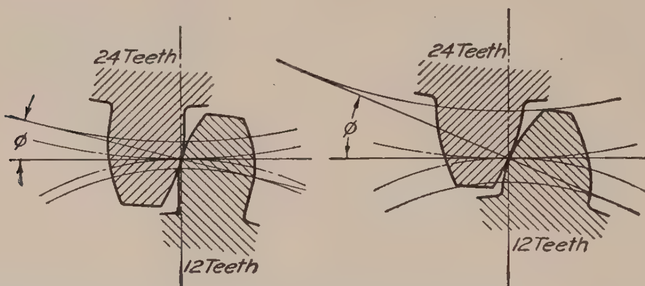


Fig. 342. Comparison of 14½ Deg. Composite Involute Teeth and Maag (Unequal-Addendum) Teeth.

composite gears of 12 and 24 teeth, respectively, with a similar pair of gears cut by the Maag system. Pinions may be produced with as few as 5 teeth for special applications.

It is obvious from the foregoing remarks that in the Maag system, one standard cutter cannot be used to generate a whole series of gears, as in the standard systems, and the gears of various transmission ratios are not interchangeable. This has limited the use of this tooth system, even though it has excellent properties.

291. Internal Gear. A ring with teeth on the inside is called an *internal* or *annular* gear. The pitch circles of the gear and its pinion are now tangent internally, as shown in Fig. 343. The teeth of an internal gear point toward the center with the addendum and dedendum in a reverse position from that of an external gear. Since the curvature of both pitch circles is in the same direction, there is a longer arc of action; therefore there are more teeth in contact.

Referring to Fig. 343, it is evident that if interference with the flanks of the pinion teeth is to be avoided, the addendum of an internal gear should not extend beyond a, the point of tangency of the base

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circle of the pinion and the line of action. In practice, the portion of the gear addendum extending inside this point of interference, as shown in Fig. 343, is trimmed off and relieved in the cutting operation; or the internal diameter of the gear is made equal to a diameter which passes through point *a*.

If the pinion approaches the internal gear in size, the pinion teeth may not be able to move out of the tooth spaces without interference. If this condition is suspected, a layout should be made. According to the Fellows Gear Shaper Company, the smallest permissible difference between the number of teeth in an internal gear and its pinion, to give proper tooth action without considerable modification of tooth shape, is 7 teeth for 20 deg. stub-tooth form, and 12 teeth for full-length 14½ deg. involute form.

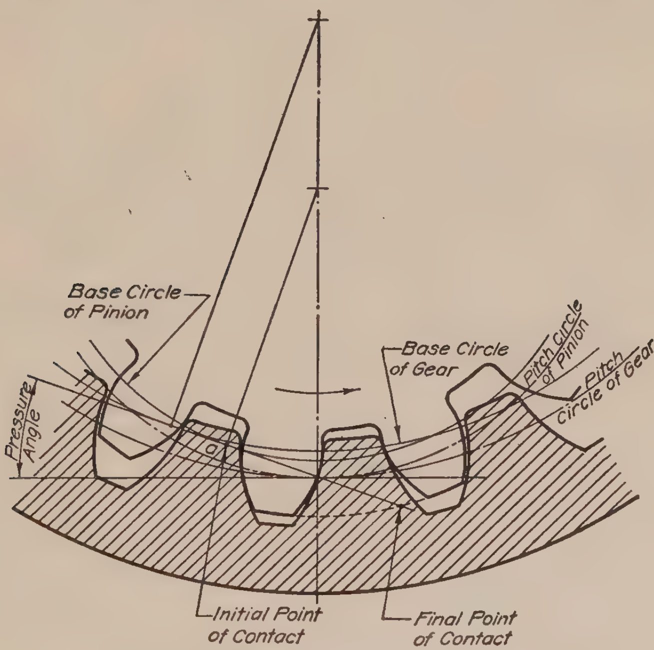
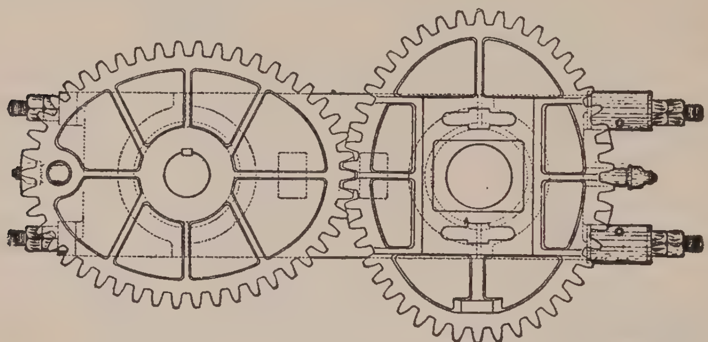


FIG. 343. INTERNAL GEAR.

An internal gear, properly applied, has a number of desirable characteristics. The tooth form is much stronger at the base than it would be on external gears. There is less sliding action, less wear, higher efficiency, more teeth in contact, and smoother operation. A more compact design is also possible due to a shorter center distance than with external gears of the same size.

Internal gears are used quite extensively for the rear-axle drives of trucks and tractors, and in planetary-gear combinations for increasing or reducing speed.

292. Non-Circular Gears. Occasionally it is necessary to transmit a variable motion from one shaft to another by means of gear wheels. With certain limitations, it is possible to select a cycle of velocity ratios, and from this cycle, to determine the required pitch lines and tooth profiles of the mating gears. The most common arrangement consists of two equal elliptical spur gears, each gear rotating about one of its foci. The distance between the centers of rotation is equal to the major axis of either ellipse.



The National Supply Co., Toledo, Ohio

FIG. 344. ELLIPTICAL GEARS.

Figure 344 shows an interesting application of a pair of elliptical gears used in an oil-well drilling attachment. In this device, the gears rotate about their centers and not about their foci. One of the gears is mounted on the crankshaft, while the other is bolted to the pitman. A frame carrying the bearings connects the two gears.

Rotation of the crankshaft gear causes the pitman gear to travel around it at one-half crankshaft speed. This mechanism gives a modified motion, the acceleration being decreased at the beginning and end of the stroke.

293. Cast-Tooth Gears. Cast-tooth gears are frequently used in slow-speed machinery, such as agricultural implements, crushers, mills, conveyors, etc. They are generally made of cast iron, malleable iron, or cast steel, from gear patterns, or by the machine molding process (see page 410). Machine-molded gears are more accurate than pattern-molded gears and can be run at higher speeds. The pitch-line speed of cast-tooth gears made from patterns should not exceed 600 ft. per

min., whereas the pitch-line speed of machine-molded gears may be as high as 900 ft. per min.

294. Selection of Pitch. The selection of the pitch of a gear to operate under given conditions is governed by (a) the magnitude and the character of the load to be transmitted, (b) requirements for resistance to wear, (c) velocity ratio, if an exact center distance must be maintained, and (d) smooth operation. Usually the first two factors determine the pitch.

295. Strength of Gear Teeth. When a driven tooth is first coming into contact, as in Fig. 345, the total load W acting at the pressure

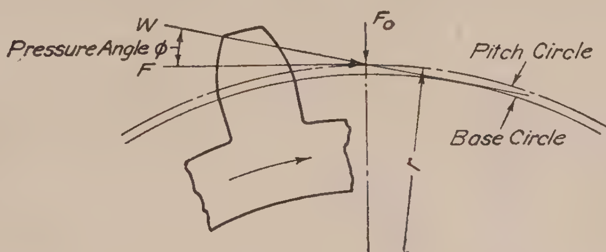


FIG. 345. FORCES ACTING ON GEAR TOOTH.

angle ϕ may be carried at the tip of the tooth. If this load is resolved into two components, a tangential force F and a radial force F_0 at the pitch line of the gear, the resultant torque on the gear is Fr, where r is the pitch radius in in. The tangential component on the tip of the tooth is less than F but F is used as the bending force on the tooth for convenience and safety.

It will be noted that the gear tooth is loaded as a cantilever beam and must sustain a bending stress, a direct compressive stress, and a direct shear stress. The shear stress is negligible and the direct compressive is less than 10 per cent of the bending stress, hence it also may be neglected. The bending-stress relationship has been derived by Lewis in the following manner.

The total load is assumed to come on one tooth at the tip (Fig. 346), normal to the tooth profile. Where this force intersects the center line of the tooth it is divided into vertical and horizontal components, the vertical component being hereafter neglected. The tooth is considered as a cantilever beam of variable section with the maximum bending stress occurring at the point where a parabola, drawn through the point of loading and representing a beam of uniform strength, is tangent to the tooth outline.

Thus the weakest section is located at d, and all material outside of the parabola simply contributes excess strength at other sections

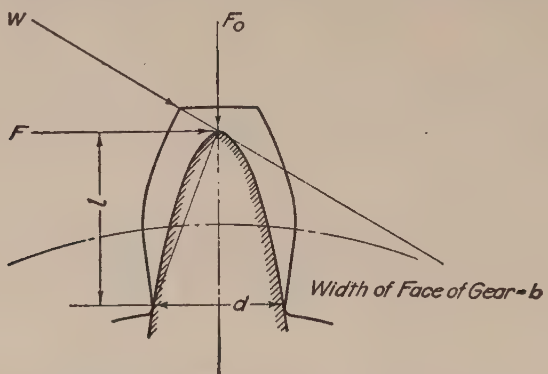


Fig. 346. Forces Acting on Gear Tooth.

of the tooth. Setting the external bending moment equal to the internal resisting moment, we have

$$Fl = \frac{bd^2}{6}s, \qquad F = bs\frac{d^2}{6l},$$

where F is the tangential force in lb., b is the width of the gear face in in., and s is the allowable stress in p.s.i. Since d and l are variables which depend upon the shape of the tooth and the circular pitch p_c in in., the quantity $d^2/(6l)$ is replaced by $p_c y$, where y is a numerical factor called the *form factor* of the tooth. Its value is independent of the pitch, and for a given tooth system varies only with the number of teeth. The final form of the strength formula is then

$$(1) \qquad F = sbp_c y, \qquad \text{or} \qquad F = \frac{\pi sby}{p_d},$$

where p_d is the diametral pitch. The numerical values of y are obtained by direct measurement from enlarged drawings. Values for three standard tooth forms are listed in Table 46.

The Lewis formula is based upon certain assumptions which are safe. It neglects, however, the concentration of stress at the root of the tooth, which may, under certain conditions, be over twice the calculated stress. By increasing the fillet radius at the root of the tooth, this stress concentration is reduced.

The simple formula given above is convenient to use when the pitch diameter of the gear is known and the force F readily obtainable. When the number of teeth is known, the pitch diameter may be ex-

TABLE 46

STRENGTH FORM FACTOR, y (BUCKINGHAM)

NUMBER OF TEETH	14½° INVOLUTE COMPOSITE	20° FULL-DEPTH INVOLUTE	20° STUB INVOLUTE
12	0.067	0.078	0.099
13	0.071	0.083	0.103
14	0.075	0.088	0.108
15	0.078	0.092	0.111
16	0.081	0.094	0.115
17	0.084	0.096	0.117
18	0.086	0.098	0.120
19	0.088	0.100	0.123
20	0.090	0.102	0.125
21	0.092	0.104	0.127
23	0.094	0.106	0.130
25	0.097	0.108	0.133
27	0.099	0.111	0.136
30	0.101	0.114	0.139
34	0.104	0.118	0.142
38	0.106	0.122	0.145
43	0.108	0.126	0.147
50	0.110	0.130	0.151
60	0.113	0.134	0.154
75	0.115	0.138	0.158
100	0.117	0.142	0.161
150	0.119	0.146	0.165
300	0.122	0.150	0.170
Rack	0.124	0.154	0.175

pressed as N/p_d, where N is the number of teeth. Since the torque on the gear shaft is equal to Fr, and the pitch radius $r = N/(2p_d)$, by multiplying both sides of equation (1) by r and substituting, we have

$$(2) \qquad M_t = \frac{sNky\pi^2}{2p_d^3}, \qquad \text{or} \qquad s = \frac{2M_tp_d^3}{k\pi^2yN},$$

in which M_t is the torque in in. lb. and b is expressed as kp_c or $k\pi/p_d$. In this form the torque and the number of teeth may be used directly in determining the induced stress.

296. Allowable Tooth Stresses. The allowable stress for gear teeth depends upon the material, pitch-line speed, and load conditions. Buckingham suggests that the factors of safety given in Table 47 be used with the ultimate strengths listed in Table 48 to obtain the safe static stress, s_0, under different operating conditions. The safe static stress is the allowable stress at zero velocity.

In order to allow for the influence of speed through impact and wear, a velocity factor is used. The original one was suggested by

TABLE 47

FACTORS OF SAFETY

For steady load on a single pair of gears	3
For suddenly applied loads on single pairs	4
For steady loads on gears of a train beyond the first mesh	5
For suddenly applied loads on gears of a train beyond the first mesh	6

TABLE 48

ULTIMATE STRENGTH OF GEAR MATERIALS

MATERIAL	ULTIMATE STRENGTH, P.S.I.
Cast iron	24,000
Semi-steel	36,000
Bronze	36,000
Cast steel (S.A.E. 1235)	45,000
Forged carbon steel (S.A.E. 1030)	60,000
Forged carbon steel (S.A.E. 1045)	90,000
Forged nickel-chromium steel (S.A.E. 3245)	120,000

Barth and is expressed as follows:

$$s = s_0\left(\frac{600}{600 + V}\right), \tag{3}$$

where s is the allowable stress in p.s.i., s_0 is the safe static stress in p.s.i., and V is the pitch-line velocity in ft. per min. This factor is still widely used, especially for ordinary commercial gears, although it is unnecessarily safe for high-grade gears. For very carefully cut gears, Buckingham recommends

$$s = s_0\left(\frac{1200}{1200 + V}\right). \tag{4}$$

For very accurately cut gears running at pitch-line velocities of 4000 ft. per min. and more, the A.G.M.A. (American Gear Manufacturers' Association) gives

$$s = s_0\left(\frac{78}{78 + \sqrt{V}}\right). \tag{5}$$

The same authority gives, for non-metallic gears of laminated phenol derivatives or rawhide,

$$s = s_0\left(\frac{150}{200 + V} + 0.25\right), \tag{6}$$

in which the safe static stress s_0 is usually taken as 6000 p.s.i.

297. Width of Face. According to the Lewis formula, it is assumed that the load is applied across the full breadth of the tooth and consequently the strength of the tooth would be increased in direct proportion to the width of the face of the gear. If the face is made too wide, however, inaccuracies in cutting and alignment may cause the load to be concentrated at one spot, perhaps at the corner of the tooth. The tooth would then fail by breaking diagonally across the corner. Cast teeth, for instance, should not be wider than 2 times the circular pitch. For cut gears, the width is generally between 3 and 3.5 times the circular pitch; under conditions of careful cutting and mounting, the width might be 5 times the circular pitch. The A.G.M.A. has recommended $10/p_d$, which is $3.18p_c$.

Example 1. Select the pitch of a 14½ deg. involute (composite)-tooth spur gear to transmit 15 hp. at 320 r.p.m. The load may be considered as steady. The gear is to be cast iron with cut teeth having a face width between 3 and 3½ times the circular pitch.

Let the pitch diameter be 8 in. Then

$$V = \pi \frac{8}{12} \times 320 = 672 \text{ ft. per min.}$$

From equation (3) and Tables 47 and 48, we find

$$s = \frac{24,000}{3}\left(\frac{600}{600 + 672}\right) = 3770 \text{ p.s.i.}$$

and

$$F = \frac{15 \times 33,000}{672} = 735 \text{ lb.}$$

Let $b = 3p_c = 3\pi/p_d$. From equation (1), we have

$$F = \frac{\pi s b y}{p_d}, \qquad \text{or} \qquad \frac{p_d^2}{y} = \frac{3\pi^2 s}{F}.$$

Hence

$$\frac{p_d^2}{y} = \frac{3\pi^2 \times 3770}{735} = 152.$$

From the tabulated values of y in Table 46, it is evident that a usual value of y is approximately 0.1. On this basis $p_d^2 \cong 15.2$. The nearest standard diametral pitch would therefore be 4. Then $N = 8 \times 4 = 32$ and y for this number of teeth $= 0.102$. Hence

$$\frac{p_d^2}{y} = \frac{16}{0.102} = 157.$$

This is somewhat greater than 152, hence the width of face will be increased slightly.

$$b = 3 \times \frac{\pi}{4} = 2.36; \text{ say } 2\tfrac{1}{2} \text{ in.}$$

Example 2. A rawhide pinion is to transmit 40 hp. at 1150 r.p.m. Select a standard pitch for 20° full-depth involute teeth.

Assume $N = 16$ and $k = 3.5$; then $y = 0.094$. From equation (1), § 151,

$$M_t = \frac{hp \times 63{,}024}{n},$$

where n equals the number of revolutions per minute. Hence

$$M_t = \frac{40 \times 63{,}024}{1150} = 2190 \text{ in. lb.}$$

From equation (2),

$$s = \frac{2M_t p_d^3}{k\pi^2 yN},$$

so that

$$s = \frac{2 \times 2190 p_d^3}{3.5\pi^2 \times 0.094 \times 16} = 84.3 p_d^3,$$

and from equation (6), the allowable stress is

$$s = 6000 \left(\frac{150}{200 + V} + 0.25 \right).$$

In these last two equations, both p_d and V are unknown, so it will be necessary to make some assumption. Since V depends upon p_d, we will assume as a first trial a diametral pitch of 3. The pitch diameter $D = 16/3 = 5.33$ in., and

$$V = \frac{\pi \times 5.33 \times 1150}{12} = 1610 \text{ ft. per min.}$$

The allowable stress is

$$s = 6000 \left(\frac{150}{200 + 1610} + 0.25 \right) = 1990 \text{ p.s.i.},$$

and the induced stress, $s = 84.3 \times 3^3 = 2276$ p.s.i. Since the induced stress is higher than the allowable stress, it is evident the tooth is too small. Assuming as a second trial, $p_d = 2\frac{1}{2}$, $D = 16/2.5 = 6.4$ in., and $V = \pi \times 6.4 \times 1150/12 = 1930$ ft. per min., we find

$$\text{allowable } s = 6000 \left(\frac{150}{200 + 1930} + 0.25 \right) = 1920 \text{ p.s.i.}$$

$$\text{induced } s = 2.5^3 \times 84.3 = 1320 \text{ p.s.i.}$$

This is amply strong, but it probably is not the finest pitch that can be used. With $p_d = 2.75$, which is a standard pitch but commercially not so easily obtainable as 2.5, the allowable stress is 1962 p.s.i. and the induced stress 1750 p.s.i. This pitch would be preferable and gives a face width

$$b = 3.5 \times \frac{\pi}{2.75} = 4.01 \text{ in., say 4 in.}$$

298. Dynamic Tooth Loads. The A.G.M.A. has adopted the dynamic tooth-load theory of Buckingham, which is based upon the idea that the actual tooth load is composed of the applied or transmitted load and an increment load. The increment load is caused by the changes in velocity of the gear resulting from inaccuracies in the tooth profiles and tooth spacing. The driven gear suffers a variation in velocity depending upon the amount of error and upon the masses and

flexibility of the revolving parts. As the method of calculation is involved and is applied principally to high-speed gears, reference is made to the treatment in Buckingham's *Spur Gears*, or the publications of the A.G.M.A. For average conditions, the A.G.M.A. gives the following equation:

$$F_d = \frac{0.05V(bC + F)}{0.05V + (bC + F)^{1/2}} + F, \tag{7}$$

where F_d is the dynamic tooth load in lb., V is the pitch-line velocity in ft. per min., b is the face width of gear in in., F is the total applied or transmitted load, and C is a constant. The values of C as given in Table 49 depend upon the material and the error e in tooth action in in.

TABLE 49

VALUES OF C

MATERIAL	TOOTH FORM	e (in.) 0.0005	e (in.) 0.001	e (in.) 0.002	e (in.) 0.003
Gray iron and gray iron.	14½° involute	400	800	1600	2400
Gray iron and gray iron.	20° full depth	415	830	1660	2490
Gray iron and gray iron.	20° stub	430	860	1720	2580
Gray iron and steel.	14½° involute	550	1100	2200	3300
Gray iron and steel.	20° full depth	570	1140	2280	3420
Gray iron and steel.	20° stub	590	1180	2360	3540
Steel and steel.	14½° involute	800	1600	3200	4800
Steel and steel.	20° full depth	830	1660	3320	4980
Steel and steel.	20° stub	860	1720	3440	5160

NOTE. For other values of e, C may be found by direct proportion.

The dynamic load should not exceed the static strength of the tooth based upon static stress s_0 of the material.

299. Wear of Gear Teeth. Gear teeth may be amply strong to withstand the bending stresses induced by the applied load, but when subjected to continuous operation, they may fail to resist the wear caused by the continued sliding of the teeth and variation of the load. Teeth with proper clearance and efficient lubrication may fail by *pitting*, which is a breaking away of small particles from the surface, so that small holes or pits are left. This action is believed to be the result of fatigue failure under alternating compressive stresses and is most evident when the material is stressed beyond the elastic limit. Its effect is generally more severe near the pitch line, at which point one tooth is usually carrying the full load. Buckingham has developed equations based upon the stresses produced on curved surfaces in contact, using the radii of tooth curvature at the pitch line. These relations have

been adopted by the A.G.M.A. in the following form:

$$F_w = D_1 bKQ, \tag{8}$$

where F_w = equivalent static load in lb.
D_1 = pitch diameter of smaller gear in in.
b = face width of gears in in.
K = stress factor $= \dfrac{s^2 \sin \phi}{1.4}\left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)$.
Q = ratio factor $= \dfrac{2N_2}{N_1 + N_2} = \dfrac{2D_2}{D_1 + D_2}$.
s = maximum fatigue limit compressive stress, p.s.i.
ϕ = pressure angle.
E_1, E_2 = moduli of the materials.
N_1 = number of teeth in pinion.
N_2 = number of teeth in gear.
D_2 = pitch diameter of larger gear in in.

The accompanying Table 50, from Buckingham, gives values of K for different materials for 14½ deg. and 20 deg. teeth and is based upon smooth tooth surfaces. These values of K are limiting values and should be reduced to allow for a margin of safety. A reduction of 25 per cent is generally satisfactory.

TABLE 50

VALUES OF FATIGUE CONSTANT, K

Material	Max. Specific Compressive Stress Assumed	K (Breakdown) for 14½° Tooth	K (Breakdown) for 20° Tooth
Cast steel and cast steel	60,000	43	59
Forged steel and cast steel	65,000	50	68
Forged steel and forged steel	80,000	76	104
Hardened steel and cast steel	90,000	96	131
Forged steel and semi-steel	80,000	114	156
Hardened steel and phosphor bronze	85,000	135	185
Hardened steel and semi-steel	90,000	145	198
Heat-treated steel and heat-treated steel	120,000	171	234
Phenolic laminated and metal	32,000	189	259
Semi-steel and semi-steel	90,000	193	264
Hardened steel and heat-treated steel	130,000	201	275
Hardened steel and hardened steel	220,000	576	790

NOTE. For cast iron use same values as for semi-steel.

EXAMPLE 3. In an automotive type of gear box the second-speed gear shaft is to be driven from the main shaft with a velocity ratio of 1.5 to 1. The main shaft transmits 60 hp. at 3000 r.p.m. The shaft center distance must not exceed 3½ in.

Determine a satisfactory pitch and the number of teeth for 20 deg. stub-tooth gears of heat-treated steel (S.A.E. 3245).

Knowing the center distance l and the velocity ratio R, the required pitch diameters are readily obtained from the formulas

$$l = \frac{D_1 + D_2}{2} \quad \text{and} \quad R = \frac{D_2}{D_1},$$

from which $D_1 = 2l/(1 + R)$; therefore $D_1 = 2 \times 3.5/(1 + 1.5) = 2.8$ in., and $D_2 = RD_1 = 1.5 \times 2.8 = 4.2$ in.

It is desirable to have about 20 teeth in the smaller gear, and with this number the diametral pitch p_d would be $20/2.8 = 7.1$, say 7. This pitch will be used in the computation although the more commonly used 6 pitch may be more desirable. With a diametral pitch of 7, the pitch diameter is 2.86 in., which may be excessive. If 19 teeth are used, then $D_1 = 19/7 = 2.71$ in. and $D_2 = 1.5 \times 2.71 = 4.06$ in. The pitch-line velocity of the pinion is

$$V = \frac{2.71\pi \times 3000}{12} = 2130 \text{ ft. per min.},$$

and the tangential force $F = 60 \times 33{,}000/2130 = 930$ lb. Selecting a factor of safety of 4 from Table 47 and a stress of 120,000 p.s.i. from Table 48, the working stress for accurately cut gears is found by substituting in equation (4):

$$s = \frac{s_0}{4}\left(\frac{1200}{1200 + V}\right),$$

to be

$$s = \frac{120{,}000}{4}\left(\frac{1200}{1200 + 2130}\right) = 10{,}800 \text{ p.s.i.}$$

From Table 46, the form factor y for 19 teeth = 0.123. Then the width of face is given by

$$b = \frac{F}{yp_cs} = \frac{930}{0.123 \times (\pi/7) \times 10{,}800} = 1.57 \text{ in.}$$

Automotive gears for such service probably would have a pitch of 6 and a face width of about 3/4 in. Such gears are not required to operate continuously at full load,* and with this width, the temporary bending stress would be only 20,000 to 40,000 p.s.i., depending upon the impact of the applied load. This stress would not be prohibitive unless the number of engagements at full load would be great enough to cause fatigue failure. To obtain a light and compact assembly, such modification of the design would be accepted.

To investigate the original design for resistance to wear according to formula (8), the values of the factors are as follows: $D_1 = 2.71$, $b = 1.57$. From Table 50, $K = 234$ for heat-treated steel on heat-treated steel and 20 deg. teeth. The number of teeth in the pinion and gear are 19 and 1.5×19, say 29. (The transmission ratio need not be exactly 1.5 to 1.) Then

$$Q = \frac{2N_2}{N_1 + N_2} = \frac{2 \times 29}{19 + 29} = 1.21.$$

Substituting in (8), $F_w = D_1bKQ$, we have

$$F_w = 2.71 \times 1.57 \times 234 \times 1.21 = 1200 \text{ lb.}$$

* See report on tests by Ross, American Machinist, vol. 56, p. 515, or C. A. Norman, *Principles of Machine Design*, p. 331.

This limiting value of F_w would indicate a margin of safety of about 25 per cent when operating at full load. If b were decreased as discussed previously, the conditions would be unfavorable for resistance to wear and could be corrected only by the use of materials that would give a higher value to K. This greater resistance could be obtained, for instance, by oil hardening S.A.E. 3445 steel or by using case-hardened S.A.E. 3220. Under these conditions factor K would be increased to perhaps 300.

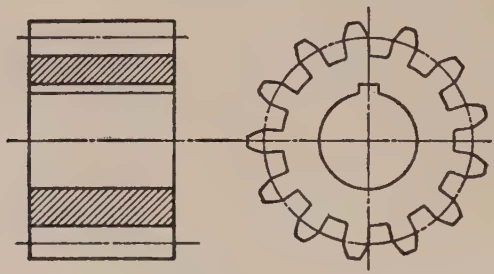


FIG. 347. PINION.

300. Gear-Wheel Proportions. The general proportions of gears are determined by empirical methods as a satisfactory analysis is difficult. The gear should be sufficiently rigid to resist excessive dis-

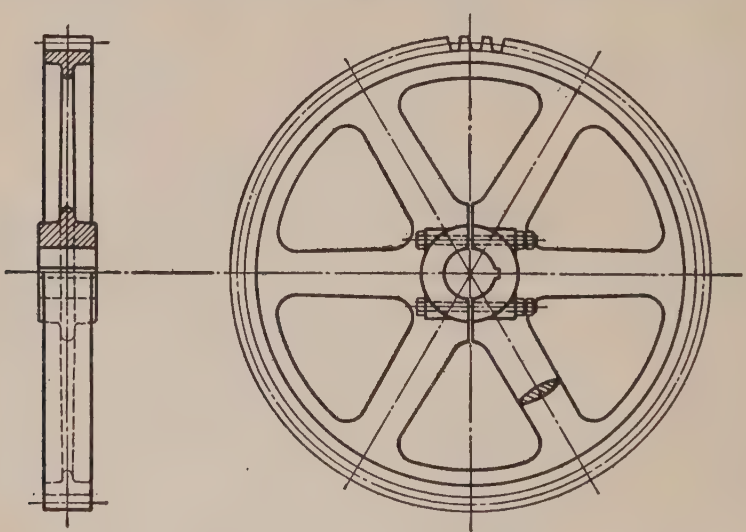


FIG. 348. GEAR WITH SPLIT HUB.

tortion under working loads and all parts must be capable of transmitting the applied forces.

Gears are made solid (Fig. 347), or with arms connecting the hub

and rim (Fig. 348), or with a disc connecting the hub and rim (Fig. 349). The solid type (usually pinions) is used where the rim diameter closely approaches the hub diameter. According to the proportions used by the R. D. Nuttall Co., the thickness of metal over the keyway of such a pinion should be not less than

$$t = \frac{\sqrt{0.2 \times \text{no. teeth}}}{p_d}. \tag{9}$$

Gears having a diameter considerably greater than the hub are provided with arms. Generally six arms are used, although four may be used on small gears (under 20 in.) and eight on large gears (over

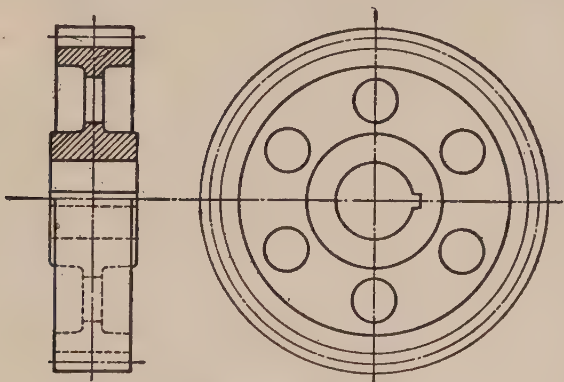


FIG. 349. WEBBED GEAR.

120 in.). The arms may be computed safely on the assumption that the stalling torque is distributed equally among them. The stalling torque is the torque that could be transmitted safely by the teeth at zero velocity; hence it is equal to the applied torque divided by the velocity factor. The arms are considered as cantilever beams, fixed at one end and free at the other, although, if joined to a heavy rim, they may approach the condition of a cantilever with one end free but guided. Arms having an elliptical cross-section are commonly used for small and medium-sized gears provided the face is not too wide. For large gears and wide faces, the H or + sections provide a better distribution of metal, resulting in a stronger arm and better lateral support to the rim. The allowable stress in the arms is taken equal to the value of the stress used for the teeth.

The hub diameter is made from 1½ to 2 times the diameter of the bore (Nuttall uses 1.8), and the hub length about 1.25 times the diam-

eter of the bore but never less than the face width. Figure 350 and Table 51 show representative practice of the R. D. Nuttall Co.

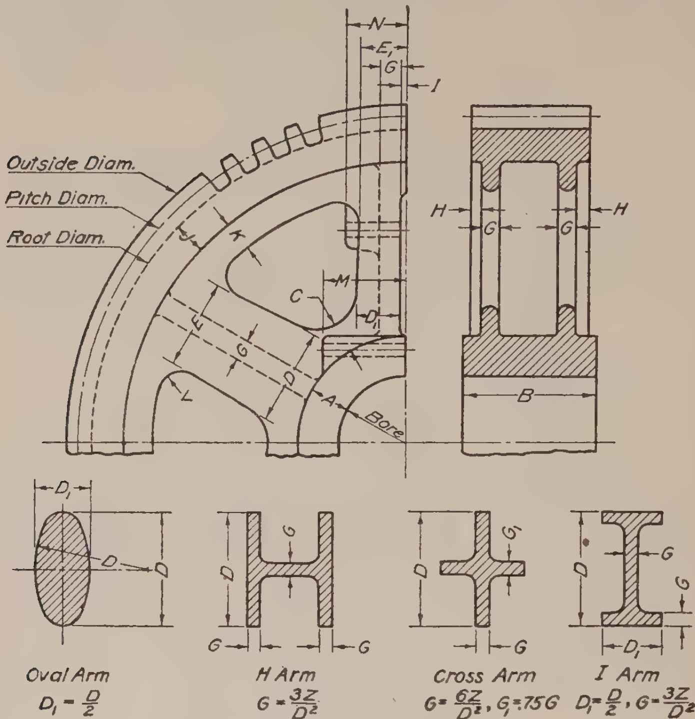


FIG. 350. GEAR DIMENSIONS.

EXAMPLE 4. Design a pair of 14½ deg. gears with a velocity ratio of 5 to 1 for the second reduction in a crane which has a capacity of a 5-ton rope pull on the drum. The first gear reduction (at the drum) is 4 to 1; the drum diameter is 26 in. and has a circumferential speed of 120 ft. per min. To conserve space, the shaft-center distance should be kept as small as possible.

In order to determine the minimum size of pinion, the size of the shaft necessary to withstand torsion and bending must be determined. Not having the layout from which to obtain the bending moments on the shafts, we shall estimate the size required from the proportions given in Table 24 for head shafts:

$$D = \sqrt[3]{\frac{133.7 \times hp}{n}}.$$

The horsepower required at the drum is $10{,}000 \times 120/33{,}000 = 36.4$ hp. Allowing 90 per cent efficiency for the first reduction, we find that the horsepower at the second reduction is $36.4/0.9 = 40.4$ hp. The gear shaft turns at $(120 \times 12 \times 4)/(26\pi) = 70.5$ r.p.m. The pinion shaft turns at $70.5 \times 5 = 352.5$ r.p.m. Therefore

$$D = \sqrt[3]{\frac{133.7 \times 40.4}{352.5}} = \sqrt[3]{15.3} = 2.46; \text{ say } 2\tfrac{1}{2} \text{ in.}$$

TABLE 51

GEAR DIMENSIONS

Stalling load $= s_0 p_c b y$, where

s_0 = stress in material at zero velocity, p.s.i.
p_c = circular pitch, in.
b = width of face, in.
y = Lewis form factor (see Table 46).

If Z is the section modulus of the arms, Z is determined by the relation

$$Z = \frac{\text{stalling load} \times \text{pitch radius}}{\text{no. arms} \times \text{stress}} = \frac{p_c b y \times \text{pitch radius}}{\text{no. arms}}.$$

If D is the width of the arms as shown in Fig. 350, we have

$$D = 4\sqrt[3]{\frac{Z}{\pi}}.$$

The other dimensions indicated by letters in Fig. 350 have values as follows:

$A = 0.4 \times$ bore, hence hub diameter $= 1.8 \times$ bore.
$B = 1.25 \times$ bore. (For a gear with a face greater than $1.25 \times$ bore, make B, however, equal to the face, and for split gears make B large enough to accommodate bolts.)
$C = 0.55A$, hence bead diameter $= 2.24 \times$ bore.
$E = D - \frac{3}{4}$ in. taper per ft.
$E_1 = 0.5E$ (for split gears and oval arms).
$H = 0.1 \times$ face (for H or cross arms).
$I = 0.25G$ (for split gears).

$$J = \frac{\sqrt[3]{\dfrac{0.5 \times \text{no. teeth}}{\text{no. arms}}}}{p_d}, \text{ where } p_d = \text{diametral pitch.}$$

$K = 1.25J.$

$$L = \frac{p_d}{4.25 \times \text{no. arms}}.$$

$M = 2A.$
$N = E_1 + \frac{1}{4}$ in.

For this size of shaft, a 5/8 in. square key would be used (Table 26). To determine the minimum metal thickness over the keyway by formula (9), the number of teeth and the diametral pitch must be known. Assume the pinion to have 16 teeth and to be made from forged steel (S.A.E. 1030) and the gear to be of cast steel (S.A.E. 1235). As the gear material is only three-fourths as strong as the pinion material (Table 48), the pitch required will be determined, in all probability, by the strength of the gear teeth. The gear torque is given by

$$M_t = 10{,}000 \times \frac{26}{2} \times \frac{1}{4} \times \frac{1}{0.9} = 36{,}100 \text{ in. lb.}$$

The gear will have $16 \times 5 = 80$ teeth with a value of $y = 0.115$ from Table 46. Assume k, the ratio of width of face to circular pitch, as 3.5. From formula (2), the

induced stress will be

$$s = \frac{2M_t p_d^3}{k\pi^2 y N} = \frac{2 \times 36{,}100 p_d^3}{3.5\pi^2 \times 0.115 \times 80} = 227 p_d^3.$$

Assume $p_d = 3$. Then $s = 227 \times 27 = 6140$ p.s.i. The allowable stress is given by formula (3):

$$s = s_0 \left(\frac{600}{600 + V} \right).$$

The gear-pitch diameter is $D = 80/3 = 26.67$ in., so that $V = 26.67\pi \times 70.5/12 = 494$ ft. per min. From Tables 47 and 48, the allowable stress is

$$s = \frac{45{,}000}{5} \left(\frac{600}{600 + 494} \right) = 4940 \text{ p.s.i.}$$

Since the induced stress is higher than the allowable stress, a readjustment in proportions is necessary. If k is increased to $3.5 \times 6140/4940 = 4.35$ (which will be satisfactory if alignment is good and the speed is low), the induced stress will be reduced. The width of face will then be $4.35 \times \pi/3 = 4.55$, say 4.5 in.

The pinion should now be checked to determine if there is sufficient metal between keyway and root of tooth. The pinion-pitch diameter is $16/3 = 5.33$ in., and, according to formula (9), there should be as a minimum, $t = \sqrt{0.2 \times 16}/3 = 0.6$ in. The actual thickness is the difference between pitch radius of the pinion and the shaft radius less the sum of the dedendum and the depth of the keyway. Therefore

$$t = \frac{5.333 - 2.5}{2} - (0.386 + 0.3125) = 0.718 \text{ in.},$$

which is sufficient.

Checking the pinion teeth for stress, we find from formula (1)

$$s = \frac{36{,}100 \times 2 \times 3}{26.67 \times 4.5 \times \pi \times 0.081} = 7090 \text{ p.s.i.}$$

The allowable stress is

$$s = \frac{60{,}000}{5} \left(\frac{600}{600 + 494} \right) = 6580 \text{ p.s.i.}$$

It is customary to make the face width of the pinion slightly greater than face width of the gear, especially where there may be a certain amount of endwise motion to the shafts. If the pinion face is increased to 5 in., the induced stress is 6370 p.s.i.; therefore the design may be considered satisfactory. The gears selected are as follows:

	Material	Diametral Pitch	No. of Teeth	Pitch Diameter	Width of Face
Gear	*Cast Steel* S.A.E. 1235	3	80	26.67	4.5
Pinion	*Forged Steel* S.A.E. 1030	3	16	5.33	5

The proportions of the gear are now to be determined.

Number of arms = 6, selected.

$$\text{Bore} = \sqrt[3]{\frac{133.7 \times 40.4}{70.5}} = 4.24, \text{ say } 4\tfrac{1}{4} \text{ in.}$$

Hub diameter = 1.8 × 4.25 = 7.65, say 7¾ in.
Hub length = 1.25 × 4.25 = 5.31, say 5½ in.

$$\text{Rim thickness} = \frac{\sqrt[3]{\dfrac{0.5 \times \text{no. teeth}}{\text{no. arms}}}}{p_d} = \frac{\sqrt[3]{\dfrac{0.5 \times 80}{6}}}{3} = 0.63, \text{ say } 5/8 \text{ in.}$$

Outside diameter, $D_0 = 26.67 + 2 \times 0.333 = 27.33$ in.

Arms. Assume the arms to be free at the rim end and of elliptical cross-section having a width twice the thickness. With an allowable stress equal to the allowable stress in the teeth we may calculate the dimensions necessary to withstand the bending action caused by stalling load.

Bending moment on each arm equals

$$\frac{\text{Stalling torque}}{\text{No. arms}} = \frac{36,100}{6} \times \frac{1094}{600} = 11,000 \text{ in. lb.}$$

$$\text{Section modulus, } Z = \frac{M_t}{s} = \frac{11,000}{4940} = 2.2 \text{ in.}^3.$$

For the elliptical section of the proportions assumed, $Z = \pi a^3/64$, or

$$a = \sqrt[3]{\frac{64Z}{\pi}} = \sqrt[3]{\frac{64 \times 2.2}{\pi}} = 3.55 \text{ in.}$$

As this value has been computed as though the arms extended to the center of the gear, a section 3½ in. × 1½ in. will be used where the arms join the hub. The arms are tapered 3/4 in. per ft. in width and 3/8 in. per ft. in thickness.

301. Maximum Transmission Ratio. Gear Trains. Theoretically, the transmission ratio between a pair of gears can have any value between 1 and infinity (or zero), the latter being the transmission ratio of a gear operating with a rack. Practically, there is a limit set to the transmission ratio by the fact that a minimum tooth number is required on the pinion and that for very large transmission ratios the diameter of the mating gear often becomes inconveniently large.

For ordinary straight-tooth spur gears, transmission ratios usually cannot greatly exceed 5. For ratios greater than this, gear trains become necessary. Such gear trains are shown, for instance, on the crane trolley, Fig. 389, page 429. The transmission ratio of a gear train as a whole is obtained by multiplying together the ratios of the individual gear sets of which it is constituted.

For instance, a train made up of three gear sets of individual ratios R_1, R_2, and R_3 has an overall ratio of $R_1 \times R_2 \times R_3$. For a train made up of three sets with ratios 1 : 3, 1 : 4, and 1 : 5, the overall ratio is 1/3 × 1/4 × 1/5 = 1 : 60.

In arranging gear trains it is well for the designer, if possible, to have the gears on the same shaft fairly far apart, since the elastic distortion of the long shaft will cushion impacts to some extent. Such impacts may come simply from tooth inaccuracies.

PROBLEMS

1. A machine-cut cast-iron gear with 192 teeth of 6 in. circular pitch and a 30 in. face, runs at 10 r.p.m. Determine the outside diameter of the gear and the hp. transmitted if the teeth are of the 14½ deg. composite form.

2. A bronze pinion of 16 teeth, 14½ deg. composite form, rotating at 600 r.p.m., transmits power with a velocity ratio of 4 to 1 to a cut cast-steel gear under steady-load conditions. The width of face of the gear is 3 in. and the diametral pitch is 3. Determine the horsepower that this pair of gears will transmit safely. Is the gear as strong as the pinion?

3. The thrust on a drill press spindle is 4200 lb. If a 15-tooth pinion driving a rack collared on the spindle is used to feed the drill into the work, determine the standard diametral pitch of the mild-steel pinion if the width of face is 1¾ in. Solve for both 14½ deg. and 20 deg. full-depth involute teeth and indicate which design is the better, giving standard pitch used and pitch diameter.

4. A pair of cast-iron spur gears transmit 33 hp. The driver has a pitch diameter of 24 in. and a face width of between 3 and 4 times the circular pitch, and it rotates at 100 r.p.m. The velocity ratio of driver to driven is 3 to 4. Determine the finest standard diametral pitch for 14½ deg. composite teeth, the number of teeth on the gears, and the outside diameters.

5. A motor develops a torque of 1500 in. lb. at 1200 r.p.m. This torque is transmitted to a shaft operating at 400 r.p.m. through a cast-iron pinion and spur gear having 20 deg. stub teeth. The center distance between shafts must be maintained at 9 in. If the load application is smooth and continuous, determine the satisfactory standard diametral pitch which will give the greatest number of teeth. Check for dynamic loading and for wear.

6. A mild-steel spur-gear pinion of 20 deg. full-depth involute teeth meshes with a gear of high-grade cast iron. The pinion has 15 teeth and rotates at 610 r.p.m. The gear reduction is 5 to 1. The gears are to transmit 24 hp. and the width of face is to be approximately 3 times the circular pitch. Find the diametral pitch and the width of face of the gears.

7. A 20-tooth, 20 deg. full-depth involute mild-steel pinion on the shaft of a 15 hp., 1200 r.p.m. motor drives a 140-tooth gear (cast iron) on a hoist drum. The width of face is to be from 3 to 3.5 times the circular pitch. Determine the necessary diametral pitch, pitch diameters, and outside diameters of the gear and pinion.

8. A pair of 20 deg. stub-tooth gears are to transmit 35 hp. at 900 r.p.m. of the pinion with a velocity ratio of 6¼ to 1. The pinion is made of S.A.E. 1030, heat-treated to a Brinell hardness number of 250, and the gear is high-grade cast iron. Determine the necessary diametral pitch and width of face for satisfactory strength and wear.

9. An electric motor develops 50 hp. at 900 r.p.m. and delivers it through a train of spur gears to a shaft which operates at 80 r.p.m. (a) Select suitable reductions for the pairs of gears in the train and determine the number of teeth in each gear. (b) Calculate the diametral pitch and width of face of each gear if made of cast iron with 20 deg. involute teeth. Check for dynamic loading and for resistance to wear.

10. Two parallel shafts, center distance 20 in., are connected by 14½ deg. composite-tooth spur gears to make 50 and 150 r.p.m., respectively. If 10 hp. is to be transmitted, determine the finest standard pitch and the width of face if the gears are of forged steel, untreated.

11. Two cast-iron $14\frac{1}{2}$ deg. composite-tooth gears are to transmit 11 hp. at 900 r.p.m. with a velocity ratio of $2\frac{2}{3}$ to 1 and with a center distance of 5.5 in. Select the standard pitch and the width of face.

12. Two cast-iron $14\frac{1}{2}$ deg. composite cut-tooth gears are to transmit 8 hp. at 100 r.p.m. of the pinion. The velocity ratio is 2 to 1 and the center distance is 17.50 in. Select a standard diametral pitch to satisfy these conditions.

13. A mild-steel pinion turning at 11 r.p.m. transmits 7.5 hp. to a cast-iron gear with a velocity ratio of 2.75 to 1. Determine the standard diametral pitch of $14\frac{1}{2}$ deg. composite teeth for a center distance of 27.50 in.

14. A cut cast-iron pinion is to transmit 1/2 hp. at 300 r.p.m. If 30 teeth are used and the pinion is made 1/2 in. wide, what $14\frac{1}{2}$ deg. composite standard pitch should be used?

15. A forged-steel pinion is to have 2 pitch, $14\frac{1}{2}$ deg. composite teeth. The pinion is keyed to a 5 in. shaft and should have not less than 12 teeth. Fix upon the number of teeth to insure sufficient material between the keyway and the root of the teeth.

16. A mild-steel pinion on a $2\frac{15}{16}$ in. commercial shaft transmits 125 hp. to a cast-steel gear that rotates at 50 r.p.m. The velocity ratio is 8 to 1. (a) Select a 20 deg. stub-tooth standard pitch. (b) Check all dimensions of the pinion and give complete specifications. (c) Fix upon and check the wheel hub and rim dimensions. (d) Determine the dimensions of the wheel arms if of elliptical cross-section, the major axis being twice the minor.

17. A cast-steel gear has 90 teeth of $1\frac{1}{2}$ pitch, 20 deg. full-depth involute form. Determine the proportions of the rim, hub, and H-section arms.

CHAPTER 19

HELICAL GEARS

302. General Description. Helical gears are essentially equivalent to a number of very thin spur gears, placed side by side, and offset relatively to one another, so that there is at all times at least one pair of teeth in contact at any point on the path of contact. This uninterrupted, gradual engagement of teeth eliminates sudden jumps from contact at one part to contact at an entirely different part on the path of contact, as is inevitable in straight-tooth spur gears. Formerly, so-called *stepped* gears, as shown in Fig. 351, were actually used. In a modern helical gear, however, the teeth are cut on an angle with reference to the axis; hence the teeth curve as helices over the gear face as shown in Fig. 352.

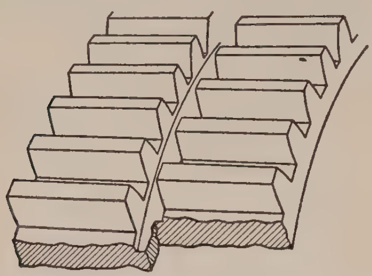


FIG. 351. STEPPED-TOOTH GEAR.

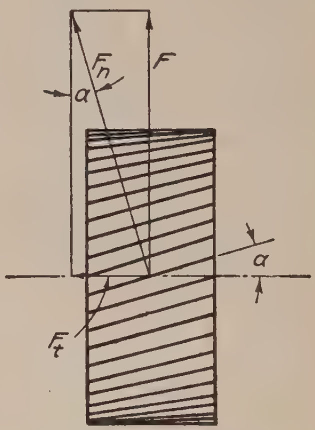


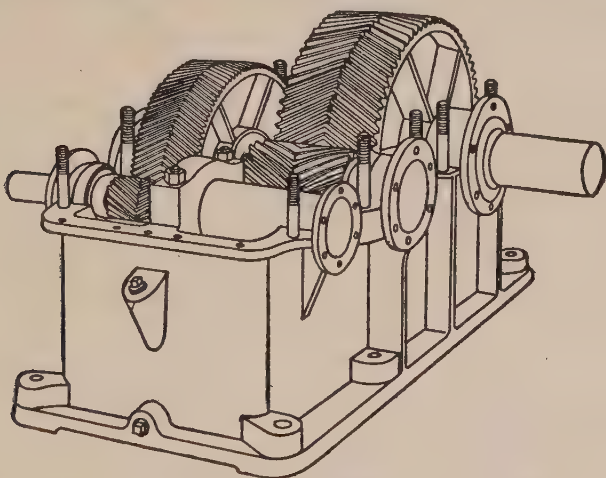
FIG. 352. FORCES ACTING ON HELICAL TOOTH.

303. Helix Angles and Width of Face. To secure continuous action throughout the path of contact, it is necessary that the teeth overlap, that is, any axial line along the face must cut at least two teeth. Consequently, the minimum width must be $p_c/\tan \alpha$, where p_c is the circular pitch and α the helix angle. This overlapping depends upon the width of the gear and the helix angle. Helix angles commonly used are 23 deg. and 30 deg., although, for very high speeds, 45 deg. may be used. For the 23 deg. angle, the minimum width required is $2.36p_c$ and for the 30 deg. angle, $1.74p_c$. The American Gear Manufacturers' Association recommends a minimum face width

15 per cent greater than $p_c/\tan \alpha$. Actually, widths of at least 3 times the circular pitch are preferred.*

304. End Thrust. If friction is neglected, the pressure between a pair of teeth is normal to the tooth surfaces at the point of contact, as shown in Fig. 352. If the tooth pressure is F_n, its component F_t along the axis of the gear is called the *end thrust.* If α is the helix angle, this end thrust has the value $F_n \sin \alpha$ or $F \tan \alpha$, where F is the tangential force at the pitch line.

If the helix angle is made small enough, say less than 20 deg., the end thrust may be carried without inconvenience by a thrust ball or roller bearing, or even by the end of a plain bearing. Such small angles are not suitable for quiet, high-speed helical gears. End thrust in high-speed gears should be balanced by using two adjoining gears with opposed helix angles, as shown in Fig. 353.



The Farrell-Birmingham Co., Buffalo, N. Y.

FIG. 353. DOUBLE-REDUCTION HERRINGBONE-GEAR SPEED REDUCER.

305. Herringbone Gears. A gear combination of opposed gears, mounted as a unit, is known as a *herringbone gear.* Cut herringbone gears were first introduced by De Laval in connection with high-speed steam turbines to make possible a gear transmission at reduction ratios and circumferential speeds entirely beyond the capacity of ordinary spur gears. Transmission ratios as high as 15 to 1 are possible and pitch-line speeds even in excess of 12,000 ft. per min. have been attained.

* A very detailed discussion of the conditions to be observed in order to obtain quietly running helical gears at high speed was given by W. P. Schmitter at the meeting of the A.G.M.A., Sept., 1936. Reference should be made to this paper for a more complete analysis.

At present, herringbone as well as helical gears are often used simply for the purpose of obtaining exceptional quietness and smoothness of operation, as, for instance, in automobile transmissions. However, as satisfactory results cannot be obtained unless the gears are very accurately made, the cutting of this type of gearing is quite specialized.

The first cut herringbone gears were made by fastening together two separately made gears of opposite helix angles. With the subsequent development in gear-cutting practice, the teeth were cut on the same blank with a considerable clearance space where the teeth would meet, or a smaller space if the teeth were staggered. By the tooth-generating method introduced by Sykes, a herringbone gear may be cut with the teeth meeting at the apex without an intervening space, thus forming a solid continuous tooth (Fig. 353). The minimum face width of a herringbone gear is twice that of a single helical gear plus the width of the groove between the helices, if such a groove is used.

306. Strength and Wear of Helical Gears. The formulas used for the computation of the load capacity of helical gears have been developed independently of those for straight-tooth gears and have in the past often differed from them fundamentally. Thus, according to Gerald Stoney * of the firm of Charles Parsons, the beam strength of the tooth was not computed at all. The permissible pressure between the teeth was determined by very simple formulas containing the pinion diameter but not the tooth dimensions.

We shall here adhere to the formula for beam strength adopted by the American Gear Manufacturers' Association, and to the formula for wear and tooth pressure developed by W. E. Sykes of the Farrell-Birmingham Company. A comprehensive paper on the load carrying capacity of helical and herringbone gears was recently published by W. P. Schmitter † of the Falk Corporation. Sykes' formula is simpler and gives results that agree closely with Schmitter's over a wide range of conditions.

The American Gear Manufacturers' Association has recommended a maximum pressure angle in the plane of rotation of 25 deg. with a minimum of 15 deg. 23 min.; a maximum helix angle of 45 deg. and a minimum of 20 deg.; a maximum addendum height of 1/diametral pitch and a minimum addendum of 0.7/diametral pitch.

* Original article by Stoney in The Engineer, November 28, 1919, p. 534. Contents reproduced in Power, January 20, 1920, p. 116, and Norman, *Principles of Machine Design*, p. 349.

† Schmitter's paper originally appeared in the June and July issues of Machine Design of 1934. A complete reprint is distributed by the Falk Corporation.

For these conditions the association has proposed the following formula for the permissible tooth load, based on beam strength:

$$F = \frac{\pi ysbK}{p_d C_l}, \tag{1}$$

where F = tangential force at pitch line in lb.,
b = active face width in in.,
p_d = diametral pitch in the plane of rotation,
s = fiber stress in p.s.i., suitable values being given in Table 52,
K = speed factor having a value $78/(78 + \sqrt{V})$, where V is the pitch-line speed in ft. per min.,
C_l = wear and lubrication factor, which may be taken equal to 1.15 for enclosed gears with proper grade of lubricant,
y = tooth proportion factor as given in Table 46, page 359.

TABLE 52

FACTOR s IN THE A.G.M.A. FORMULA FOR HELICAL GEARS

Material	Factor s
High carbon or alloy steel, heat-treated to an elastic limit of approximately 60,000 p.s.i.	15,000
0.40 to 0.50 carbon steel, heat-treated to an elastic limit of approximately 50,000 p.s.i.	12,500
0.40 to 0.50 carbon steel, untreated with an elastic limit of approximately 40,000 p.s.i.	10,000
Cast steel, A.S.T.M. Class B. Elastic limit approximately 36,000 p.s.i.	7,500
Cast iron, tensile strength approximately 24,000 p.s.i.	4,000
Bronze (88 copper, 10 tin, 2 lead), tensile strength approximately 27,000 p.s.i.	4,000

NOTE. It should be noted that the teeth of herringbone gears are cut after the material is treated. Higher physical characteristics than those quoted above for steels may result in material too hard to cut.

Helical gears computed by the A.G.M.A. formula will have teeth of ample size from a standpoint of wear if they are adequately lubricated. In the case of gears with a great number of small teeth the formula completely neglects the fact that the load is spread over several teeth; hence the carrying capacity is underestimated. The capacity of high-speed gears, which are usually made with a large number of comparatively small teeth (30 to 40 or more), is almost always limited by the crushing and wearing down of the surfaces and not by the beam strength of the teeth.

307. Wear Formula for Helical Gears. The wear formulas of Sykes and Schmitter take into consideration the crushing strength of

the teeth. The wear formula of Sykes can be reduced to the following form (not given in his original article):

$$(2) \qquad F_w = 100C_rD_pbq,$$

where F_w = tangential force at the pitch line in lb. for satisfactory wear, C_r = ratio factor having the value $R/(R+1)$, R = gear ratio D_g/D_p, D_g = gear diam. in in., D_p = pinion diam. in in., b = face width in in., and q = service factor, having the values given in Table 53.

TABLE 53

Service Factor q in Formula (2) for Helical Gears

Type of Machinery	Value of q
Industrial steam turbines for continuous service	0.6
Steam turbine units for war ships	0.9
Heavy-duty reciprocating pumps, continuous service	0.7
Intermittent duty reciprocating pumps	0.9
Oil-well pumping equipment	1.0
Cranes and hoisting machinery	1.5 to 2.85
Airplane-engine gears for driving the propeller, transport service	4.0
Same, racing service	6.5

The Sykes formula pertains only to the material which has proved to be the most satisfactory for this type of gearing and is also the one most commonly used, namely, forged steel of not too great hardness. The various grades of steel, which may be included in this classification, must possess good machining properties and must have adequate wear resistance. In large gear sets, the gear is generally made of cast steel, but the pinion, which is subject to greater wearing action, is invariably made of forged steel. The practice of such firms as Parsons of England and the Westinghouse Electric & Mfg. Co. of Pittsburgh seems to indicate that no speed allowance is necessary for such material. For the general designer this treatment may be sufficient, but for those who may wish to make allowances for speed and different kinds of materials, reference should be made to Schmitter's original article.

308. Comparison of the A.G.M.A. and the Sykes Formulas. The variation in results obtained by the use of formulas (1) and (2) will be compared in the following example.

Problem. Determine the hp. capacity of a pair of helical turbine gears having a transmission ratio of 10 : 1. The teeth are 20 deg. full-depth involute, 6 diametral pitch. The pinion has 25 teeth, and rotates at 5000 r.p.m. The active face width is 3 in. and the material is 0.40 per cent carbon steel, untreated.

A.G.M.A. Formula (1).

The pinion diameter = 25/6 = 4.167 in.

The pitch-line velocity $= \pi \times 4.167 \times 5000/12 = 5450$ ft. per min.
The speed factor $K = 78/(78 + \sqrt{5450}) = 0.513$.
From Table 52, $s = 10{,}000$ p.s.i.
From Table 46, page 359, y for 25 teeth $= 0.108$.
With good lubrication, factor $C_l = 1.15$.
We have, from equation (1),

$$F = \frac{\pi y s b K}{p_d C_l},$$

$$F = \frac{\pi \times 0.108 \times 10{,}000 \times 3 \times 0.513}{6 \times 1.15} = 758 \text{ lb.},$$

and

$$hp = \frac{758 \times 5450}{33{,}000} = 125.$$

Sykes' Formula (2).
Ratio factor $C_r = 10/(10 + 1) = 10/11$.
Minimum value of q from Table 53 $= 0.6$.
We have from equation (2), $F_w = 100 C_r D_p b q$,

$$F_w = 100 \times (10/11) \times 4.167 \times 3 \times 0.6 = 682 \text{ lb.}$$

and

$$hp = \frac{682 \times 5450}{33{,}000} = 113.$$

In this example the results are in fairly close agreement. If, however, we now reduce the size of the tooth without change in diameters, the value given by the A.G.M.A. formula is reduced very appreciably, whereas the value given by the Sykes formula will remain unchanged. Consequently, for gears with numerous small teeth, the A.G.M.A. formula probably underestimates the capacity considerably, at least for carefully cut gears.

309. Formula for First Lay-Out. The Sykes formula given in (2) may be less convenient to use for first lay-outs than the original formula given in Schmitter's paper, from which the following formula is derived:

(3) $$C^2 b = \frac{hp\,(R + 1)^2\, 315}{(r.p.m.)\, C_r q}.$$

Here C is the center distance in in. and hp is the horsepower. The *r.p.m.* in the denominator refers to the speed of the pinion. All other terms are the same as in formula (2). The product $C^2 b$ is a quantity that must be kept constant for a given problem; hence the center distance C and face width b must be chosen accordingly. These two factors may be varied to suit the space conditions of the gear set.

For instance, from the preceding problem,

$$C^2 b = \frac{113(10 + 1)^2 \times 315 \times 11}{5000 \times 10 \times 0.6} = 1580.$$

With a center distance of $2.083 + 20.835 = 22.918$ in., the value for b is equal to $1580/22.918^2 = 3$ in., as assumed. If this face width should not be suitable, the center distance must be changed until acceptable proportions are found.

According to Gerald Stoney in the article previously referred to, long and slender pinions should be checked for distortion. The maximum permissible tooth deflection * resulting from either bending or torsion should not exceed 0.001 in. in the length of the pinion, computed by assuming the pitch diameter as the effective resisting diameter.

310. Speed Reducers. Speed reducers employing helical and herringbone gears are very commonly used and are manufactured as standard units in a wide range of speed ratios and capacities. Single-pair units are ordinarily made with gear ratios up to 10 or 11 to 1. Double-pair and triple-pair reducers are provided for the higher reductions, the latter being furnished for ratios as high as 300 to 1. The internal arrangement and construction of a herringbone gear reducer is shown in Fig. 353.

NOTE. New A.G.M.A. standards, issued after the above text was written, contain formulas for pitch based on surface hardness. It is regretted that their inclusion here is impossible.

PROBLEMS

1. A continuous-tooth herringbone forged chrome-nickel-steel pinion and cast-steel gear is rated at 13 hp. per inch of face at 1200 r.p.m. of the pinion. The teeth are 20 deg. full-depth with a 30 deg. helix angle. The pinion has 17 teeth of 3 pitch. Transmission ratio is 3 to 1. Check this rating for strength and wear.

2. A continuous-tooth herringbone cast-steel gear with 30 deg. helix angle is to transmit 35 hp. at 1200 r.p.m. If the gear has 24 teeth, determine the necessary diametral pitch, pitch diameter, and width of face for 20 deg. full-depth teeth.

3. A pair of equal-diameter herringbone gears of cast iron have a center distance of 2.50 in. and are to transmit 6 hp. at 900 r.p.m. Select the proper pitch, pitch diameter, helix angle, and width of face for hoisting service for 20 deg. full-depth teeth.

4. A pair of parallel shafts at 5 in. center distance are to be connected by a pair of helical gears having a velocity ratio of $1\frac{1}{2}$ to 1. Design the gears for 20 deg. stub teeth of heat-treated alloy steel to transmit 100 ft. lb. torque from the pinion. Pinion speed is 3000 r.p.m. and drive is under steady load. What is the end thrust on the pinion?

5. A pair of helical gears connecting parallel shafts is to transmit 300 ft. lb. torque at 2800 r.p.m. of the pinion. The teeth are to be 20 deg. stub of heat-treated alloy steel. The width of face is limited to $1\frac{1}{2}$ in. The driven gear rotates at 1800 r.p.m. For a helix angle of 30 deg., select the necessary pitch and check for wear. What is the axial thrust developed?

* An example showing the computation of such deflections is given in Norman's *Principles of Machine Design*, p. 355.

6. A pair of cast-iron helical gears is to connect two parallel shafts with a velocity ratio of 2.25 to 1. The pinion delivers 4 hp. at 1200 r.p.m. Select the 20 deg. stub-tooth standard pitch and width of face necessary for a 45 deg. helix angle and check for wear.

7. A helical gear 16 in. in diameter is carried on a shaft between two bearings, 10 in. from one and 15 in. from the other. The driving and driven shafts are in the same horizontal plane and transmit 4000 in. lb. torque. The helix angle is 45 deg. and in such a direction as to put the thrust on the inside of the far bearing. The pressure angle is 20 deg. in the plane of rotation. (a) Determine the end thrust and the reaction on each bearing. (b) Determine the necessary diameter of commercial steel shafting for steady load.

8. A 20 in. diameter helical gear transmits 25 hp. at 200 r.p.m., being driven from a pinion above. The helix angle is 30 deg. and is in such a direction as to put the thrust on the inside of the nearer bearing. The gear is between bearings, 10 in. from one, 20 in. from the other. The pressure angle is 14½ deg. in the plane of rotation. (a) Determine the end thrust and the reactions on the bearings. (b) Determine the necessary diameter of commercial steel shafting for steady load.

9. In a heavy hoisting machine the pinion is cut on the forged end of a steel shaft limiting the maximum pitch diameter of the pinion to 3 in. The r.p.m. of the pinion being changeable, determine the pitch and number of cut 20 deg. stub teeth in a cast-steel gear to transmit 30 hp. at 75 r.p.m. if the velocity ratio is between 13 and 14 to 1. Use a 23 deg. helix angle and keep the width of face between 3 and 3.5 times the circular pitch.

CHAPTER 20

BEVEL GEARS

311. Straight Bevel Gears. If the axes of two shafts intersect, motion can be transmitted from one to the other by means of two bevel gears. When the teeth of a gear converge to the apex of a cone along straight lines, it is called a *straight-tooth* bevel gear (Fig. 354).

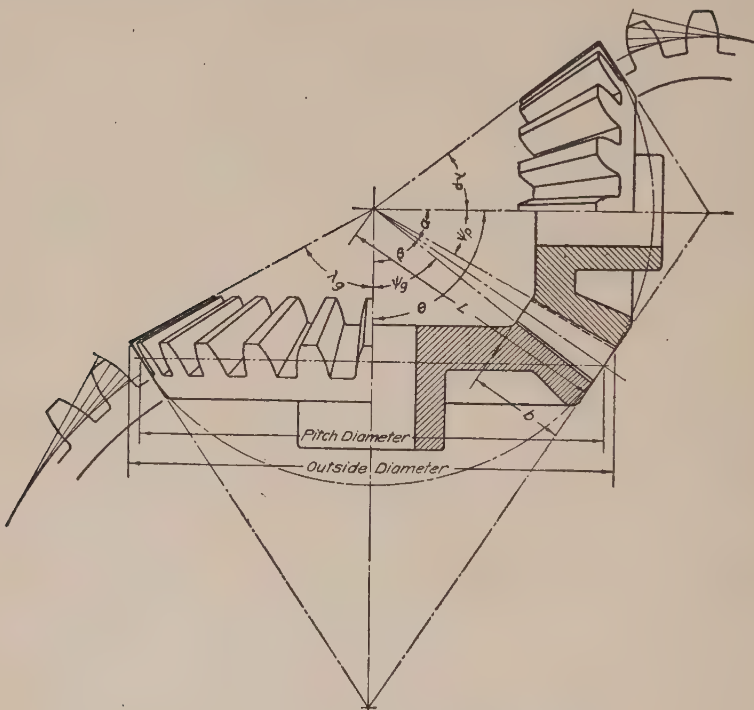


FIG. 354. STRAIGHT-TOOTH BEVEL GEARS.

θ = center angle of gears.
α = pitch angle of pinion.
β = pitch angle of gear.
λ_p = face angle of pinion.
λ_g = face angle of gear.
ψ_p = root (cutting) angle of pinion.
ψ_g = root (cutting) angle of gear.
b = face width.
L = cone distance.

The teeth in this case are formed upon the rolling pitch surfaces of two cones which have the same axes as the gears and have a common apex.

Theoretically the tooth profiles should be laid out on the surface of a sphere drawn through the common pitch point, as shown in the illustration. Since a spherical surface cannot be developed into a plane, it is sufficiently accurate to determine the outline of the teeth of each gear on the surface of a so-called back cone, tangent to the sphere at the pitch circle, as shown. The development of this conical surface forms a portion of a circle on which the tooth curve is determined for the large end of the gear. The number of teeth in this imaginary spur gear, which has a pitch radius equal to the length of the back-cone side, is called the formative number of teeth of the bevel gear. Obviously, the number of teeth in this formative gear always will be greater than the actual number of teeth in the bevel gear (see page 388).

312. Spiral Bevel Gears. *Spiral bevel* gears have their teeth curved (Fig. 355). Due to this curvature, there is a gradual engagement of teeth and an overlapping of contact, which results in smooth, quiet operation. A larger number of teeth are in mesh, and at the pitch line, where the tooth action is pure rolling, there is a continuous pitch-line contact. The load is transferred gradually from one tooth to another without sudden impacts or changes in tooth pressure. In straight-tooth gears, especially at high pitch-line velocities, it is the sudden engagement of teeth striking in full-line contact that produces noise and vibration.

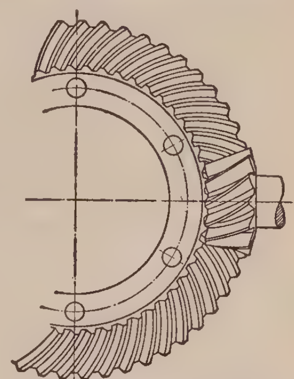


Fig. 355. Spiral Bevel Gears.

The almost universal adoption of the spiral bevel gear for the rear axle drive of automobiles, and their increasing use in high-grade machinery, are the best evidences of their superiority over straight-tooth bevel gears. They are especially applicable for large ratios at high speed, where the size of the installation must be kept as small as possible.

313. Applications of Bevel Gears. The most extensively used combination of bevel gearing is one in which the axes of two straight bevel gears intersect and form a right angle, as shown in Fig. 356a. A pair of bevel gears of the same size on shafts intersecting at right angles are called *miter* gears (Fig. 356b). Under certain conditions, when oblique drives cannot be avoided, the angle between the shafts may be greater or less than 90 deg., as illustrated in Fig. 356c and d. If

the pitch angle of a bevel gear becomes 90 deg., as shown in Fig. 356e, the gear is called a *crown* gear. The crown gear in bevel gearing cor-

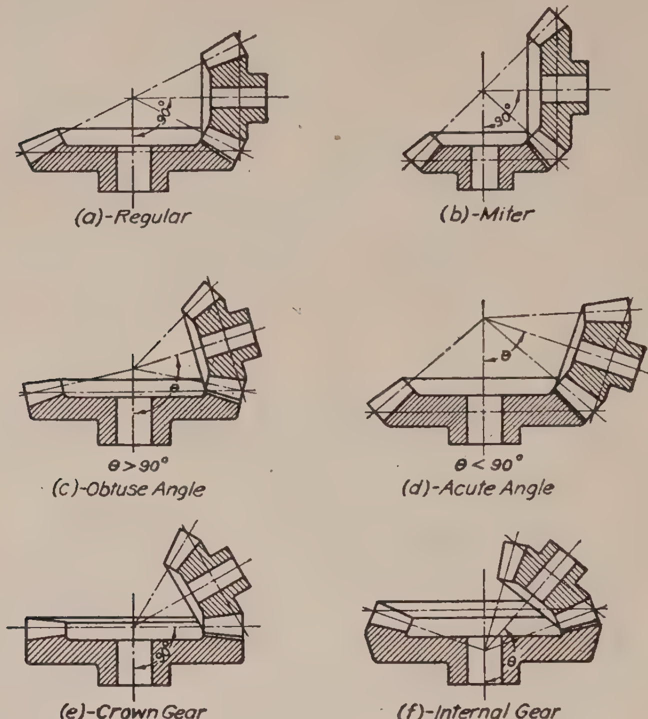


FIG. 356. DIFFERENT COMBINATIONS OF BEVEL GEARS.

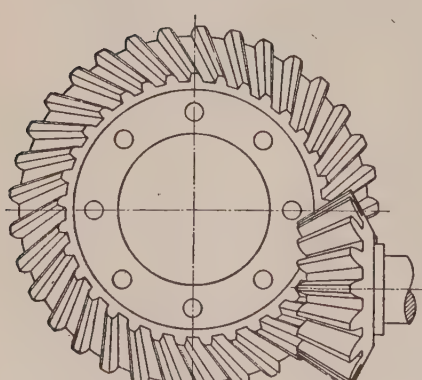


FIG. 357. SKEW BEVEL GEARS.

responds to the rack in spur gearing. Figure 356f is an internal bevel gear drive.

314. Skew Bevel Gears. If the axes of two shafts do not intersect and are not parallel, a constant velocity ratio may be obtained by means of *skew* bevel gears (Fig. 357). The pitch surfaces are rolling hyperboloids and the pitch line contact is a straight line. The tooth profiles are difficult to machine; consequently the gears are seldom used.

315. Hypoid Gears. A recent development and an improvement over the skew bevel gear is the type shown in Fig. 358, called the

hypoid gear. It is similar to the spiral bevel gear and has many of its characteristics, the main difference being that the pinion is offset from the center of the gear axis as in skew bevel gears. The teeth are curved and can be cut in practically the same manner as spiral bevels, although there is usually less spiral on the gear and more on the pinion. When used with a gear of the same size, a hypoid pinion will be from 20 to 30 per cent larger than a spiral bevel pinion of the same number of teeth.

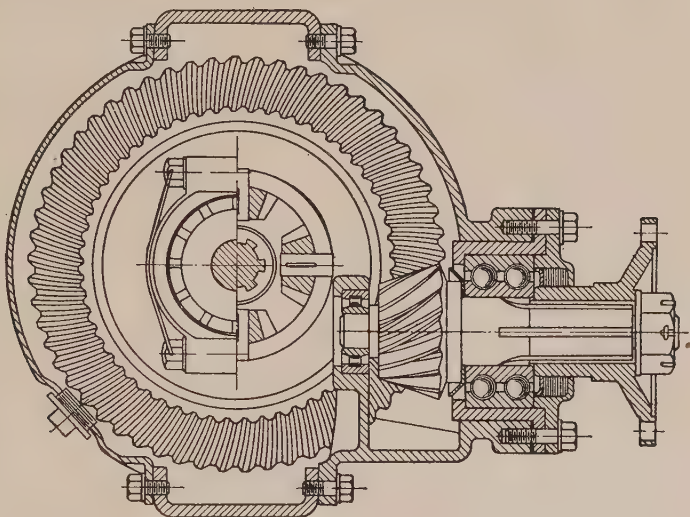


FIG. 358. APPLICATION OF HYPOID GEAR IN REAR AXLE DRIVE OF AUTOMOBILE.

Hypoid gears have of late found extended application, particularly for the rear axle drive of automobiles. The offset drive shaft with hypoid gearing permits the use of a lower body construction without sacrifice of road clearance. There may also be advantages in the form of greater quietness, a greater pinion diameter, and a greater possible pinion-shaft diameter than could be accommodated by spiral bevel gears. There is, however, an increased end thrust of the pinion on forward drive, and lubrication is more difficult on account of greater sliding action between the teeth.

316. Strength of Bevel-Gear Teeth. The methods of computing the strength of bevel-gear teeth which are in use at the present time seem to be somewhat in a state of change. This situation arises from the fact that along with the development of improved meth-

ods for producing gear teeth, particularly by generating processes, studies have been made of tooth forms, of wear on teeth, impact, strength, etc.

The American Gear Manufacturers' Association now seems strongly inclined to adopt certain tooth forms and methods of computation introduced by the Gleason Works of Rochester, N. Y., although these forms and methods may not be universally applicable.

For a long time the prevailing attitude was to adopt the same gear-teeth outlines for bevel gears as were used for spur gears, in order to obtain interchangeability, or rather applicability to the whole range of transmission ratios.

In the case of bevel gears, however, interchangeability is affected by a variation in the cone apex angle. A pinion suitable for a 1 : 4 ratio, for instance, will not mesh properly with any other ratio, if the shafts are to remain at right angles. Consequently, every pair of bevel gears really presents an individual problem and the gears may well be designed for maximum strength, smoothness of action, and minimum wear for this particular combination, rather than for applicability of the individual gears in other combinations. Because of this individual design, teeth of unequal addendum have been adopted more readily for bevel gears than for spur gears. The Gleason Works has been particularly influential in this connection, although others may have been working along similar lines.

It may be advisable, at the present stage of development, still to refer to methods of computation based on standard spur-gear practice; but it certainly would be unwise to disregard evident trends and to neglect to consider the more specialized phases of bevel-gear practice recently introduced.

Fortunately, the Lewis type of formula is still used to calculate tooth strength, even for the newer tooth forms. The basic formula is merely modified to allow for the fact that, while a bevel gear is specified by the diameter and the pitch at the *large end*, the strength of the converging teeth is manifestly less than for spur-gear teeth of the same pitch.

To allow for this condition, Lewis * proposed the following formula for the strength of bevel-gear teeth:

$$F = sbp_cy\frac{D_s}{D_l}, \tag{1}$$

* Proceedings of Engineers' Club of Philadelphia, 1892. Formulas combining *sy* into one load factor *C*, suited for the pinion, have been widely used and are convenient. For values, see Norman, *Principles of Machine Design*, p. 333.

where

F = tangential force in lb. at pitch radius of large end,
s = bending stress in p.s.i.,
y = form factor,
b = face width of gear in in.,
p_c = circular pitch at large end in in.,
D_s = pitch diameter at small end in in.,
D_l = pitch diameter at large end in in.

The introduction of the factor D_s/D_l evidently compensates for two things: first, that the tangential force F is determined as if it were concentrated at the large-end diameter, while actually it is spread along the tooth; second, that the circular pitch p_c is taken as the large-end pitch, while actually, as already emphasized, the teeth converge to a smaller pitch at the small end.

If the resultant of the tangential force could be regarded as concentrated on the mean diameter $(D_s + D_l)/2$ and at the mean pitch $p_c(D_s + D_l)/2D_l$; then, the tooth strength being put proportional to the first power of the circular pitch, the Lewis formula for spur gears as applied to bevel gears would give $2FD_l/(D_s + D_l) = sbp_cy(D_s + D_l)/2D_l$, or $F = sbp_cy(D_s + D_l)^2/4D_l^2$. This value is somewhat greater than D_s/D_l values in common use. If it is assumed, as would appear to be necessary, that straight bevel-gear teeth must remain straight and converging on the cone apex even when under load, then it can be proved that the load resultant should be assumed to be concentrated at a diameter somewhat larger than the mean diameter.*

Under ordinary conditions the ratio D_s/D_l should be not less than 2/3, that is, the face of the gear should be not more than 1/3 of the cone distance. The face is limited to this length to facilitate cutting the teeth. The cutting tools must pass through the tooth spaces at the small end of the gear and, if the face were made too long, the cutters would be slender and weak.

To accord with the prevailing practice of specifying gears by diametral pitch instead of by circular pitch, the American Gear Manufacturers' Association substitutes π/p_d for p_c, and combines π and y into one factor, which is denoted by Y. They also introduce the cone distance L and the face b instead of the diameters. Their formula thus takes the form:

(2) $$F = \frac{sbY(L - b)}{p_dL} = \frac{sb\pi y(L - b)}{p_dL}.$$

For cut gears the Association recommends for s the formula

(3) $$s = s_0\left(\frac{1200}{1200 + V}\right),$$

* See Norman, *Principles of Machine Design*, p. 360. Formula (1) is therefore in every way on the safe side.

where V is the tangential velocity in ft. per min. at the large end. For the so-called static stress s_0, the bevel-gear standards merely state that it should not exceed two-thirds of the ultimate strength. A better limiting value for the static stress seems to be that given by the A.G.M.A. for spur gears, which is that the stress should never exceed the endurance strength in fatigue. Values based on the endurance strength are given in Table 54, as are also certain more conservative values given by Buckingham. If there is no particular reason to go to extremes, the latter values should not be exceeded.

TABLE 54

s_0 VALUES FOR VARIOUS MATERIALS[a]
(According to Buckingham, and Am. Gear Mfrs.' Ass'n.)

Material	Buckingham		A.G.M.A.	
	S.A.E. No.	s_0 p.s.i.	Brinell Hardness	s_0 p.s.i.
Cast iron		8,000	160	12,000
Semi-steel		12,000	200	18,000
Bronze		12,000		
Phosphor bronze			100	24,000
Steel, cast	1235	15,000		
Steel			150	36,000
Steel	1030	20,000	200	50,000
Steel	1045	30,000	240	60,000
Steel	3245	40,000	280	70,000
Steel			320	80,000
Steel			360	90,000
Steel			400	100,000

[a] These values should be regarded as maximum values. The Buckingham values correspond to a factor of safety of 3 and are applicable to steady loads on a single pair of gears. For suddenly applied loads on single pairs, the factor of safety, according to Buckingham, should be 4, and the values should be reduced to 3/4 of those given. For gears of a train beyond the first mesh, the factors should be 5 for steady loads and 6 for sudden loads, that is, in the latter case only half of the values listed should be used. There is every reason to reduce the A.G.M.A. values in a similar ratio.

If standard spur-gear tooth forms are used, the form factor y may be selected from Table 46 on page 359 for spur gearing. It means to err on the safe side if y is selected for the actual number of teeth in the bevel gear. If it is desired to arrive at the smallest possible tooth, then the formative tooth number should be used. This number is the actual number of teeth divided by the cosine of the apex angle, or the actual number of teeth multiplied by L/r, where L is the cone distance and r the pitch radius, not of the gear itself, but of the mating gear. For instance, the actual tooth number of the smaller gear should be multiplied by $2L/D$, where D is the pitch diameter of the larger gear.

TABLE 55—"Y" FACTORS FOR GLEASON TYPE OF TEETH

Number of Teeth in Pinion	Ratios														
	1.00 to 1.25	1.25 to 1.50	1.50 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	3.75 to 4.00	4.00 to 4.50	4.50 to 5.00	5.00 to ∞
	Spiral Bevel Gears														
5	.297	.322	.343	.361	.376	.388	.398	.406	.411	.416	.420	.424	.431	.438	.450
6	.310	.332	.353	.372	.386	.398	.406	.414	.419	.424	.428	.431	.436	.443	.452
7	.318	.333	.347	.360	.373	.384	.392	.398	.405	.410	.415	.419	.426	.432	.439
8	.298	.320	.336	.348	.357	.366	.373	.379	.384	.388	.392	.394	.397	.400	.405
9	.292	.313	.327	.338	.346	.352	.357	.363	.367	.370	.373	.376	.380	.384	.388
10	.315	.338	.353	.363	.371	.345	.326	.342	.351	.357	.363	.367	.371	.374	.377
11	.316	.335	.343	.325	.327	.333	.338	.344	.350	.356	.361	.367	.375	.384	.390
12	.298	.318	.333	.343	.351	.357	.363	.368	.372	.377	.379	.381	.384	.386	.388
13	.302	.320	.334	.343	.351	.358	.365	.371	.376	.381	.384	.386	.388	.391	.393
14	.306	.322	.334	.345	.354	.362	.369	.374	.378	.382	.386	.389	.391	.393	.395
15	.314	.330	.342	.352	.360	.368	.374	.380	.385	.389	.392	.394	.397	.399	.402
16	.322	.335	.347	.358	.367	.374	.381	.386	.390	.394	.397	.400	.402	.404	.406
17–18	.329	.343	.354	.364	.373	.382	.389	.394	.398	.400	.403	.406	.407	.409	.410
19–21	.339	.351	.362	.373	.382	.389	.396	.401	.405	.407	.410	.411	.412	.414	.415
22–25	.351	.363	.373	.382	.391	.398	.403	.407	.410	.412	.413	.414	.415	.417	.418
26–30	.364	.374	.384	.393	.399	.404	.407	.410	.412	.414	.415	.416	.417	.418	.419
	Straight Bevel Gears														
10	.231	.260	.280	.294	.305	.315	.324	.332	.340	.347	.353	.358	.365	.371	.377
11	.268	.264	.273	.286	.296	.303	.309	.315	.320	.324	.328	.332	.336	.340	.342
12	.248	.265	.281	.295	.308	.318	.328	.335	.341	.345	.348	.351	.353	.355	.356
13	.264	.278	.291	.280	.278	.286	.291	.295	.298	.299	.301	.303	.305	.307	.310
14	.242	.254	.263	.272	.281	.288	.294	.299	.304	.307	.310	.313	.316	.318	.319
15	.248	.258	.266	.274	.283	.290	.296	.301	.305	.308	.312	.315	.318	.319	.320
16	.252	.261	.269	.277	.285	.292	.298	.304	.308	.312	.314	.317	.319	.321	.323
17–18	.257	.265	.273	.281	.288	.295	.302	.307	.311	.315	.318	.320	.322	.325	.326
19–21	.265	.272	.279	.286	.294	.300	.307	.312	.317	.320	.324	.326	.328	.330	.332
22–25	.274	.281	.288	.295	.301	.307	.314	.319	.324	.327	.331	.332	.335	.337	.338
26–30	.284	.291	.297	.304	.310	.317	.322	.327	.332	.336	.339	.342	.344	.346	.347

If the Gleason Works tooth forms are used, then the form factor Y may be taken directly from Table 55 and used in formula (2), which gives $F = sbY(L - b)/(p_d L)$. It is not necessary to give here all of the details of these tooth forms Suffice it to say that the pressure angle varies from 14½ deg. to 20 deg., and that the gear addendum varies for straight-tooth gears from $1/p_d$ for a ratio of 1 : 1 to $0.540/p_d$ for a ratio of 1 : ∞; and for spiral gears from $0.850/d_p$ for a ratio of 1 : 1 to $0.460/d_p$ for 1 : ∞. For straight-tooth bevel gears in this system, the minimum number of teeth in the pinion is 10, and for spiral bevel gears as low as 5.

317. Strength of Spiral Bevel Gears. In spiral bevel gears the force normal to the tooth is at an angle to the tangential force which transmits the torque, and consequently is greater than the tangential force. Also for equal circular pitch, the spiral tooth is thinner than the straight tooth. On the other hand, the spiral tooth is longer than the straight tooth and there are more teeth in contact. It is therefore regularly assumed that spiral bevel gears are at least as strong as straight-tooth bevel gears of the same pitch.

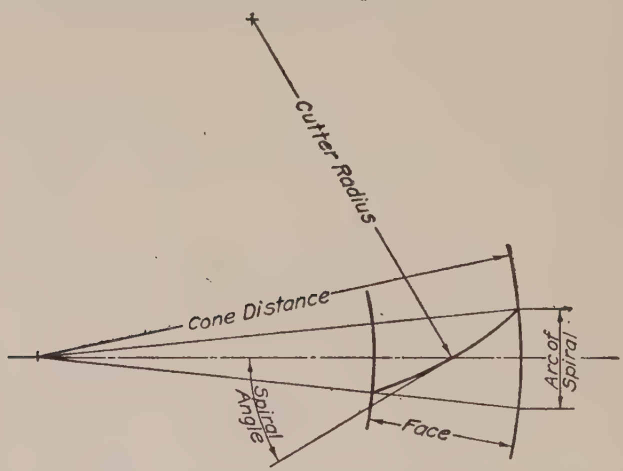


Fig. 359. Method of Measuring the Spiral Angle of Spiral Bevel-Gear Teeth.

The spiral angle is measured, as shown in Fig. 359, for bevels with teeth on the arc of a circle. This angle should be large enough to make the arc of spiral not less than 1.15 to 1.40 times the circular pitch to obtain sufficient overlapping of teeth. A pair of gears are considered right-hand if the teeth of the pinion slope in the same direction as the threads of a right-handed screw. The hand of the spiral should be selected to make the member having the greater axial

thrust tend to move out of mesh under load. Spiral angles vary between 20 deg. and 35 deg., with 30 deg. as the average. For gear ratios of 4 to 1 or more, 10 teeth are recommended as the practical minimum for satisfactory operation. A smaller number of teeth can be used to reduce the size of the installation if extreme quietness is not essential.

318. Wear Resistance of Bevel-Gear Teeth. The A.G.M.A. has adopted the formula for wear resistance as proposed by the Gleason Works, which is as follows:

(4) $$F_w = 376\, bK_mK_s\sqrt{\frac{N}{p_d}}$$ for straight bevel gears,

(5) $$F_w = 470\, bK_mK_s\sqrt{\frac{N}{p_d}}$$ for spiral bevel gears,

where F_w = tangential load allowable for wear in lb. at large end, b = face width in in., N = number of teeth in pinion, K_m = material factor given in Table 56, and K_s = service factor given in Table 57.

TABLE 56

MATERIAL FACTOR, K_m

Material		Material Factor
Pinion	Gear	
Cast iron or unhardened steel	Cast iron	0.30
Heat treated steel	Heat-treated steel	0.35
Case-hardened steel	Cast iron	0.40
Oil-hardened steel	Cast iron	0.40
Case-hardened steel	Unhardened steel	0.45
Oil-hardened steel	Unhardened steel	0.45
Case-hardened steel	Heat-treated steel	0.50
Oil-hardened steel	Heat-treated steel	0.50
Oil-hardened steel	Oil-hardened steel	0.80
Case-hardened steel	Oil-hardened steel	0.85
Case-hardened steel	Case-hardened steel	1.00

The above values are based upon steels of the properties given below:

	Hardness		Steels Commonly Used
	Brinell	Scleroscope	
Unhardened steel	160–190	25–26	S.A.E. 1035
Heat-treated steel	200–260	30–36	S.A.E. 2335, 3140
Oil-hardened steel		70–80	S.A.E. 3245
Case-hardened steel		80–90	S.A.E. 2315, 2512, 3312X

TABLE 57
SERVICE FACTOR, K_s

TYPE OF SERVICE	NATURE OF LOAD	SERVICE FACTOR
Intermittent	Non-pulsating Light shock Heavy shock	1.30 1.00 0.65
Continuous	Non-pulsating Light shock Heavy shock	1.00 0.75 0.50
Starting	Infrequent and of short duration	1.50

319. Example of Bevel-Gear Calculation. The vertical drill spindle of a drilling machine is to be driven by means of a pair of straight bevel gears with the pinion mounted on the horizontal drive shaft. The approximate speed ratio is 2.25 to 1 and the drill must have a torque of 3500 in. lb. at 414 r.p.m. Determine the proportions of the gears.

In our first computation we shall assume 20-deg. stub teeth, a face width of 1/4 to 1/3 of the apex distance, and S.A.E. 1035 steel.

As a preliminary step we may further assume either tooth numbers and a pitch, or a set of diameters. If we select tentatively for the pinion 24 teeth and 6 pitch, the pitch diameter would be $24/6 = 4$ in. The number of teeth in the gear would be $24 \times 2.25 = 54$; but it would be better to use a hunting tooth and use either 53 or 55, say 53. The diameter of the gear would then be $53/6 = 8.83$ in. The torque on the pinion is equal to $3500 \times 24/53 = 1585$ in. lb.

The apex distance $L = \frac{1}{2}\sqrt{4^2 + 8.83^2} = 4.85$ in.

Face $b = L/4$ to $L/3 = 1.21$ to 1.62, say 1.5 in.

$$D = \text{formative diameter of pinion} = \frac{D_p \times L}{r_g} = \frac{4 \times 4.85}{4.41} = 4.39 \text{ in.}$$

N = formative number of teeth in pinion $= 4.39 \times 6 = 26.3$.

From Table 46, page 359, for spur gears, with 26 stub teeth, $y = 0.135$.

Tangential force $F = 1585/2 = 792.5$ lb.

$V = \pi \times 8.83 \times 414/12 = 958$ ft. per min.

Allowable stress (with s_0 from Table 54 equal to 20,000 p.s.i.)

$$s = 20{,}000\left[\frac{1200}{(1200 + 958)}\right] = 11{,}100 \text{ p.s.i.}$$

Actually induced stress $= Fp_dL/(by\pi(L - b))$

$$= \frac{792.5 \times 6 \times 4.85}{1.5 \times 0.135 \times 3.14(4.85 - 1.5)} = 10{,}800 \text{ p.s.i.}$$

Therefore, with 20-deg. stub teeth, the actually induced stress is less than the permissible stress.

If, on the other hand, Gleason-type teeth were used, we would select from Table 55 for straight-tooth bevel gears, a Y value $= 0.301$, and would then find

$$s = \frac{Fp_dL}{Yb(L - b)} = \frac{792.5 \times 6 \times 4.85}{0.301 \times 1.5(4.85 - 1.5)} = 15{,}300 \text{ p.s.i.}$$

This stress is too high and a larger tooth would have to be used.

Checking these gears for wear, we have the permissible tangential load for straight-tooth bevels

$$F_w = 376 b K_m K_s \sqrt{\frac{N}{p_d}}.$$

Taking $K_m = 0.30$ from Table 56, and $K_s = 1.00$ from Table 57, we have

$$F_w = 376 \times 1.5 \times 0.30 \times 1.00 \sqrt{\frac{24}{6}} = 338 \text{ lb.}$$

This value is only about half the tangential force the gears are required to transmit. By the use of oil-hardened steel S.A.E. 3245, the factor K_m would be increased to 0.80 and the required resistance to wear would be attained.

320. Axial Thrust of Bevel Gears. A complication in the use of bevel gears arises from the fact that the normal forces on the teeth have components along the gear axes. These components must be

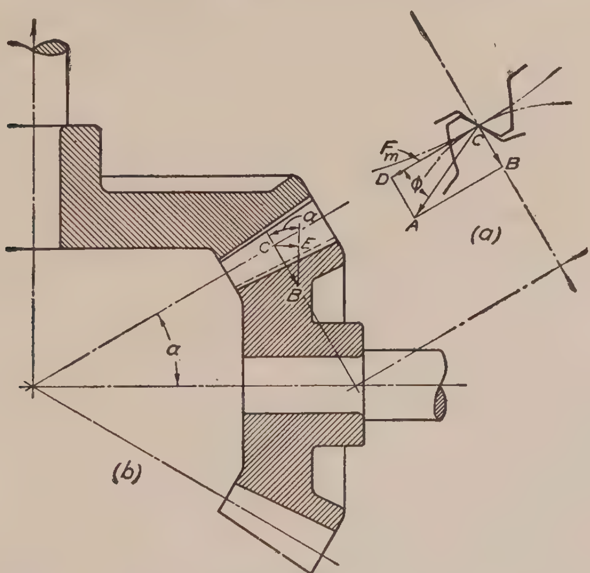


Fig. 360. Axial Thrust of Straight-Tooth Bevel Gears.

taken up either by thrust bearings or by radial bearings with thrust provisions.

In straight-tooth bevel gears the normal force between the teeth, CA in Fig. 360a, is inclined to the tangent CD at the pitch circle by the amount of the pressure angle ϕ. With reference to Fig. 360b, this inclination gives rise to a force $CB = F_m \tan \phi$, which in turn has a component $CE = F_m \tan \phi \sin \alpha$ parallel to the gear axis, where F_m

is the mean tangential force. This end thrust, arising from the angularity of the plane of the tooth normal, decreases with decreasing pressure angle, and would vanish if the pressure angle were zero.

Spiral bevel gears have an end thrust due to the spiral angle γ which does not vanish, even if the pressure angle is zero. As shown in Fig. 361, the normal force CA, which, with zero pressure angle,

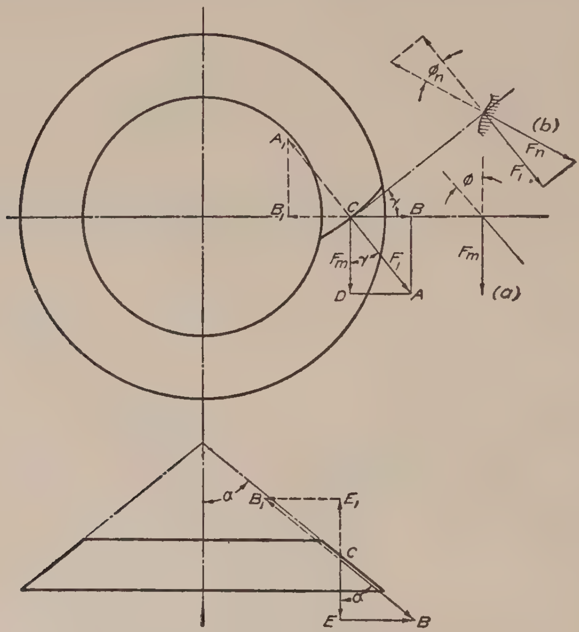


Fig. 361. Axial Thrust of Spiral Bevel Gears.

lies in a plane tangential to the pitch cone, has a component $CB = F_m \tan \gamma$ along the face of the gear, and this force in turn has a thrust component $CE = F_m \tan \gamma \cos \alpha$ along the axis.

This component changes sign to CE_1, if the direction of rotation is reversed. It then has a tendency to pull the gear off the shaft and necessitates fastening the gear by a nut, as shown in Fig. 253.

In addition to the thrust arising from the spiral angle, spiral bevel gears also have the thrust arising from the pressure angle. If the pressure angle, ϕ in Fig. 361a, is measured in a plane containing the tangential turning force F_m, the thrust arising from this source will be $F_m \tan \phi \sin \alpha$, exactly as for straight-tooth bevel gears. If, however, the pressure angle, ϕ_n in Fig. 361b, is measured in a plane normal to the tooth, then the end thrust is $F_m \tan \phi_n \sin \alpha/\cos \gamma$, since $\tan \phi$ is equal to $\tan \phi_n/\cos \gamma$, or F_1 in Fig. 361 is equal to $F_m/\cos \gamma$.

Following the practice of the American Gear Manufacturers' Association and using ϕ_n rather than ϕ, we have for the end thrust of spiral bevel gears the following formulas:

$$F_p = F_m \left(\frac{\tan \phi_n \sin \alpha}{\cos \gamma} \pm \tan \gamma \cos \alpha \right), \tag{6}$$

$$F_g = F_m \left(\frac{\tan \phi_n \sin \beta}{\cos \gamma} \pm \tan \gamma \cos \beta \right). \tag{7}$$

The symbols have the following meaning:

F_p = end thrust of pinion,
F_g = end thrust of gear,
F_m = tangential turning force at mean radius,
ϕ_n = pressure angle in a plane normal to the tooth,
α = cone apex angle of pinion,
β = cone apex angle of gear,
γ = spiral angle.

The plus sign indicates thrust away from the cone apex, the minus sign thrust toward the cone apex. With reference to the pinion, the minus sign applies for right-hand pinions running clockwise as viewed from the large end, and for left-hand pinions running counter-clockwise. For the gear, the minus sign applies to gears mating with left-hand pinions running clockwise, and with right-hand pinions running counter-clockwise.

321. Bearing Reactions. Shafts at an Acute Angle. In the vast majority of bevel-gear applications, the shafts are at right angles. Once in a while, however, this is not the case. An important method of bevel-gear cutting is based on the fact that a gear of any apex angle can be made to mesh with a crown gear, if only the axis of the gear to be cut is inclined at the right angle to the axis of the crown gear. We shall here compute an example of bevel gearing with shafts at an acute angle, to illustrate how bearing reactions, end thrusts, etc., may be obtained.

Example. Two shafts intersecting at an angle of 75 deg. are connected by straight-tooth bevel gears to give a velocity ratio of 3 to 1. The pinion has a mean pitch diameter of 8 in. and delivers 10 hp. at 400 r.p.m. The teeth have a $14\frac{1}{2}$ deg. pressure angle in the normal plane. (To emphasize the desired points the forces will be considered as acting at the mean radius and the effect of friction will be neglected.) The mean plane of the gear is 5 in. from the right bearing and 4 in. from the left bearing, as shown in Fig. 362b. Determine the pitch angles of the gears, the forces on the gear bearings, the bending moment on the gear shaft, and the necessary diameter of the shaft.

Gear forces. The law of sines applied to Fig. 362*a* to determine the cone angles gives

$$\frac{12}{\sin\beta} = \frac{4}{\sin\alpha}, \qquad \text{or} \qquad \beta = 58^\circ\,30', \quad \alpha = 16^\circ\,30'.$$

The tangential force $F_m = \dfrac{hp \times 33{,}000 \times 12}{\pi D n} = \dfrac{10 \times 33{,}000 \times 12}{\pi \times 8 \times 400} = 395$ lb.

$\tan\phi = \tan 14\tfrac{1}{2}^\circ = 0.259$, $\sin\beta = \sin 58^\circ\,30' = 0.853$, $\cos\beta = \cos 58^\circ\,30' = 0.523$.

$$F_x = 395 \times 0.259 \times 0.853 = 87.3 \text{ lb.}, \qquad F_y = 395 \times 0.259 \times 0.523 = 53.5 \text{ lb.}$$

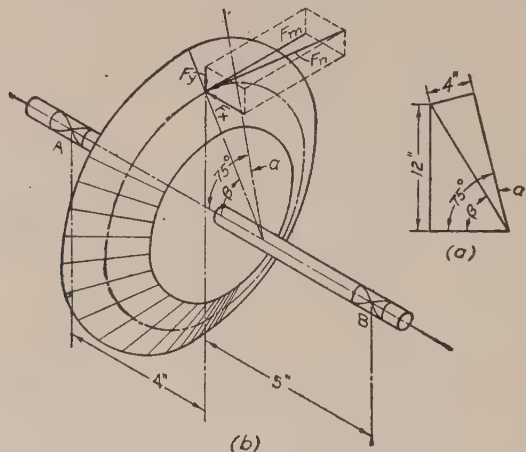


Fig. 362. Forces Acting on Straight-Tooth Bevel Gear.

Bearing reactions. Horizontal reactions due to F_m:

At bearing A, $\qquad R_{AF} = \dfrac{5}{9} \times 395 = 219.5$ lb.

At bearing B, $\qquad R_{BF} = \dfrac{4}{9} \times 395 = 175.5$ lb.

Vertical reactions due to F_x:

$$R_{AF_x} = \frac{12}{9} \times 87.3 = +116.4 \text{ lb. (up).}$$

$$R_{BF_x} = \frac{12}{9} \times 87.3 = -116.4 \text{ lb. (down).}$$

Vertical reactions due to F_y:

$$R_{AF_y} = \frac{5}{9} \times 53.5 = 29.8 \text{ lb. (up).}$$

$$R_{BF_y} = \frac{4}{9} \times 53.5 = 23.7 \text{ lb. (up).}$$

R_{AV}, vertical component of reaction at $A = R_{AF_x} + R_{AF_y} = 116.4 + 29.8 = 146.2$ lb.
R_{AH}, horizontal component of reaction at $A = R_{AF} = 219.5$ lb.
R_A, total reaction at $A = \sqrt{(R_{AV}^2 + R_{AH}^2)} = \sqrt{(146.2^2 + 219.5^2)} = 264$ lb.
M_b, total bending moment in shaft $= R_A \times 4 = 264 \times 4 = 1056$ in. lb.
M_t, torque $= 395 \times 12 = 4740$ in. lb.

Assuming the shaft to be made of cold-rolled steel of a tensile strength of about 75,000 p.s.i. with a factor of safety of 6, we can calculate the necessary diameter for strength. The load is considered to be of a steady character.

$$\frac{\pi D^3}{32} s_t = \sqrt{(M_b^2 + M_t^2)},$$

$$\frac{\pi D^3}{32} \times \frac{75{,}000}{6} = \sqrt{(1056^2 + 4740^2)} = 4856,$$

$$D^3 = 3.96,$$

$$D = 1.58 \text{ in., say } 1\tfrac{5}{8} \text{ in.}$$

PROBLEMS

1. Two cut cast-steel miter gears with $14\frac{1}{2}$ deg. composite involute teeth have a face width of $2\frac{1}{2}$ in., a pitch diameter of 9 in., and a diametral pitch of 3. Compute the outside diameter of the gears. What hp. can be transmitted at 200 r.p.m.?

2. A bevel-gear speed reducer has 18 teeth on the pinion and 99 teeth on the gear, both being of a hardened alloy steel. The diametral pitch is 4 and the width of face is $2\frac{1}{4}$ in. The rating listed is 56 hp. at 900 r.p.m. of the pinion. Determine the probable material and form of tooth. Is this rating conservative?

3. A bevel-gear speed reducer with 14 and 49 teeth, $14\frac{1}{2}$ deg. composite form, is rated at 30 hp. at 200 r.p.m. of the pinion. The diametral pitch is $1\frac{1}{2}$ and the width of face is $4\frac{5}{8}$ in. If the gear is cast iron, what would be a safe continuous horsepower rating?

4. A 16-in. diameter bevel gear running at 500 r.p.m. drives a 15-tooth pinion. The gears have 20 deg. stub teeth of 5 pitch and a width of face of 2.5 in. What materials are necessary for the gear and pinion if the gears are to transmit 20 hp.?

5. A bevel-gear pinion of 24 teeth, rotating at 900 r.p.m., meshes with a bevel gear of 72 teeth. Both gears are alloy steel, the teeth being 20 deg. full-depth involute, 6 pitch. The width of face is 2 in. What safe hp. will these gears transmit?

6. Two shafts at right angles are driven through cast-iron bevel gears with an angular-velocity ratio of driver to driven of 4 to 3. The driver transmits 3 hp. at 1100 r.p.m. Assuming a pitch diameter of 3 in. for the driver and a width of face equal to $\frac{1}{3}$ the slant height of the pitch cone, determine the necessary standard diametral pitch of the gear for $14\frac{1}{2}$ deg. composite teeth.

7. A pair of cast-iron straight-tooth Gleason bevel gears are to have a velocity ratio of 3.25 to 1. For the transmission of 22.5 hp. at 1250 r.p.m. of the pinion, select the standard diametral pitch of the teeth for satisfactory strength and wear. Give also the width of face and the diameters.

8. A spiral-bevel-gear speed reducer has 26 teeth on the pinion and 104 teeth on the gear. If the teeth are 20 deg. full-depth of 12 pitch and $1\frac{1}{16}$ in. face, what should be the horsepower rating at 1200 r.p.m. of the pinion? The gears are made of S.A.E. 3245, hardened.

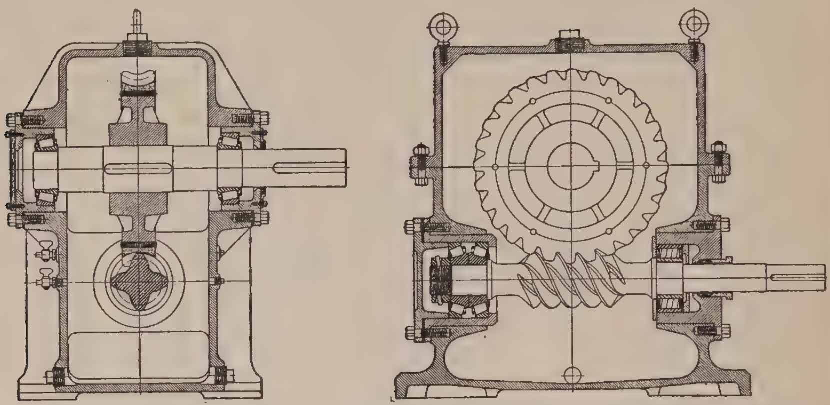
9. What diametral pitch and width of face are necessary for Gleason spiral bevel gears, transmitting 40 hp. at 2500 r.p.m. of the pinion with a velocity ratio of 4 to 1? The pinion has 10 teeth and is made of S.A.E. 1045 steel.

10. A pair of bevel gears of 12 and 25 teeth, respectively, have 20 deg. full-depth teeth and a pitch of 5. They transmit 4 hp. at 1200 r.p.m. of the pinion. If the pinion is a right-hand spiral bevel with a spiral angle of 40 deg. and the pinion rotates clockwise, determine the forces acting on the bearings.

CHAPTER 21

WORM GEARS

322. Definition and Description. A simple worm-gear combination consists of a screw meshing with a helical gear. The screw is called the *worm* and the gear the *worm gear*. The worm shaft is



W. A. Jones Foundry & Machine Co., Chicago, Ill.

FIG. 363. WORM-GEAR SPEED REDUCER.

usually at right angles to the gear shaft, as shown in Fig. 363, but other angular combinations are occasionally found in practice. For

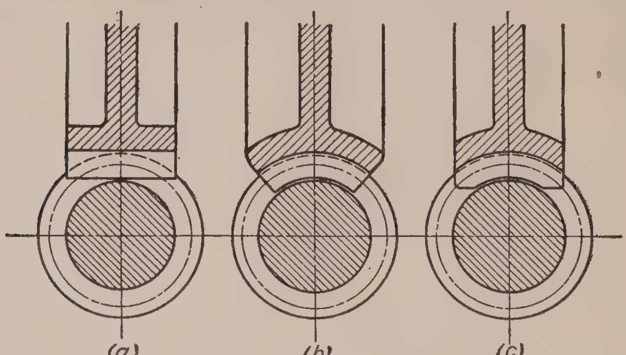


FIG. 364. DIFFERENT FORMS OF WORM-GEAR RIMS.

small capacities, the worm-gear teeth may be cut on a cylindrical surface and the gear is then identical to a helical gear. Figure 364a

shows this arrangement in a section at right angles to the worm axis. Usually the gear teeth are formed to fit the curvature of the worm to some extent, as shown in transverse section in Figs. 364b and 364c. An arrangement similar to the one in Fig. 364c is no doubt the most commonly used type for high-grade gearing suitable for fairly high speeds.

The thread outline of the worm in axial section is that of a rack and is usually made straight-sided. The *helix angle* is the angle between the *pitch helix* and a plane at right angles to the worm axis (Fig. 365). The *lead* is the distance the thread advances in one revolution. The *linear pitch* is the distance between similarly located points on adjacent threads of the worm in axial section, and this distance is equal to the circular pitch of the gear. The lead and the pitch are equal if there is only one thread on the worm. If there are several threads in parallel, then the lead is equal to the pitch times the number of threads in parallel. According to the number of threads in parallel, the worm is called a single-, double-, triple-, or quadruple-threaded worm, and higher multiples may occur.

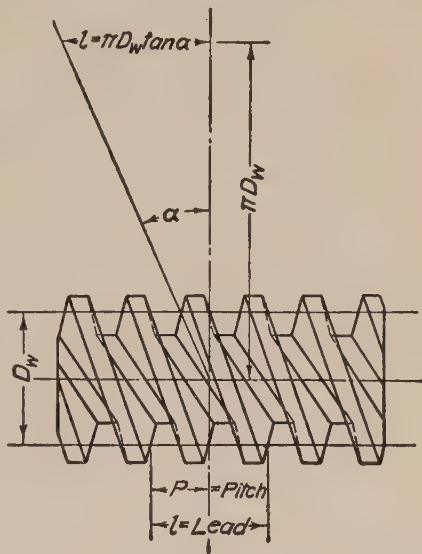


FIG. 365. DOUBLE-THREADED WORM.

Referring to Fig. 365, and calling the lead l, the helix angle α, and the worm-pitch diameter D_w, we have $l = \pi D_w \tan \alpha$. This value is the distance the worm thread advances in one revolution, and is only a part of the gear circumference expressed by the fraction $\pi D_w \tan \alpha / \pi D_g$, where D_g is the gear-pitch diameter. The transmission ratio is therefore

$$(1) \qquad \frac{n_w}{n_g} = \frac{N_g}{N_w} = \frac{\pi D_g}{\pi D_w \tan \alpha} = \frac{D_g}{D_w \tan \alpha},$$

where n_w is the r.p.m. of the worm, n_g the r.p.m. of the gear, N_g the number of teeth on the gear, and N_w the number of threads in parallel on the worm.

The fact that the transmission ratio depends not only on the diameter ratio, which has its limitations, but on tan α, which can vary

from zero to infinity, makes it possible to obtain a very wide range of speed ratios with worm gearing. As an example, the ratios of standard gear sets manufactured by a leading firm range from 3⅝ to 1 to 100 to 1. By the use of two sets of gears in series, the speeds of high-speed electric motors can be transformed to speeds of one or two revolutions per minute. Such reductions are needed, for instance, in driving rotary kilns.

323. Efficiency. For a long time worm gearing was used only for very great transmission ratios. It was taken for granted that the efficiency would be very low, because in many cases the gearing had to be *self-stopping*. Thus, in a worm-gear hoist it was desired that the load should not descend if left to itself, even if no special brake were provided. Just as in screws, this requirement meant that the helix angle of the worm thread had to be less than the friction angle. The friction force was at least of the same order of magnitude as the force necessary to hoist the load, and in consequence the efficiency was about 50 per cent or less.

It is obvious that if the helix angle, which moves the gear ahead, is considerably larger than the friction angle, the useful work will be much greater than the friction work, and the efficiency will be quite high. If we apply, for a first approximation, the formula for the efficiency of a square screw thread given on page 129 to the efficiency of worm gearing, we have

$$\text{(2)} \qquad \text{Efficiency} = \frac{\tan \alpha}{\tan (\alpha + \phi)} = \frac{1 - f \tan \alpha}{1 + f/\tan \alpha},$$

where f is the coefficient of friction and is equal to $\tan \phi$, where ϕ is the friction angle.

It will be seen that for α equal to zero, the efficiency is zero. Likewise, the efficiency becomes zero when $f \tan \alpha = 1$, or when $\tan \alpha = 1/f$. If f is very small, this value gives an α close to 90 deg. If $f = 0.05$, it gives a value of about 87 deg.; and if $f = 0.10$, a value of about 84 deg. The variation of the efficiency for $f = 0.05$ is shown by the curve in Fig. 366. It runs very flat at about 90 per cent efficiency from 30 to 60 deg., and remains above 80 per cent for a much wider range. This curve contains no allowance for the fact that the worm thread is not square, but angular, nor for the friction loss in the bearings. With modern ball and roller bearings and with the worm dipping in oil so as to produce so-called film lubrication, the friction loss is very low, even for helix angles of 15 to 20 deg. In fact, trouble has been experienced in producing self-stopping worms even with thread

angles of but a few degrees. With worms of ordinary rack-shaped section, efficiencies * as high as 98 per cent have been measured.

In view of the fact that such efficiencies may be attained by worms having the ordinary rack-shaped section, there is little need to consider worms of hour-glass shape (Hindley Worms), or other modified shapes, which are more difficult to produce and were originally introduced to improve efficiency. Some modifications of thread forms may be introduced to increase load capacity, and at the present time considerable research is being done in this direction.

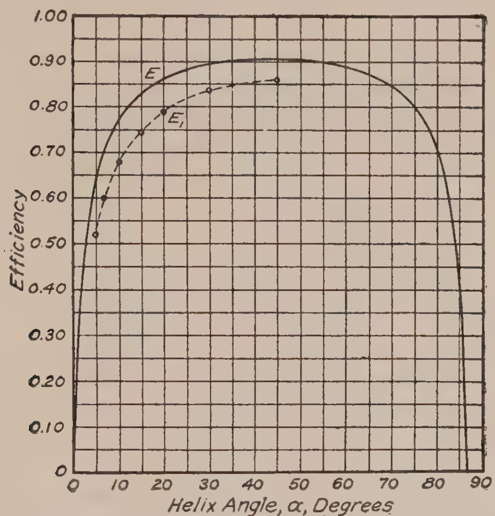


FIG. 366. EFFICIENCY OF WORM GEARING. CURVE E DERIVED FROM FORMULA (2) WITH COEFFICIENT OF FRICTION 0.05. CURVE E_1 PLOTTED FROM EXPERIMENTS BY LEWIS. (Transactions A. S. M. E., vol. 7.)

324. Load Capacity of Worm Gearing. For the rating of worm-driven speed reducers of high-grade workmanship, the American Gear Manufacturers' Association recommends the following empirical formula:

$$hp = \frac{n}{R} KQm, \tag{3}$$

where n = r.p.m. of the worm, R = gear ratio, K = pressure constant to be taken from Table 58, Q = ratio constant having the approximate value $R/(R + 2.5)$, and m = velocity factor dependent on center distance, transmission ratio, and worm speed.

* Kennerson, Transactions A.S.M.E., vol. 34, 1912, p. 919.

The A.G.M.A. standards supply a number of diagrams * giving these factors in a not altogether simple manner. An investigation shows that for average conditions, and with an approximation on the conservative side, we may set $m = 450/(450 + V + 3V/R)$, where V is the circumferential velocity of the worm at the pitch line in ft. per min. This statement is based on the assumption recommended by the A.G.M.A. that the worm-pitch diameter be made equal to $C^{0.875}/2.2$, where C is the center distance between the worm and the gear in in. The recommended face width of the gear is $C^{0.875}/3$, that is, the face width is 2.2/3, or 0.73 × worm-pitch diameter.

TABLE 58

Pressure Constant K in Formula (3)

Center Distance in Inches	K	Center Distance in Inches	K
1	0.0125	10	1.20
2	0.0125	15	4.0
3	0.04	20	8.0
4	0.09	30	29.0
5	0.17	40	66.0
6	0.29	50	120.0
7	0.45	60	200.0
8	0.66	70	320.0
9	0.90		

Other recommendations are:

1. All worms must be case-hardened to a surface hardness of not less than 70 Shore on the threaded portion.

2. All worms must be accurately ground.

3. All gear rims must be cast bronze † free from impurities and according to one of the following analyses.

S.A.E. No. 65

Copper	88 to 90%
Tin	12 to 10%
Phosphorus	0.10 to 0.30%
Lead, zinc, and other impurities	0.50% max.

Nickel Bronze

Copper	87 to 88%
Tin	10.75 to 10.25%
Nickel	1.25 to 1.75%
Lead	0.25% max.
Phosphorus	0.25% max.
Zinc	Trace

4. All gear rims are to be hobbed to at least 50 per cent initial bearing as determined by running the mating parts in their proper relationship with red lead, or some similar marking material.

* The formulas for Q and m are given mainly because it is impossible to reproduce all of the various diagrams of the A.G.M.A. standards. It is stated by the Association that the diagrams represent actual design practice.

† Aluminum bronze has often been used for automotive worm gears.

The gearing is assumed to be encased in a housing of the type shown in Fig. 363. The surface area of this housing, exclusive of base, flanges, and fins, is to be not less than $0.3C^{1.7}$ sq. ft.

If overheating is to be avoided, the horsepower * developed in such a housing when merely air-cooled must not exceed $9.5C^{1.7}/(R + 5)$. This value applies to worm speeds of less than 2000 r.p.m. For speeds above 2000 r.p.m. artificial cooling must be provided.

The above formulas are applicable to reducers in service from 8 to 10 hours per day without regularly recurring shocks. For 24-hour service under conditions involving shock, the rating obtained from the formulas should be divided by 1.33, and, of course, the thermal rating must not be exceeded. For intermittent service of not more than 15 minutes every two hours, the rating from the formulas may be multiplied by 1.43 and the thermal rating may be neglected. Intermediate conditions may be corrected for according to the designer's judgment within these limits.

For worm speeds below 100 r.p.m., a torque rating figured on the basis of the horsepower rating at 100 r.p.m. may be used. This torque in in. lb. is equal to (hp. at 100 r.p.m.) $\times$ 63,024/100 $\doteq$ 630 $\times$ (hp. at 100 r.p.m.). To obtain the torque output at the gear shaft, it is suggested that an efficiency of 97 per cent be assumed for transmission ratios of 6 to 1 and less, and that for higher ratios, the efficiency be assumed equal to 100 per cent minus half the transmission ratio. Thus for a ratio of 10 to 1 the assumed efficiency would be $100 - 10/2 = 95$ per cent.

325. Tooth Loads and Strength. The speed-reducer formula of the American Gear Manufacturers' Association contains no allowance for tooth size and therefore does not enable the designer to determine the pitch. In the determination of tooth size and strength, it is safe to assume that the worm gear should govern the design rather than the worm. The continuous section of the worm thread offers greater resistance to bending than the gear tooth, and furthermore the worm is almost invariably made of steel, whereas the gear is usually made of cast iron or bronze.

For computing tooth size, a modification of the Lewis formula is commonly used, in which the strength of the worm-gear tooth is determined in the same manner as for a spur gear. In worm gearing two or more teeth are usually in contact, but as the actual tooth loads are somewhat indefinite, only one tooth is assumed to transmit the power, and lower working stresses are used. It is the practice of

* This formula is obtained by evaluating a chart.

the Foote Bros. Gear & Machine Co. to use a static stress of 5300 p.s.i. for cast iron and 8000 p.s.i. for bronze, with a speed factor $600/(600 + V_g)$. With these values, the Lewis equation takes the form

$$F = s_0 \left(\frac{600}{600 + V_g} \right) b p_c y, \tag{4}$$

in which F = tangential turning force in lb. at the pitch radius of gear, s_0 = static stress of the gear material, p.s.i., V_g = pitch-line velocity of the gear in ft. per min., b = face width of gear in in., p_c = circular pitch of gear (linear pitch of worm) in in., and y = form factor of gear tooth (see Table 46).

The capacities of worm gears as determined by formula (4) are recommended for average conditions, where loads are fairly uniform and service is intermittent. For continuous service, the ratings should be reduced one-third.

As previously explained, the efficiency of worm gearing is dependent on a minimum of sliding between the teeth and a large helix angle. Obviously, a large helix angle can be attained only by keeping the worm diameter as small as possible. The minimum worm diameter is limited by the size of worm shaft, which must resist a bending deflection due mainly to the side thrust of the teeth. According to Unwin, this deflection should not exceed two- or three-thousandths of an inch. Good results may be obtained if the pitch diameter of the worm D_w is determined by the empirical equation

$$D_w = 3p_c. \tag{5}$$

The length L of the threaded portion of worm according to the A.G.M.A. is given as

$$L = p_c \left(4.5 + \frac{N}{50} \right), \tag{6}$$

where N is the number of teeth on the worm gear.

326. Tooth Computation. Formulas (3) and (4) are based on the supposition that the units are fully assembled, of high-grade workmanship, encased and adequately lubricated, with the housings arranged for the gears to run in oil. It has been found that the quality of the lubricant has a considerable influence on the capacity; that is, overheating will take place sooner with one kind of oil than with another. Formulas for tooth size should perhaps be based on pressure per unit of contact area between the teeth, but this area is difficult to determine. In automotive applications, it has been customary to allow about 2000 p.s.i. of projected area of tooth overlap. This projected area is

equal to the working depth of the tooth times the arc width of gear measured on the pitch line.

For the general run of commercial worm gearing not encased in housings and not adequately lubricated, much lower tooth loads are advisable. An idea of such loads may be obtained by the formulas of Kutzbach, which are as follows:

(7) $$F = bp_cC',$$

in which the terms are the same as before and C' has the following values:

(8) for cast iron on cast iron, $$C' = \frac{226{,}000}{400 + V_s},$$

(9) for steel on bronze, $$C' = \frac{340{,}000}{400 + V_s}.$$

Here V_s is the sliding velocity between worm and gear tooth in ft. per min. If the circumferential velocity of the worm at the pitch line is V, then V_s is $V/\cos\alpha$.

An example will show the application of these different formulas and the variation of results obtained.

Example. Determine the proportions of a worm gear set to transmit 10 hp. from a shaft running at 1700 r.p.m. to one running at 85 r.p.m., the transmission ratio being 20.

Problem (a). Let us first consider this gear set as a speed-reducer unit, encased in a housing, and apply the A.G.M.A. formula (3). If we tentatively assume a gear diameter of 12 in. and a worm diameter of 2½ in., the center distance is $6 + 1.25 = 7.25$ in. We find $C^{0.875} = 5.66$; hence the pitch diameter of the worm is $5.66/2.2 = 2.57$ in., and the face width of the gear $5.66/3 = 1.89$ in. If the worm diameter is made 2.6 in. for convenience and the face 1.9 in., the corrected center distance is 7.3 in. From Table 58, we find the pressure constant $K = 0.51$.

The ratio constant $Q = 20/(20 + 2.5) = 20/22.5$. The worm pitch-line velocity V is $\pi \times 2.6 \times 1700/12 = 1160$ ft. per min. The value of the velocity constant m is $450/(450 + V + 3V/R) = 450/(450 + 1160 + 3 \times 1160/20) = 0.252$.

From formula (3) we have $hp = nKQm/R$, so that

$$hp = \frac{1700 \times 0.51 \times 20 \times 0.252}{20 \times 22.5} = 9.8.$$

This result is close enough to the required 10 hp.

Problem (b). To determine the tooth size by formula (4) we must first compute the tangential force. We shall be on the safe side if we assume a worm efficiency of 100 per cent and consider 10 hp. actually transmitted to the gear teeth. The gear pitch-line velocity $V_g = \pi \times 12 \times 85/12 = 268$ ft. per min., and the tangential force $F = 10 \times 33{,}000/268 = 1230$ lb.

The form factor y depends on the pressure angle of the tooth and the number of teeth in the gear. We shall select a pressure angle of 20 deg. to agree with the solution under (a). The number of teeth will have to be a multiple of the transmission

ratio 20, for instance, 20 for single-threaded worm, 40 for double-threaded, etc. As a first trial, we will assume a double-threaded worm.

From Table 46 on page 359, y for 40 teeth 20 deg. pressure angle = 0.124. The face width b as already found in problem (a) equals 1.9 in. We have from formula (4)

$$p_e = \frac{F(600 + V_g)}{s_0 \times 600 \times b \times y},$$

so that

$$p_c = \frac{1230(600 + 268)}{8000 \times 600 \times 1.9 \times 0.124} = 0.945 \text{ in.}$$

We would select 1 in. as the nearest standard circular pitch. The pitch diameter of the gear would then be $40 \times 1/\pi = 12.73$ in., and the center distance $6.365 + 1.3 = 7.665$ in. This value is quite close to the center distance of 7.3 established in problem (a). Actually, if we use 0.945 in. as the pitch, the center distance happens to be 7.3 in., but such a pitch is not standard. Checking the worm diameter by formula (5), $D_w = 3p_c$, or $D_w = 3 \times 1$ in. = 3 in. This value is in fairly close agreement with the diameter of 2.6 in., as found in problem (a).

If we had assumed a single-threaded worm instead of a double-threaded worm, p_c, due to the higher tangential load, would be 1.15 in. The center distance would then be entirely too small and incidentally the unit would have a low efficiency. A triple-threaded worm gives a pitch of 0.870 in., but the center distance is now too large. Consequently, a double-threaded worm is the only one that will satisfy the proportions as established in problem (a), that is, a gear diameter of approximately 12 in.

Problem (c). For comparison, it is found that the following dimensions may be obtained by the use of formula (7) for a commercial, industrial gear set without housing:

Gear-Pitch Diameter	22.3 in.
Worm-Pitch Diameter	4 in.
Circular Pitch	1¾ in.

327. Radial and Thrust Loads on Bearings. Referring to Fig. 367, if the normal force on the gear tooth is F_n, and the pressure angle in the plane normal to the tooth is δ_n, the component in a plane tangential to the pitch cylinder of the worm is $F_n \cos \delta_n$, and the tangential turning force in this plane is $F = F_n \cos \delta_n \cos \alpha$, where α is the helix angle. There is, however, in the same plane, a horizontal side thrust $F_h = F_n \cos \delta_n \sin \alpha = F \tan \alpha$. This side thrust is increased by the component $F_n f \cos \alpha = Ff/\cos \delta_n$ of the friction force $F_n f$ along the thread, where f is the coefficient of thread friction. The total horizontal force is then $F(\tan \alpha + f/\cos \delta_n)$. A common practice for adequately lubricated worm drives is to add to the helix angle α about 3 deg. to allow for the friction angle.

There is also a vertical thrust $F_v = F_n \sin \delta_n = F \tan \delta_n/\cos \alpha$ normal to the worm axis and normal to the pitch circles of the worm and the gear at the point of contact. This force tends to separate the worm from the gear.

The horizontal thrust F_h and vertical thrust F_v cause radial reactions at the worm-shaft bearings. These reactions are found by taking moments about the bearing centers in the usual manner. The tangential force F causes a tipping moment $FD_w/2$ on the worm shaft and is held in equilibrium by a couple R_wL_w, where R_w is the reaction at the bearings and L_w is the distance between bearing centers.

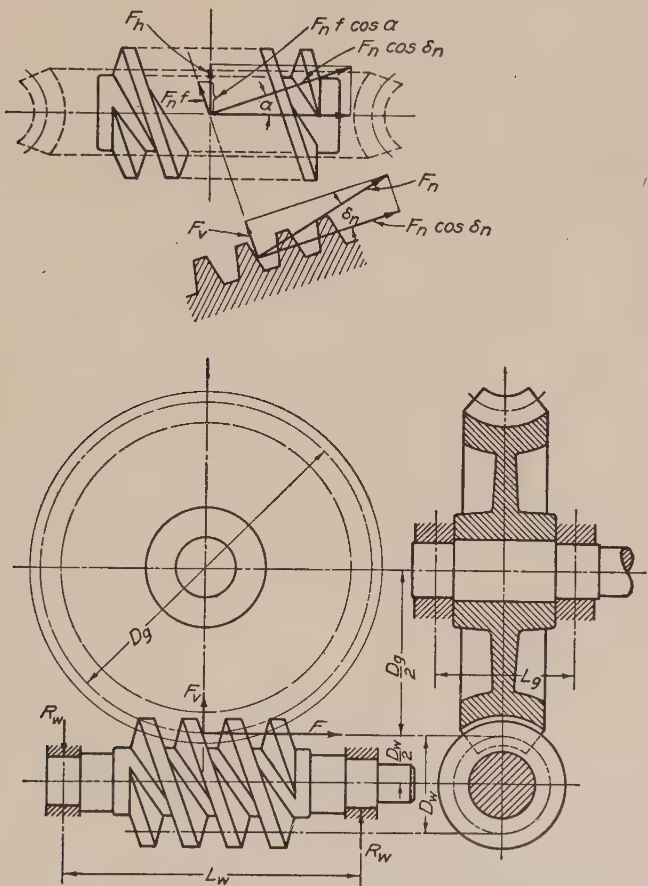


FIG. 367. FORCES ACTING ON WORM AND WORM GEAR.

The tangential turning force F and the vertical thrust F_v cause radial reactions on the gear-shaft bearings. The horizontal thrust F_h causes a tipping moment $F_hD_g/2$ on the gear shaft and is balanced by a couple R_gL_g, just as in the case of the worm.

All the radial forces at each bearing should be compounded and the resultant force thus determined is the radial load on the bearing.

In addition to the radial loads, there are end-thrust loads on the bearings. The end thrust on the worm-shaft bearings is the reaction to the tangential force F. The end thrust on the gear-shaft bearings is the reaction to the horizontal thrust F_h.

328. Screw Gearing. This form of gearing as shown in Fig. 368 is commonly misnamed "spiral gearing," but essentially it is the same as worm gearing and is used for the same type of service. The diameters, however, are usually more nearly equal than they are in ordinary worm gearing, and with so many threads or teeth in parallel, only a fraction of a complete lead or helix turn is found on the gears. It is apparent from the figure that a pair of these gears actually consists of a pair of helical gears mounted on shafts at an angle.

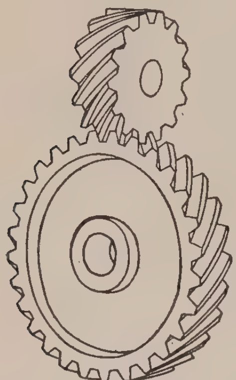


FIG. 368. SCREW GEARS.

It is customary to specify the helix angle for screw gears, α, as the angle between the tooth and a plane at right angles to the axis of the shaft, rather than as the angle between the tooth and a plane through the axis of the shaft, as in helical gears. With this practice, the formula for the transmission ratio will be the same as for a worm drive. If we denote the diameter of the driver as D_1, its helix angle as α, the diameter of the driven gear as D_2, and the number of revolutions per minute of the driver and driven as n_1 and n_2, respectively, we have

$$\frac{n_1}{n_2} = \frac{D_2}{D_1 \tan \alpha}, \tag{10}$$

where the driver D_1 takes the place of the worm in worm gearing.

Screw gears of this type are suitable for comparatively light service, as compared to worm gearing, on account of the small contact area involved and the highly localized pressure between teeth. Gearing used in this manner may have to be computed for loads perhaps not more than half as high as the very conservative loads given in formulas (7), (8), and (9).

At present, according to Trautschold,* worm gearing of low transmission ratios is being substituted for screw gearing. Worm gears of this type are computed for the same tooth loads as apply on any other regular worm-gear combinations.

* See Trautschold, *Standard Gear Book*, McGraw-Hill, p. 121, 1935.

Note.—The following pressure angles and pitches have been suggested by A.G.M.A. for industrial worm gears: For single- and double-threaded worms 14½ deg. composite teeth with an addendum of $0.318p_c$, and for triple- and quadruple-threaded worms 20 deg. teeth with an addendum of $0.286p_c$. The latter are regarded as stub teeth with form factors as given on p. 359. Standard pitches are: 1/8, 5/16, 3/8, 1/2, 5/8, 3/4, 1, 1¼, 1½, 1¾, and 2 in. All pressure angles are normal angles, i.e., they are measured in a plane at right angles to the thread.

PROBLEMS

1. A worm gear 15 to 1 reduction unit is rated at 10.5 hp. at 1150 r.p.m. of the worm. The worm is double-threaded and has a pitch diameter of 3.000 in. and a circular pitch of 1.125 in. The worm wheel has a face width of 2 in. The worm is made of heat-treated nickel steel and the worm wheel of bronze. Compare this rating for smooth loading with calculated values.

2. A worm and wheel are to have a reduction of 10 to 1 and a center distance of 13.50 in. If four threads are used on the worm, select the material and fix upon the circular pitch, helix angle, and width of face necessary to transmit 100 hp. at 3600 r.p.m. of the worm. Will the unit function under continuous load?

3. A worm gear 20 to 1 reduction unit is to transmit 1 hp. at 1750 r.p.m. of the worm under continuous shock-load service. If the center distance is to be 3.00 in., determine the necessary proportions of the drive.

4. A worm and wheel combination has the following specifications: ratio 12¼ to 1, rated 2.35 hp. at 1750 r.p.m. of the worm. Worm has four threads, 7 diametral pitch, 20 deg. pressure angle, center distance, 5.1875 in. Determine the forces producing radial and thrust loads on the bearings for both worm and wheel. What is the probable efficiency?

5. A worm of 3 threads, 1 in. pitch, drives a 63-tooth bronze wheel at 50 r.p.m. The distance between shaft centers is 12.95 in. For a torque of 10,000 in. lb. applied to the wheel, determine if the gear is satisfactory.

6. A worm and wheel are used to transmit power between shafts at right angles and 13.00 in. between centers. The worm rotates at 1200 r.p.m. and the torque on the wheel is 35 times the torque on the worm, neglecting friction losses. For a bronze wheel of 1 in. pitch, determine the hp. (Note: The diameter of the worm should be about 3 times the circular pitch.)

7. Design a worm and wheel unit to meet the following requirements: center distance = 10.00 in., velocity ratio = 14 to 1, worm speed = 900 r.p.m. (Suggestion: Use a triple-threaded worm and a cast-iron wheel.) What maximum hp. may be transmitted if the coefficient of friction is 0.15?

8. A worm and wheel unit is to deliver 10 hp. at 400 r.p.m. of the worm with a velocity ratio of 23 to 1. The worm is to be triple-threaded and the coefficient of friction is taken as 0.15. (a) Fix upon the circular pitch and the width of face of the wheel. (b) What is the center distance? (c) What is the hp. input?

9. Two shafts at right angles and 10.00 in. between centers are to transmit 20 hp. with a velocity ratio of 14 to 1. Design a worm and wheel unit for these conditions.

CHAPTER 22

METHODS OF FORMING AND FINISHING GEAR TEETH

329. Methods Used. The fundamental principles used in the formation of gear teeth and the ingenious methods that have been devised in the development of automatic gear-cutting and finishing machines can be described but briefly in a general textbook on machine design. The principal methods of tooth formation are: *casting*, *form cutting*, *point cutting from a master templet*, and *generating*. In the generating process, a rack-shaped cutter, pinion-shaped cutter, or hob may be used. The principal methods of finishing are: *burnishing*, *shaving*, *grinding*, and *lapping*.

330. Cast Teeth. The teeth may be formed in the *casting* process by using a complete pattern for the gear, in which case the pattern must be provided with sufficient draft to permit its withdrawal from the sand. This process when applied to spur gears results in teeth which are thicker at one end than at the other. Considerable backlash may be expected due to irregularities in the pattern, variations in molding, and unevenness of the tooth surface.

In the machine molding process, the teeth are formed individually by using a single-tooth pattern, the spacing being controlled by an accurate indexing device. This method allows the withdrawal of the pattern radially, thus eliminating the taper mentioned above. A machine-molded gear is more accurate than a pattern-molded gear and can be run at higher speeds.

Very accurately formed gears may be obtained by the die casting process and, for lighter applications, sheet metal stampings are used.

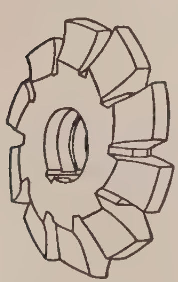


Fig. 369. Formed Milling Cutter.

331. Formed Teeth. Gear teeth may be *formed* to shape by feeding a rotary milling cutter (Fig. 369) across the gear blank. The contour of the cutter is the shape of the space between the teeth. After one tooth is cut the blank is indexed to the next tooth and the cutting operation repeated. As the tooth shape varies with the number of teeth in a series of gears of the same pitch, a set of eight cutters is ordinarily used in the involute system to cut the entire range of spur gears

from 12 teeth to a rack. The contour of the cutter approximates the tooth outline for the tooth numbers as designated below, being correct for the lowest number in the range.

No. Teeth	12–13	14–16	17–20	21–25	26–34	35–54	55–134	135–Rack
Cutter No.	8	7	6	5	4	3	2	1

While helical gears may be cut with formed cutters, this method cannot be applied satisfactorily to bevel gears because of the change in pitch as the teeth converge towards the apex.

332. Point Cut Teeth. The method of *point cutting from a master templet*, as originated by the Bilgram Machine Works, is applied to the cutting of bevel gears. A cutter sufficiently narrow to pass through the small end of the tooth space is reciprocated across the face of the blank, always cutting along a line passing through the cone-apex point of the gear (Fig. 370). The stroke of this cutter is guided by a large master templet of the correct tooth outline, thus reproducing an accurate tooth form at all points along the gear face.

Fig. 370. Method of Planing Bevel-Gear Teeth by Means of a Master Templet.

333. Generated Teeth. The *generating* method is applicable to spur, helical, worm, and bevel gears. The characteristic feature of generated teeth is that only a single basic cutter is required to produce theoretically correct tooth profiles on all mating gears of the same pitch, regardless of the number of teeth.

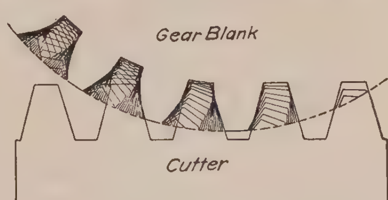
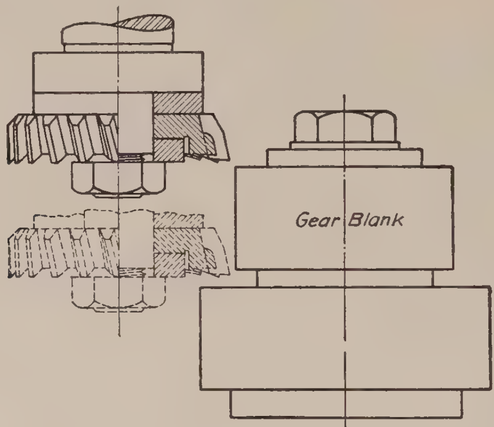


Fig. 371. Rack-Type Cutter Used in Generating Gear Teeth.

In generating spur and helical Maag gears, the Niles-Bement-Pond Co. use a *rack-shaped cutter* (Fig. 371). This cutter is given a reciprocating motion across the face of the gear blank while between cutting strokes the blank is rolled along the face of the cutter. The rolling action between cutter and blank corresponds to the engagement between a rack and a gear, the cutter in this case removing all inter-

fering metal, thus generating teeth. Because of the length limitations of the rack cutter, the blank is returned periodically without rotation to the starting point. A cutter of this type with straight-sided rack teeth can be made very accurately.



The Fellows Gear Shaper Co., Springfield, Vt.

FIG. 372. HELICAL GEAR-SHAPER CUTTER.

The method used by the Fellows Gear Shaper Co. is similar to that just described except that a *pinion-shaped cutter* is used. The cutter is accurately ground by a flat-sided wheel simulating the side of a rack tooth. Helical gears can be cut with a helical cutter (Fig. 372), if the cutter ram is turned through the helix angle by a cam as the cutter reciprocates. The shaper generating method is especially useful for cluster gears and internal gears. It can be adapted also to the generation of splines and special shapes. The Sykes generator employs two opposed pinion-shaped helical cutters that are given alternate cutting strokes extending to the middle of the face of the blank, thus producing a herringbone gear with a solid uninterrupted tooth.

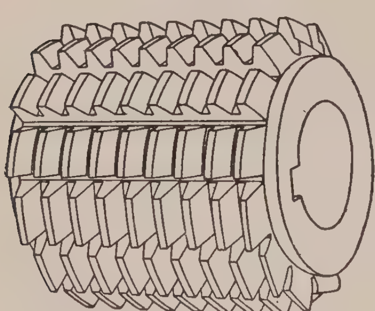


FIG. 373. HOB USED IN GENERATING GEAR TEETH.

In the *hobbing* method, the cutter (Fig. 373) is called a *hob* and has the form of a worm with teeth of rack-shaped cross-section, gashed to provide cutting edges. In cutting spur gears, the hob axis is dis-

placed from a perpendicular position to the gear axis by the amount of the helix angle of the hob. The hob is set to the proper depth of cut and is fed across the face of the blank while blank and hob are rotated at the correct velocity ratio.

Hobs for worm gears are frequently made with one end tapered. In the cutting operation the hob is set at the same center distance as that of the worm when assembled with the gear. By feeding the hob along its axis into the blank, more accurate tooth forms are generated.

Straight-tooth bevel gears are generated by the cutting action of two reciprocating tools having their cutting edges set to represent sides of adjacent teeth of an imaginary crown gear. When the tools and gear blank are rotated about their axes, as shown in Fig. 374, the action is the same as if a crown gear and pinion were in mesh.

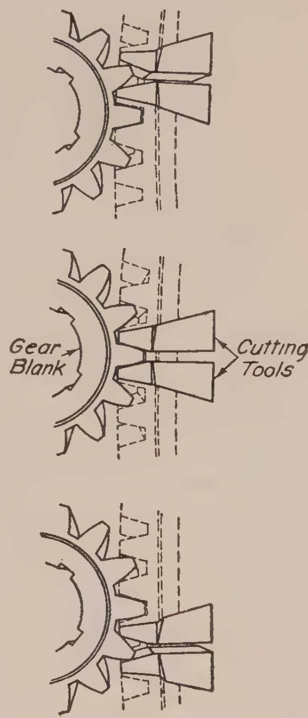


Fig. 374. Method Used in Generating Straight-Tooth Bevel Gears.

In spiral bevel and hypoid gears the theoretical curve of a tooth element is a logarithmic spiral. To simplify the generation of these gears, the curvature of the spiral at the center of the tooth is approximated by one of constant radius (Fig. 359), which permits the use of individual rack-shaped tools inserted in a holder to form a circular cutter (Fig. 375).

In the Gleason machine, the cutting edges of the cutter correspond to the tooth outline of a crown gear. In the cutting process, the tooth surfaces are generated by rolling the blank and the revolving cutter together with the same relative motion as would occur between a crown gear and pinion. The blank is indexed for each tooth. The same general principle is used in the Cleveland Rigidhobber, except that the cutters are arranged in a spiral instead of a circle and are graduated in height. By synchronizing the rotation of cutter and blank, the teeth may be cut continuously without intermittent indexing.

334. Comparison of Forming Methods. According to O. W. Boston,* the generating methods of cutting teeth are considered more

* *Engineering Shop Practice*, vol. 2, by O. W. Boston.

accurate than the forming methods. From a standpoint of accuracy, the rack generator is placed first, the hob second, and the pinion cutter third. For rapidity of production, the hob is placed first, the pinion and rack cutters second.

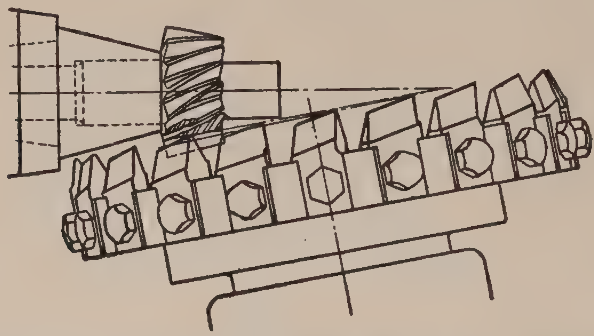


FIG. 375. CIRCULAR TYPE CUTTER EMPLOYED IN GENERATING SPIRAL BEVEL AND HYPOID GEARS.

335. Finishing Gear Teeth. Smooth, quiet action and freedom from high impact loads can be obtained only if gears have their teeth accurately spaced, correctly formed, and free from cutter marks. Consequently, after the teeth have been cut, other finishing operations are frequently employed as a means of securing greater accuracy.

Burnishing is a cold forming process of smoothing tooth surfaces by running the cut gear under load with one or more hardened and ground master gears.

Shaving is a light cutting operation in which from 0.005 to 0.010 of an inch of metal is removed in a series of cuts by means of a rack, or pinion-shaped, cutter. The transverse tool marks are removed and any marks that remain are perpendicular to the tooth elements along the direction of sliding.

Burnishing and shaving operations are applied to gears before heat treatment and in many instances the results obtained are considered satisfactory for quiet running. Heat-treatment, however, usually results in some distortion, the amount depending upon the procedure and care used. Authoritative * opinion seems to be that final accuracy is best controlled by the care used in production prior to heat-treatment. After heat-treatment, the tooth forms may be corrected by *grinding* or *lapping*.

* *Methods of finishing transmission gears*, by S. O. White and M. C. Hedgeland, S.A.E. Paper, Cleveland, September, 1935.

Grinding may be accomplished by the use of a diamond-dressed formed wheel fed across the tooth profiles; it removes about 0.010 of an inch of metal. A more accurate method is to use a flat-faced wheel set at the angle of the rack tooth. The wheel is rotated and moved tangentially as a rack while the gear rotates slowly. This principle is used in the Pratt and Whitney grinder for spur and helical gears.

Lapping as applied to gears consists of running the cut gear under load with master iron gears charged with an abrasive. The light grinding action removes about 0.001 of an inch of metal, polishes the teeth, and produces a surface free from cutter marks.

336. Commercial Methods. The automotive industry offers a good example of the methods employed in producing accurate gears in quantities. The blanks are rough-machined on all surfaces, normalized, and finish-machined. The teeth are then rough cut, finish-generated, and shaved. The gears are next hardened and quenched while held in a die to prevent distortion. Spur and helical gears are then lapped or ground. A number of manufacturers use the lapping process because it is less expensive than grinding.

PROBLEMS

1. Define the term "generating" as applied to forming gear teeth. What are the generating methods?
2. Explain the difference between the Fellows and Magg generating systems.
3. Describe how bevel-gear teeth are generated.
4. Why are gear teeth given a finishing operation? What type of service requires specially finished gears?
5. What are the essential differences in the various finishing operations?
6. What precautions are necessary to secure accurate gears?

CHAPTER 23

WIRE ROPE AND HOISTING

337. Power Transmission by Wire Rope. Thirty or forty years ago, power transmission by means of wire rope was considered a method of great importance and great possibilities. In Europe especially, numerous transmission systems of this type were installed. To prevent rapid wear, direct contact between the rope and the metal of drive sheaves was avoided by having the sheave wheels lined with wood, leather, or rubber. The sheaves often were of very large size.

The advent and rapid development of electric power transmission, however, stopped this growth. The necessity of renewing the rope every few years, the large and unsightly relay towers with exposed revolving machinery, the necessity of driving around corners by means of gearing, the comparatively low power capacity, and many other factors were against the wire-rope drive when compared with this new form of power transmission. Wire rope for power transmission no longer deserves a detailed discussion in a general textbook on machine design. On the other hand, as a machine element in the field of hoisting, haulage, and similar applications, it is important. We shall therefore consider it entirely from this point of view.*

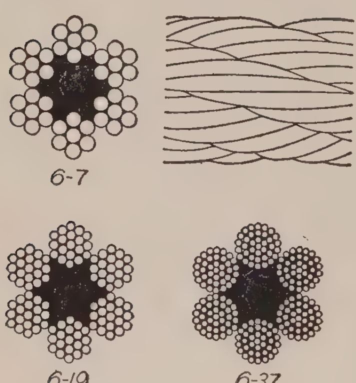


FIG. 376. REPRESENTATIVE TYPES OF ROUND WIRE ROPE, REGULAR LAY.

338. Wire-Rope Construction. Round wire rope is made up of strands of twisted wires. These strands are in turn twisted around a hemp or metal core to form the rope. The rope is designated by the number of strands in the rope and the number of wires in the strands, as, 6–7, 6–19, 8–19, 6–37, etc. (Fig. 376). The first figure denotes the number of strands, the second the number of wires in the strand.

In addition to the standard round sections, there are ropes of flattened-strand section giving increased wearing surface, steel-clad

* Much of the data in this chapter has been drawn from the book *American Wire Rope*, published by the American Steel and Wire Company, and also from a paper by James F. Howe, Transactions A.S.M.E., vol. 40, 1918, p. 1043.

ropes for exceptional resistance against wear, and finally flat ropes suitable for deep mine hoists. The flat rope will not spin, is very flexible, and can be coiled up on itself on reels requiring but little space axially.

The strands of a rope are laid by two different methods. In the regular, or common lay (Fig. 376), the wires in the strand are twisted in one direction and the strands themselves in the opposite direction. This construction results in a rope that does not untwist easily, although it does not present the best possible wearing surface. In Lang's lay (Fig. 377), the wires in the strand and the strands themselves are wound in the same direction. This rope untwists more readily and is less easy to splice, but presents a better wearing surface and therefore has longer life.

FIG. 377. WIRE ROPE, LANG'S LAY.

339. Wire-Rope Materials. Wire ropes are made either of high-grade wrought iron (Swedish iron) or of various grades of steel. The steel employed is usually called cast steel, originally to convey the impression that crucible steel was used, but now open-hearth steel is always the material employed.

Table 59 gives the average strength of various kinds of material used for wire rope. Wires of high tensile strength have relatively low ductility. Wire ropes as a whole stretch only 2 or 3 per cent before breaking. This characteristic reduces the reserve strength of the rope under impact and sudden loads. For this reason, wherever a plain steel rope can be accommodated, it is on the whole the best rope to use. Only where the size of sheaves requires a particularly small diameter, or where abrasion from ore dust, etc., demands an especially hard surface, should the high-strength steels be used. Swedish wrought iron is used by some elevator manufacturers on the plea that the steel ropes wear out the grooves in the sheaves.

TABLE 59

WIRE-ROPE MATERIALS

MATERIAL	AVERAGE STRENGTH, p.s.i.
Iron	85,000
Ordinary cast steel	150,000–200,000
Extra strong cast steel	180,000–200,000
Plow steel (high-strength open hearth)	200,000–260,000
Special extra-strong steels (of various trade names)	220,000–280,000

Thorough lubrication is recommended for wire ropes in hoisting service and this unquestionably lengthens the life of the rope considerably.* Various kinds of lubricants are manufactured or recommended by wire-rope manufacturers for this purpose. Sometimes, however, even in elevator service, the application of fresh lubricant may result in dust and dirt clinging to the rope in quantities sufficient to cause stiffness or crowding in the grooves.

The guaranteed strengths of wire ropes have been standardized by the wire-rope manufacturers and are given for the more common sections in Table 60. In many handbooks, tables are given that contain data for "working strength." Usually this "working strength" is obtained by dividing the ultimate strength by 5. The factor of safety, however, should always be adjusted to actual load conditions. In many cases a factor of safety higher than 5 is required even by law. Data for "working strength" should therefore be disregarded, unless the values are definitely understood and are such as to suit the problem.

TABLE 60

PROPERTIES OF STEEL WIRE ROPE

Kind of Rope	Diam. of Rope, In.	Wt. per Ft., Lb.	Ultimate Strength, Tons of 2000 Lb.		Advisable Drum Diam., In.	Kind of Rope	Diam. of Rope, In.	Wt. per Ft., Lb.	Ultimate Strength, Tons of 2000 Lb.		Advisable Drum Diam., In.
			Cast Steel	Plow Steel					Cast Steel	Plow Steel	
6–7	5/16	0.15	3.5	4.4	27	6–7	3/4	0.89	18.6	23.0	60
6–7	3/8	0.22	4.6	5.9	33	6–7	7/8	1.20	24.0	31.0	72
6–7	7/16	0.30	5.5	7.0	36	6–7	1.0	1.58	31.0	38.0	84
6–7	1/2	0.39	7.7	10.0	42	6–7	1 1/4	2.45	46.0	60.0	108
6–7	5/8	0.62	13.0	16.0	54	6–7	1 1/2	3.55	63.0	82.0	132
6–19	1/4	0.10	2.2	2.65	12	6–19	7/8	1.20	23.0	29.0	42
6–19	5/16	0.15	3.1	3.80	15	6–19	1	1.58	30.0	38.0	48
6–19	3/8	0.22	4.8	5.75	18	6–19	1 1/4	2.45	47.0	58.0	60
6–19	7/16	0.30	6.5	8.00	21	6–19	1 1/2	3.55	64.0	82.0	72
6–19	1/2	0.39	8.4	10.00	24	6–19	1 3/4	4.85	85.0	112.0	84
6–19	5/8	0.62	12.5	15.50	30	6–19	2	6.30	106.0	140.0	90
6–19	3/4	0.89	17.5	23.00	36	6–19	2 1/4	8.00	133.0	186.0	102
6–37	3/8	0.22	4.2	5.1	12	6–37	1	1.58	29.0	35.0	30
6–37	7/16	0.30	5.5	7.2	14	6–37	1 1/4	2.45	45.0	55.0	38.5
6–37	1/2	0.39	7.25	9.25	16	6–37	1 1/2	3.55	63.0	80.0	45.0
6–37	5/8	0.62	11.2	14.0	21	6–37	1 3/4	4.85	84.0	108.0	
6–37	3/4	0.89	17.5	21.0	22	6–37	2	6.30	105.0	130.0	
6–37	7/8	1.20	23.0	27.0	26						

* See paper of A. S. Biggart referred to on p. 420.

340. Sizes of Sheaves and Drums. Wire rope for hoisting purposes is run either on drums (Fig. 378), reels (Fig. 379), or friction sheaves (Fig. 387). It is of utmost importance, in all wire-rope appli-

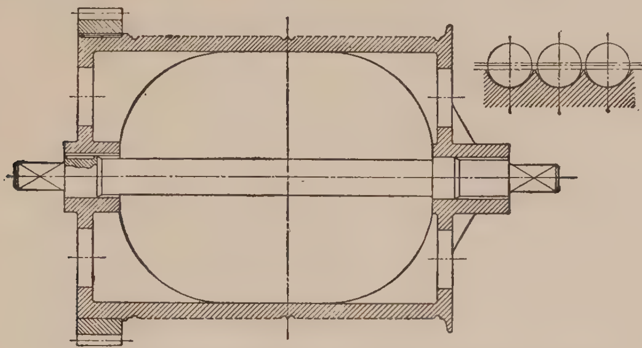


FIG. 378. CROSS-SECTIONAL VIEW OF HOISTING DRUM.

cations, not to bend the rope over too small a radius. The smaller the sheave diameter, the finer the wires should be in the rope. It is desirable to have the sheave diameter not less than 1000 times the

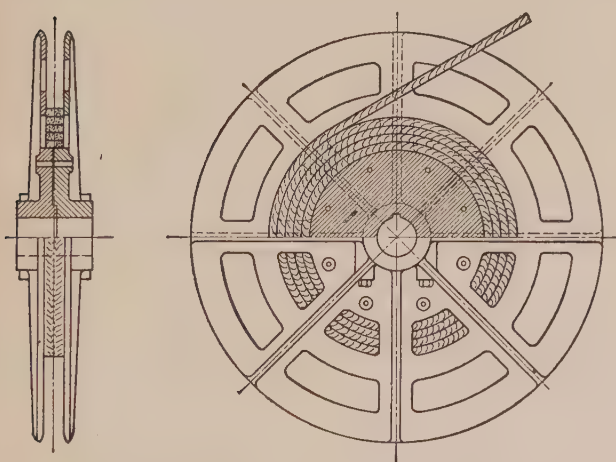


FIG. 379. REEL FOR WINDING FLAT ROPE.

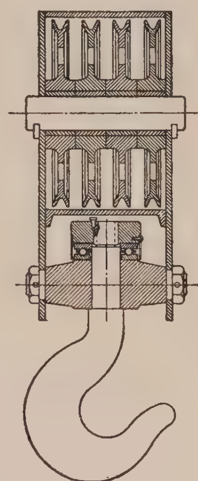


FIG. 380. HOOK SWIVEL.

diameter of the largest wire in the rope. Actually, according to J. F. Howe, of the American Steel and Wire Company,* present day American practice is about as given in Table 61. (See also Table 60.)

* Transactions A.S.M.E., vol. 40, 1918, p. 1068.

TABLE 61

WIRE-ROPE PRACTICE

Use	Make-up of Rope	Ratio Sheave Diameter to Rope Diameter
Derricks	6–19 8–19	20 to 30
Coal towers, clam-shell buckets for ore handling	6–19	40
Ladle cranes, steel mills	6–37	30
Mine hoists	6–19	60 to 100
Lift bridges	6–19	50 to 80

While all bending constitutes a stress in wire ropes, reverse bending is particularly serious on account of the reversal of stress. Tests by A. S. Biggart * show that a rope with reverse bends might be only half as strong as a rope bent in one direction only.

341. Computation of Bending Stresses. The fundamental formulas for bending are

$$s = \frac{M_b c}{I}, \qquad \frac{1}{\rho} = \frac{M_b}{EI},$$

where s is the bending stress, M_b the bending moment, c the distance from the neutral axis to the outermost fiber, I the moment of inertia, ρ the radius of curvature of the neutral axis of the bent rod, and E the modulus of elasticity. By substitution, we find

$$s = \frac{E}{\rho} c.$$

In case of a wire of radius r bent over a circular sheave of pitch radius R, we have, at least very approximately,

$$\frac{c}{\rho} = \frac{r}{R} = \frac{d}{D},$$

where d is the wire diameter and D is the sheave diameter. Hence the bending stress produced in a single wire drawn over a sheave is

$$s = \frac{Ed}{D}. \tag{1}$$

It has long been realized (for example, by Bach and others) that this formula could be applied to a wire in a rope only by modifying the proportionality factor E in some way. From experiments of Howe, the results of which agree very well with theoretical predictions, it

* Proceedings, Institution of Civil Engineers, vol. 101. The influence not only of reverse bending but also of lay, oiling, etc., on the life of the rope is given in this paper.

appears that the modulus of elasticity of a rope as a whole, even in straight tension, rarely exceeds 12,000,000 pounds per square inch of total wire section.

Due to the fact that wire rope is often made up of a number of different sizes of wires, and that as a rule the makeup of the cross-section is not given in catalogs, it is seldom practicable to compute loads and bending stresses. It is therefore necessary to rely upon published data, as given in Tables 60 and 62.

In the use of Tables 60 and 62, the procedure is as follows: Assume a factor of safety considerably higher than the real factor of safety desired. Knowing the load to be hoisted, compute the required ultimate strength with this factor of safety and select from Table 60 a

TABLE 62

BENDING LOADS FOR WIRE ROPE IN TONS (2000 LB.)

Kind of Rope	Diam. of Rope, In.	Diameter of Sheave or Drum in Ft. and In.							
		14′ 0″	12′ 0″	10′ 0″	8′ 0″	7′ 0″	6′ 0″	5′ 0″	4′ 0″
6–19	2	6.40	7.47	8.96	11.20	12.80	14.94	17.92	22.40
6–37	2	4.29	5.00	6.00	7.50	8.58	10.00	12.00	15.00
6–19	1¾	4.28	4.99	5.99	7.48	8.56	9.98	11.98	14.96
6–37	1¾	2.89	3.38	4.05	5.06	5.78	6.76	8.10	10.12
6–7	1½	5.40	6.30	7.56	9.45	10.80	12.60	15.12	18.90
6–19	1½	2.69	3.14	3.77	4.72	5.38	6.28	7.54	9.44
6–37	1½	1.80	2.10	2.52	3.15	3.60	4.20	5.04	6.30
6–7	1¼	3.12	3.64	4.37	5.46	6.24	7.28	8.74	10.92
6–19	1¼	1.56	1.82	2.18	2.72	3.12	3.64	4.36	5.44
6–37	1¼	1.04	1.22	1.46	1.83	2.08	2.44	2.92	3.66
6–7	1	1.60	1.87	2.24	2.80	3.20	3.74	4.48	5.60
6–19	1	0.80	0.93	1.12	1.40	1.60	1.86	2.24	2.80
6–37	1	0.54	0.63	0.75	0.94	1.04	1.26	1.50	1.88
6–7	⅞	1.11	1.29	1.55	1.94	2.22	2.58	3.10	3.88
6–19	⅞	0.54	0.63	0.75	0.94	1.08	1.26	1.50	1.88
6–37	⅞	0.38	0.42	0.51	0.63	0.72	0.84	1.01	1.26
6–7	¾	0.67	0.78	0.94	1.18	1.34	1.56	1.88	2.36
6–19	¾	——	0.40	0.47	0.59	0.67	0.80	0.94	1.18
6–37	¾	0.23	0.26	0.31	0.39	0.46	0.52	0.63	0.78
6–7	⅝	0.39	0.46	0.55	0.69	0.78	0.92	1.10	1.38
6–19	⅝	——	0.23	0.27	0.34	0.39	0.46	0.54	0.68
6–37	⅝	——	0.15	0.18	0.23	0.26	0.30	0.36	0.44
6–7	½	0.20	0.23	0.28	0.35	0.40	0.46	0.56	0.70
6–19	½	——	——	0.14	0.17	0.20	0.23	0.28	0.34
6–7	7/16	0.14	0.16	0.19	0.24	0.28	0.32	0.38	0.48
6–19	7/16	——	——	——	0.12	0.13	0.15	0.18	0.24
6–7	⅜	0.09	0.10	0.12	0.15	0.17	0.20	0.24	0.30
6–19	⅜	——	——	——	——	0.08	0.10	0.12	0.15
6–7	5/16	0.05	0.06	0.07	0.09	0.10	0.12	0.14	0.18
6–19	5/16	——	——	——	——	——	0.06	0.07	0.09
6–19	¼	——	——	——	——	——	0.03	0.04	0.05

TABLE 62—*Continued*

Kind of Rope	Diam. of Rope, In.	Diameter of Sheave or Drum in Ft. and In.							
		3′ 6″	3′ 0″	2′ 6″	2′ 0″	1′ 9″	1′ 6″	1′ 3″	1′ 0″
6–19	2	25.60	——	——	——	——	——	——	——
6–37	2	17.16	20.00	24.00	——	——	——	——	——
6–19	1¾	17.12	19.96	23.96	——	——	——	——	——
6–37	1¾	11.56	13.52	16.20	20.24	——	——	——	——
6–19	1½	10.76	12.56	15.08	——	——	——	——	——
6–37	1½	7.20	8.40	10.08	12.60	14.40	——	——	——
6–19	1¼	6.24	7.28	8.72	10.88	12.48	——	——	——
6–37	1¼	4.16	4.88	5.84	7.32	8.32	9.76	11.68	——
6–7	1	6.40	7.48	——	——	——	——	——	——
6–19	1	3.20	3.72	4.48	5.60	6.40	7.44	8.96	——
6–37	1	2.08	2.52	3.00	3.76	4.16	5.04	6.00	7.52
6–7	⅞	6.40	5.16	6.20	——	——	——	——	——
6–19	⅞	2.16	2.52	3.00	3.76	4.32	5.04	6.00	7.52
6–37	⅞	1.44	1.68	2.02	2.52	3.08	3.36	4.04	5.04
6–7	¾	2.68	3.12	3.76	4.72	——	——	——	——
6–19	¾	1.34	1.60	1.88	2.36	2.68	3.20	3.76	4.72
6–37	¾	0.92	1.04	1.26	1.56	1.84	2.08	2.52	3.12
6–7	⅝	1.56	1.24	2.20	2.76	3.12	——	——	——
6–19	⅝	0.78	0.91	1.08	1.36	1.56	1.82	2.16	2.72
6–37	⅝	0.42	0.61	0.73	0.92	1.04	1.22	1.40	1.84
6–7	½	0.80	0.92	1.12	1.40	1.60	1.84	——	——
6–19	½	0.40	0.46	0.56	0.68	0.80	0.93	1.12	1.36
6–37	½	0.26	0.31	0.38	0.46	0.52	0.62	0.76	0.92
6–7	7/16	0.56	0.64	0.76	0.96	1.12	1.28	——	——
6–19	7/16	0.27	0.30	0.36	0.47	0.54	0.63	0.72	0.94
6–37	7/16	——	0.21	0.25	0.31	0.39	0.42	0.50	0.62
6–7	⅜	0.34	0.40	0.47	0.59	0.68	0.79	——	——
6–19	⅜	0.17	0.20	0.24	0.30	0.33	0.40	0.48	0.60
6–37	⅜	——	0.13	0.16	0.19	0.23	0.26	0.32	0.38
6–7	5/16	0.20	0.23	0.28	0.35	0.39	0.47	——	——
6–19	5/16	0.10	0.12	0.14	0.17	0.19	0.23	0.28	0.34
6–19	¼	0.05	0.06	0.07	0.09	0.10	0.12	0.14	0.17

rope of the required ultimate strength. From Table 62 determine the bending load for this rope and add this bending load to the actual hoisted load. The ultimate strength divided by this sum is the real factor of safety, provided the effect of the weight of the rope itself and the force of acceleration may be neglected. If an allowance is to be made for these two loads, they should be included in the total before the real factor of safety is ascertained.

This method of allowing for the bending stress is based on the assumption that in the extreme fiber the tensile stress due to bending is added directly to the tensile stress derived by dividing the load by the area of the metal section. Consequently, even though the bending stress is not distributed uniformly over the total section, it must be

allowed for as if it were, provided we allow for the other stresses as total loads rather than as stresses per square inch.

342. General Arrangement and Efficiency of Hoists. In cranes, the end of the rope is anchored to a drum, as shown in Fig. 381, and the whole length of rope is wound up on this drum as the load is

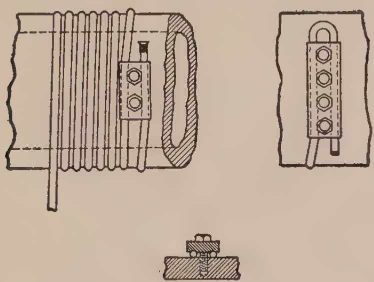


FIG. 381. METHOD OF FASTENING ROPE TO DRUM.

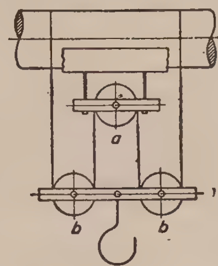


FIG. 382. TACKLE WITH FOUR ROPE PARTS.

hoisted. If only one rope were used, it is obvious that the load would shift axially along the drum. To avoid such shifting, two ropes are used in parallel, each anchored at opposite ends of the drum. Quite regularly the rope passes over one or more pulleys, or blocks, before running onto the drum. By this arrangement the load is distributed over several rope parts or lines. The load on each part is but little

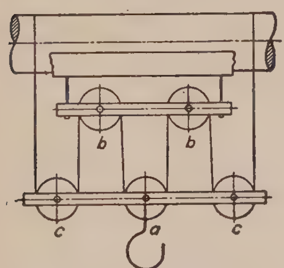


FIG. 383. TACKLE WITH SIX ROPE PARTS.

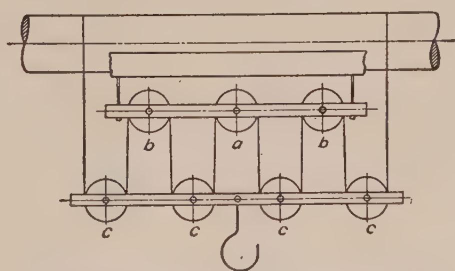
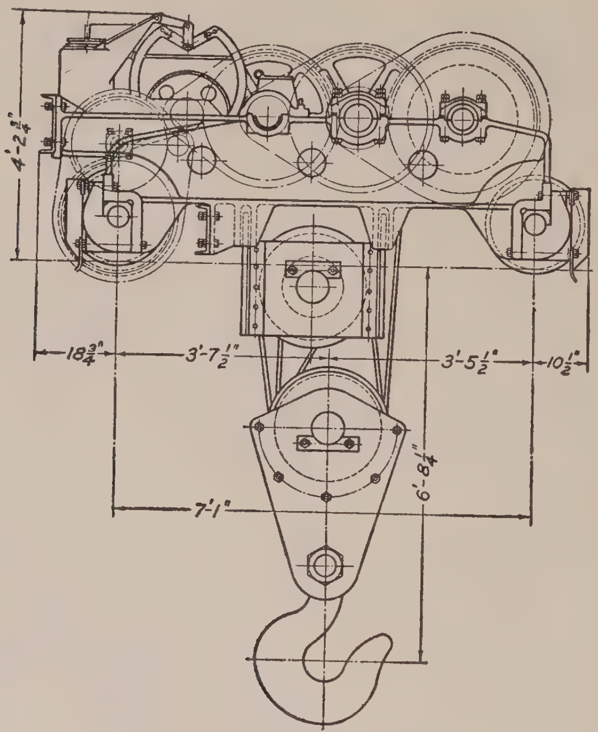


FIG. 384. TACKLE WITH EIGHT ROPE PARTS.

more than the total load divided by the number of parts, and the load on the drum is reduced.

To compute the efficiency of such a tackle, suppose the sheaves to be arranged as shown in Figs. 382, 383, and 384. In Fig. 383, for instance, the tackle consists of six parts. It is apparent that the arrangement may be extended to provide for a greater number of parts. The equalizing sheave *a* at the middle is introduced to equalize

the pull at the two sides of the tackle. It is anchored either to the trolley frame, as shown in Figs. 382 and 384, or to the hook frame, as shown in Fig. 383. In actual design, all the working sheaves are on the same shaft, and the equalizing sheave may also be arranged on this shaft, although this arrangement introduces a cross-over of the rope, as shown in Fig. 385. The equalizing sheave may of course be arranged at right angles to the other sheaves and on a separate shaft, as shown in Fig. 386.



The Morgan Engineering Co., Alliance, Ohio

FIG. 385. FIFTY TON CRANE TROLLEY.

If there are m parts of rope in series, there must be wound up on the drum a length of rope equal to m times the hoisting height. On the other hand, the pull in the rope would be only the mth part of the load suspended by these ropes, if no allowance is made for friction. Frictional work must be done, however, in bending the ropes over the sheaves and in turning the sheaves on the shafts. Consequently, if a rope runs onto a sheave with a pull F, beyond the sheave the pull must

be CF, where C is greater than unity. If there are n sheaves in series, the formula for the pull F_d in the rope running on the drum will be

$$(2) \quad F_d = C^n \left[\frac{C-1}{C^m - 1} \right] W,$$

where W is the hoisted load.

Leutwiler * finds an average value of C of about 1.065. However, the American Hoist and Derrick Company now assume a friction loss per sheave of 3 per cent for ball and roller bearings, and 5 per cent for bronze-bushed sheaves. According to this company, Table 63 gives the ratio between the total load hoisted and the maximum pull in a single line for various numbers of lines, counting both the parts in parallel and those in series.

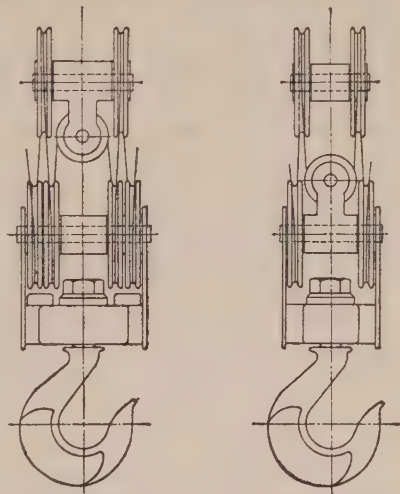


FIG. 386. CRANE HOOKS WITH EQUALIZING SHEAVES AT RIGHT ANGLES TO ROPE SHEAVES.

TABLE 63

RATIO TABLE[a]

3% AND 5% FRICTION FOR EACH ROPE

No. of Lines	3% Friction	5% Friction	No. of Lines	3% Friction	5% Friction
1	1	1	19	14.6455	12.4556
2	1.97	1.95	20	15.2062	12.8332
3	2.9019	2.8525	21	15.7501	13.1919
4	3.8234	3.7099	22	16.2777	13.5327
5	4.7085	4.5245	23	16.7895	13.8565
6	5.5671	5.2984	24	17.2859	14.1641
7	6.4000	6.0336	25	17.7674	14.7339
8	7.2079	6.7321	26	18.2345	14.9976
9	7.9916	7.3957	27	18.6876	15.2481
10	8.7518	8.0261	28	19.1271	15.4861
11	9.4892	8.6250	29	19.5532	15.7122
12	10.2045	9.1940	30	19.9665	15.9270
13	10.8983	9.7346	31	20.3674	16.1311
14	11.5713	10.2482	32	20.7663	16.3250
15	12.2241	10.7361	33	21.1533	——
16	12.8573	11.1996	34	21.5287	——
17	13.4716	11.6399	35	21.8928	——
18	14.0675	12.0582	36	22.2460	——

[a] $\frac{\text{Total Load Lifted}}{\text{Max. Single Line Pull}} = \text{Ratio.}$

* Leutwiler, *Elements of Machine Design*, p. 204.

As an example, suppose that on each side of the equalizer there are two pulleys and three lines. The total number of lines would then be six and the ratio of total load to single line pull with 3 per cent friction loss would be 5.5671. Without friction the ratio would be 6. The loss ratio is then $(6 - 5.5671)/6 = 0.0721$, or 7.21 per cent. If the load is 10,000 lb., the total rope pull going up on the drum is 10,000 lb./5.5671 = 1800 lb. on each rope.

Mine hoists and elevators operating at comparatively high hoisting speed cannot use a tackle; hence the ropes run directly on the drum. For mine hoisting, conical drums of very large diameter are used in order to compensate for the weight of the very long cables. At the beginning of the hoisting, when a great length of cable is out, it is wound up on a small diameter so as to pull on a short lever arm; as the cable is wound up and the suspended weight decreases, the lever arm and the hoisting speed are both increased.

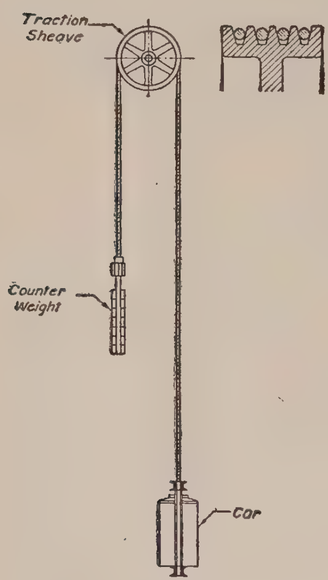


FIG. 387. ELEVATOR HOIST.

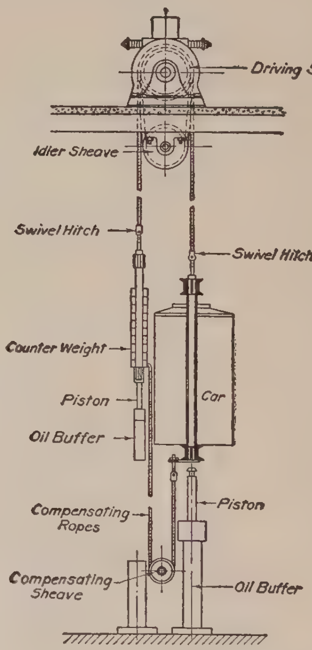


FIG. 388. ELEVATOR HOIST WITH EQUALIZING SHEAVE.

Modern elevators for tall buildings are usually equipped with friction sheaves in place of drums. The sheaves are now mostly V-grooved, as shown in Fig. 387, to increase the tractive force, as explained on page 302 in connection with V-belts. Formerly rounded grooves were used and the rope was run over an idler and back onto the main

sheave, as shown in Fig. 388, to increase the contact angle by approximately 180 deg.

The weight of the rope and the force for load acceleration must be allowed for in all computations for elevators and mine hoists. If, as in the case of a mine hoist, the load moves up a track inclined at an angle α to the horizontal, the direct pull of a weight W is only $W \sin \alpha$, but the load due to acceleration must be computed for the whole mass W/g.

Elevators are regularly counterbalanced, fully for the cage weight, and partly for the useful load. During the starting-up period, the load on the descending side is reduced by acceleration and the load on the ascending side is increased. A friction hoist should be checked to determine whether the contact angle of the sheave is sufficient to maintain the tension ratio established during the period of acceleration. For such an investigation the average friction coefficient for wire rope on iron may be assumed to be about 0.13, but according to the Trenton Iron Company it varies from 0.07 for greasy ropes to 0.17 for quite dry ropes.

The groove angle for V-groove sheaves is usually made 29 deg. (included angle), or 14½ deg. for the half angle. This angle was established simply because the sine or the tangent of 14½ deg. is very close to 1/4, so that the angle can easily be laid out without a protractor.

343. Example. Computation of an Elevator Hoist.

Let us select as an example a passenger elevator with a hoisting speed of 600 ft. per min. The elevator must attain full hoisting speed in 15 ft. and the maximum hoisting height is 200 ft. The car weighs 2500 lb. and the live load is 1800 lb. The elevator is of the *traction type*, that is, the load is lifted by the friction of the rope passing over a grooved sheave.

To comply with a common legal safety requirement, there must be at least 4 ropes in parallel, and a factor of safety of 8. We shall assume 4 ropes, and a ratio of hoisted load to the total ultimate strength of ropes of 10, so as to leave a margin for acceleration and rope weight.

The maximum useful load is $2500 + 1800 = 4300$ lb. The ultimate strength of each rope must be $10 \times 4300/4 = 10{,}750$ lb. The most commonly used elevator rope is of mild steel and of 6–19 construction. Referring to Table 60 we would select a 7/16 in. rope having a strength of 6.5 tons. The weight of this rope is 0.30 lb. per ft., and the weight of 200 feet would be 60 lb. There will be a balancing sheave at the bottom of the hoist, as shown in Fig. 388, so that the hoisted rope weight will remain constant.

The rope speed is 600 ft. per min. or 10 ft. per sec. Since this speed is to be attained in a distance of 15 ft., we have from the formula $2aS = v^2$, where a is the acceleration in ft. per sec.^{2}, S the distance in ft., and v the velocity in ft. per sec.

$$2a \times 15 = 10^2, \qquad \text{or} \qquad a = 3.33 \text{ ft. per sec.}^2.$$

The total hoisted load per rope is $4300/4 + 60 = 1135$ lb. The force necessary for acceleration is therefore $1135 \times 3.33/32.2 = 117$ lb.

If we select a sheave wheel of 36 in. diameter, the bending stress in a 7/16 in. 6–19 rope, according to Table 62, is 0.30 ton, or 600 lb. The total load per rope is therefore made up of the following items:

Useful load	1075 lb.
Weight of rope	60 lb.
Acceleration	117 lb.
Bending load	600 lb.
Total	1852 lb.

With a breaking strength of 6.5 tons or 13,000 lb., the actual factor of safety is $13{,}000/1852 = 7$. Even considering that the rope is subject to fatigue and may ultimately break, say at 40 per cent of its ultimate strength, it still has a real margin of safety of 175 per cent of the actual load. The discrepancy should be noted between the assumed factor of safety of 10 applied to the useful load alone and the real factor of safety of 7 found by making rational allowances for all the loads and stresses that actually occur.

In the case of traction elevators it is necessary to ascertain whether the friction is enough to sustain the load. The formula covering this relationship is $T_1/T_2 = e^{f\alpha}$, exactly as in the case of belts and ropes (see page 296). This equation shows that it is the tension ratio that counts and not the absolute value of the tensions.

All elevators are counterbalanced, and it has been found that the most economical use of power is obtained if the counter-weight balances the whole car weight and 30 to 40 per cent of the useful load. When a load is being lowered it takes force to accelerate it in the downward direction. The gravitational pull is reduced by the amount of the force required for this acceleration. The maximum tension ratio will be found when a car is started up with full load, since then the acceleration increases the pull on the car side and decreases it on the counter-weight side. Suppose in the case here considered the counter-weight balances one-third of the live load. It then amounts to $2500 + 1800/3 = 3100$ lb. The portion carried by one rope is $3100/4 = 775$ lb., to which should be added 60 lb. of rope weight. The total per rope is 835 lb. The force required to accelerate this weight is $835 \times 3.33/32.2 = 87$ lb. The tension ratio during acceleration is then

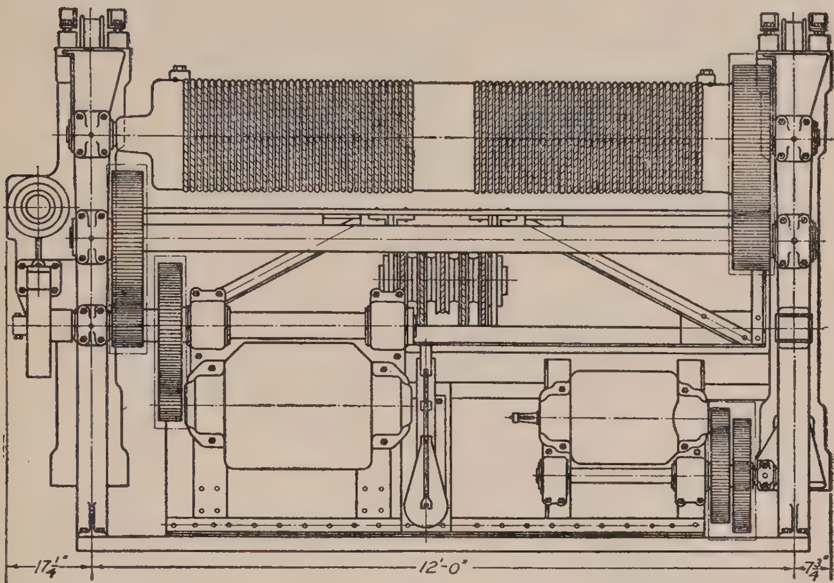
$$\frac{T_1}{T_2} = \frac{1135 + 117}{835 - 87} = 1.67.$$

Taking the coefficient of friction for greasy ropes as 0.07 (page 427), we have for an open, rounded groove without wedge action and for a contact angle of 180 deg., $e^{f\alpha} = 1.247$. Such an arrangement would then lead to slipping. With a wedge groove of 29 deg. angle, we would have (see page 302) $e^{f\alpha/\sin 14\frac{1}{2}°} = 2.42$. This value is amply sufficient.

It is for this reason that wedge-grooved sheaves have replaced sheaves with rounded grooves. It is possible, however, by running the rope over an idler and back onto the main sheave for a second half lap, to attain a tension ratio with rounded grooves sufficient for hoisting purposes.

344. Hoisting Accessories. On page 35 we have already given the strength computations for a crane hook. Strength computations for a crane drum involve bending strength, torsion, and crushing.

Usually the driving gear is shrunk and keyed to the drum, as shown in Figs. 378 and 389, so that no torsion occurs on the drum shaft, which merely serves as a support in the bearings. In computing the



The Morgan Engineering Co., Alliance, Ohio

FIG. 389. FIFTY TON CRANE TROLLEY.

bending and torsional stresses in the drum we may set the axial section modulus equal to $\pi D^2t/4$, and the torsional section modulus equal to $\pi D^2t/2$, where D is the mean diameter and t the thickness of the drum metal section, both in in. In addition the drum must be checked for crushing. If the rope is coiled on the drum under full tension, there must be a metal section of sufficient area under each rope to withstand in compression the pull exerted by the rope. Usually this compressive stress determines the wall thickness.

EXAMPLE. Let us assume that we have a cast-iron drum 16 in. in diameter and 72 in. long which is 3/4 in. thick at the bottom of the groove. On this drum two 1/2 in. ropes are to be wound under a pull of 2000 lb. per rope.

When the ropes are close to the middle of the drum, the bending moment will be $72 \times 2 \times 2000/4 = 72{,}000$ in. lb. The axial-section modulus is, approximately, $\pi \times 16^2 \times 0.75/4 = 150$ in.^{3}, and the bending stress $72{,}000/150 = 480$ p.s.i. The torque is $4000 \times 8 = 32{,}000$ in. lb., the torsional section modulus $= 300$ in.^{3}, and the torsional shear stress in consequence $32{,}000/300 = 107$ p.s.i. These stresses are practically negligible. On the other hand the compressive stress per pitch length on a longitudinal section through the center of the drum is $2000/(0.75 \times 0.625)$ $= 4300$ p.s.i. (if we allow a clearance of 1/8 in.). While this is not a serious com-

pressive stress in cast iron, it is a stress of entirely different order of magnitude from the other two stresses. The wall thickness of 3/4 in. is none too great from the point

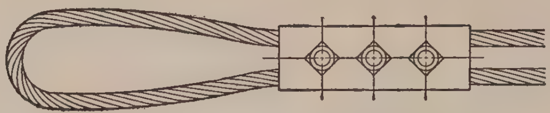


FIG. 390. BLOCK CLAMP.

of view of obtaining a sound casting, even if the stresses are not too high. The general arrangement of the drum and the method of fastening the ropes to the drum are shown in Fig. 389 and Fig. 381.

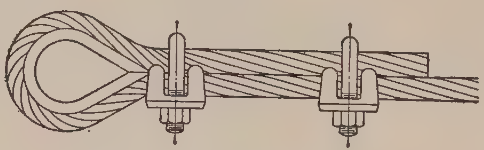
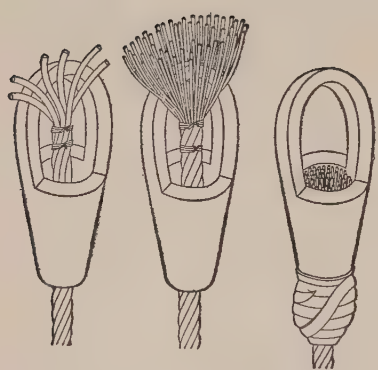


FIG. 391. CROSBY CLAMPS.



E. J. Edwards, American Machinist, Vol. 56, p. 309

FIG. 392. METHOD OF ATTACHING WIRE ROPE TO SOCKET.

In Fig. 380 is shown a detail of a hook swivel; in Figs. 390 and 391 the method of making loops by means of clamps. In Fig. 392 is shown the method of attaching a wire rope to a socket. The socket is poured full of spelter (zinc) to hold the rope in place.

PROBLEMS

1. A 2 in. 6–19 rope carries a direct load of 25,000 lb. and runs over a 4 ft. sheave. What is the overall factor of safety?

2. A 5/8 in. 6–7 rope carries a direct load of 7500 lb. What minimum-diameter sheave can be used if an overall factor of safety of 4 is to be maintained?

3. Two 3/4 in. 6–37 ropes, 300 ft. long, pass over a 2 ft. diameter sheave. The maximum acceleration is 5 ft. per sec. per sec. What may be the weight of the hoisted load for a factor of safety of not less than 6?

4. A mine-hoist cage weighs 5000 lb. and the weight of car and coal lifted per trip is 5200 lb. The maximum hoisting speed is 1800 ft. per min., attained in 4 sec. If the lift is 500 ft. and the drum diameter 7 ft., select the rope to give an overall factor of safety of 8.

5. A traction elevator with an idler sheave to give a complete turn of the rope on the drum has six 5/8 in. 6–19 mild-steel ropes. The drum diameter is 3 ft., the lift 1350 ft., the maximum hoisting speed 1250 ft. per min., accelerating time 10 sec. The cage weighs 2600 lb. and the load 2200 lb. The counter-weight is 30 per cent more than the weight of the empty car. What is the most severe load condition? What is the overall factor of safety? Is the contact angle of the rope sufficient?

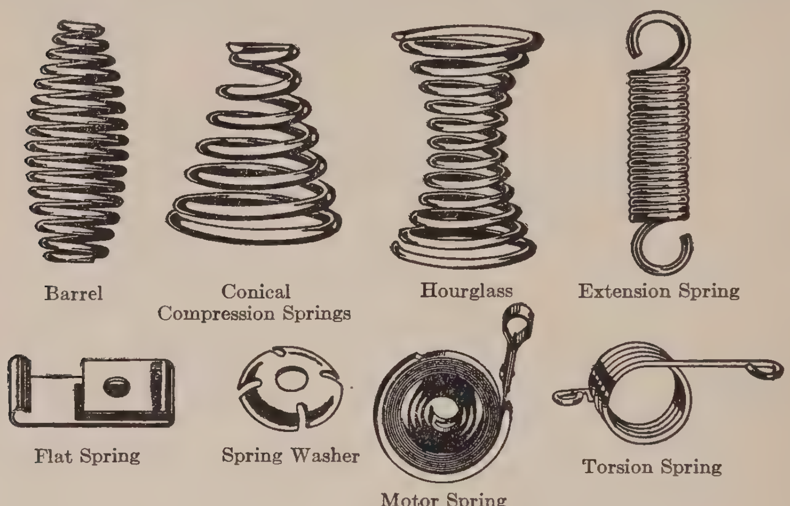
6. A hoisting tackle is arranged as in Fig. 383. What is the efficiency if ball bearings are used? If bronze-bushed bearings are used? What value is obtained from formula (2)?

7. A hoisting tackle has three pulleys and four lines on each side of an equalizer (Fig. 384). What is the loss ratio?

CHAPTER 24

SPRINGS

345. General Remarks. Springs are extensively used as machine elements, have a wide variety of applications, and are made in many different forms (Fig. 393). An important application is to absorb



Wallace Barnes Co., Bristol, Conn.

FIG. 393. VARIOUS FORMS OF SPRINGS.

energy and to cushion shocks, as in various types of vehicle springs. Springs are used also to store and redistribute energy, as in clock

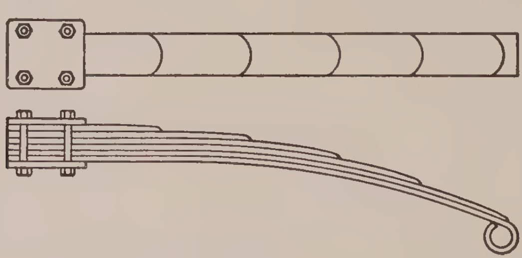


FIG. 394. CANTILEVER LEAF SPRING.

mechanisms; to apply forces, as in valve actions; and to measure forces, as in weighing scales and pressure gauges.

346. Leaf Springs. A common *cantilever* leaf spring is shown in Fig. 394. This type of spring is designed to replace a cantilever beam of uniform strength of the form shown in Fig. 395. The strength and

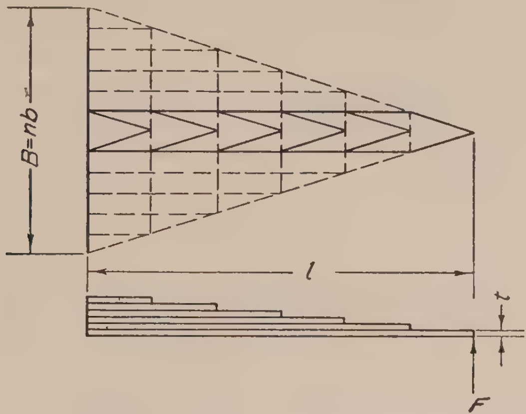


FIG. 395. BEAM OF UNIFORM STRENGTH.

deflection formulas for leaf springs, as commonly used, are derived on the assumption that a flat beam of uniform strength is divided into a number of narrow strips, which, when piled one on top of the other, will flex in the same manner as the original beam. By substituting for the base width B of the flat beam the total strip width nb of the spring package, we obtain from the formulas for a cantilever beam of uniform strength the formulas for the *simple cantilever*, or *quarter-elliptic* leaf spring. Thus we may write

$$F = \frac{nbt^2 s_b}{6l}. \tag{1}$$

Here F is the load on the spring in lb., n the number of strips, b the width, and t the thickness of the strips, both in in., l the length of the spring in in., and s_b the working stress in p.s.i. For the deflection, we have

$$y = \frac{6Fl^3}{Enbt^3} = \frac{s_b l^2}{Et}, \tag{2}$$

in which y is the deflection in in. and E the modulus of elasticity. (In the second form, F is eliminated by means of formula (1).)

The reason that the design and computation of leaf springs are based on beams of uniform strength is that in such beams the stress is uniform throughout, thus utilizing the material to its maximum capacity. While this construction results in a minimum of material expense,

it is of equal importance also to eliminate any unnecessary weight, especially in vehicle springs. With the leaves well lubricated and the friction low, the actual deflection will agree quite closely with the theoretical value. If the leaves are dry and rusty, considerable variation may be expected in the deflection, both in loading and unloading the spring.

A flat beam of uniform strength is not as light in weight as a parabolic beam of uniform strength (Fig. 396). The strength of a beam of rectangular cross-section increases with the first power of the width, but with the second power of the thickness. The weight, however, increases with the first power of each. In a parabolic beam, the increased strength toward the base is obtained by increasing the thickness, and this distribution results in a minimum weight increase. Parabolic springs should not be particularly expensive or difficult to produce with modern drop-forging methods; yet, such springs have not come into use for vehicles or similar applications.

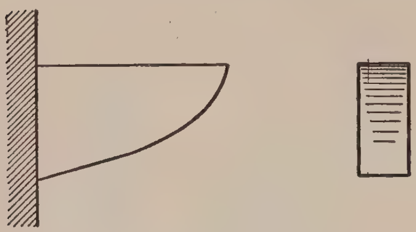


FIG. 396. PARABOLIC BEAM OF UNIFORM STRENGTH.

However, it is interesting to note that without a knowledge of the theory involved, the old bowyers developed a typical English long bow shaped somewhat like a modified parabolic beam of uniform strength. It was recognized that as much as possible of the spring force in the bow should be utilized in throwing the arrow and not in throwing the weight of the bow itself.

347. Extra Full-Length Leaves. Theoretically, a flat triangular spring of uniform strength should taper to a point, since the bending moment at the tip is zero. Actually in a leaf spring there is superimposed on the leaf package, representing the triangular spring, one or more extra full-length leaves, as shown in Fig. 397. The top full-length leaf (although more than one may be used if necessary) is usually required to take endwise forces in addition to the bending

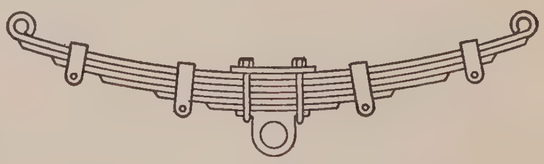


FIG. 397. SEMI-ELLIPTIC SPRING.

loads. For this purpose an extra-strong material like chromium-vanadium steel may be used. Theoretically at least, the top leaf may be thinned so as to reduce the bending stress in the extreme fiber, and thus leave a margin of strength for straight tension or compression. (Observe that within the proportional elastic limit, the bending stress in the fiber is proportional to its distance from the neutral axis.)

The formulas for springs with extra full-length leaves were first developed by Egbert R. Morrison.* The following formulas pertain to symmetrical *semi-elliptic springs*, as shown in Fig. 397, with half the load applied at each end:

$$F = \frac{2nbt^2s_b}{3L}, \tag{3}$$

$$y = \frac{3FL^3}{8Enbt^3} = \frac{L^2s_b}{4Et} \quad \text{(without extra full-length leaves)}, \tag{4}$$

$$y = \frac{3FL^3}{4(2+r)Enbt^3} = \frac{L^2s_b}{2(2+r)Et} \quad \text{(with extra full-length leaves)}. \tag{5}$$

Here F is the total load on the spring in lb., one-half of which is applied at each end, L is the total length of the spring in in., and $r = n'/n$, where n' is the number of extra full-length leaves and n the total number of leaves. It should be remembered that the number of graduated leaves contains one full-length leaf. The preceding leaf-spring formulas are merely approximations, but they are in general use and give acceptable results when combined with the experience of spring manufacturers.

348. General Remarks on Leaf-Spring Design. Since extra full-length leaves are flat-beam strips of *uniform cross-section*, they should be formed and assembled with a *reverse* camber (Fig. 398), since other-

FIG. 398. REVERSE CAMBER IN TOP LEAF.

wise the stress in these leaves for the same deflection would be 50 per cent greater than the stress in the graduated leaves of the spring package. When the spring is assembled with reverse camber, there is induced in the extra full-length leaves an initial stress of opposite sign to that which occurs when the load is applied. By selecting the

* See *Spring Engineering*, by Egbert R. Morrison, published at Sharon, Pa., p. 19, etc.

proper camber, which is usually left to the spring manufacturer, the stress may be kept uniform throughout the spring.

Leaf springs are seldom made with the ends of the graduated leaves tapered to a point. Instead, the ends are usually thinned and rounded, as shown in Fig. 394, so as to approximate the pointed condition. Eyes are generally formed at the ends of the top leaf and these afford a means of attaching the spring to shackles or brackets by bolts.

The amount of curvature, or camber, given to a leaf spring, is governed mainly by the deflection. If it is intended, for instance, that the spring shall be straight when fully loaded, then the depth of the curvature or camber should equal the deflection.

Leaf springs are provided with rebound clips (Fig. 394), in order that the graduated leaves may be stressed on the rebound and thus help to resist the rebound load. They serve also the purpose of holding the leaves in proper alignment. In addition to the rebound clips, the spring must be securely clamped at the point where it is attached to the axle or other machine member. U-bolts and clamping pads are generally used for this purpose (Fig. 394). While there is some stiffening of the spring due to this clamping action, its effect may be disregarded without serious error in the computations for strength and deflection.

349. Leaf-Spring Materials and Permissible Stresses. The important leaf-spring materials recognized by the Society of Automotive Engineers are listed in Table 64.

The heat-treatment of these steels for leaf springs is specified as follows: (1) heat to 1430° to 1500° F. for the carbon steels, 1500° to 1600° F. for the chromium-vanadium or silico-manganese steels, (2) quench in oil, (3) temper to required hardness. A Brinell hardness of 388 to 444 is recommended for the chromium-vanadium and silico-manganese steels.

Following the practice of the Pennsylvania Railroad Co. for railway-car springs, a working stress at resting load of about 50,000 p.s.i. has often been assumed as standard for vehicle springs of all kinds. Stresses exceeding this value as much as 50 per cent have been considered permissible in Germany, and Rötscher * states that actual stresses in springs for racing automobiles at maximum load are allowed to reach a value as high as 205,000 p.s.i. with special steel having an ultimate strength in the hardened condition of about 250,000 p.s.i. The safest spring as far as breakage is concerned would no doubt be one of unhardened steel having an ultimate strength of about 125,000

* *Die Maschinenelemente*, p. 53 and p. 88.

TABLE 64

LEAF-SPRING MATERIALS

S.A.E. No.	Kind of Steel	Carbon Range	Manganese Range	Chromium Range	Vanadium Range	Silicon Range	Phosphorus Max.	Sulphur Max.
1085	Carbon	0.80–0.95	0.60–0.90	—	—	—	0.040	0.055
1090	Carbon	0.85–1.00	0.60–0.90	—	—	—	0.040	0.055
1095	Carbon	0.90–1.05	0.25–0.50	—	—	—	0.040	0.055
6145	Chromium-vanadium	0.40–0.50	0.60–0.90	0.80–1.10	0.15–0.18	—	0.040	0.050
6150	Chromium-vanadium	0.45–0.55	0.60–0.90	0.80–1.10	0.15–0.18	—	0.040	0.050
9255	Silico-manganese	0.50–0.60	0.60–0.90	—	—	1.80–2.20	0.040	0.050
9260	Silico-manganese	0.55–0.65	0.60–0.90	—	—	1.80–2.20	0.040	0.050

p.s.i. and an ultimate elongation of 18 to 20 per cent. Such a material would have an endurance limit of about 60,000 p.s.i. for loads varying between zero and a maximum, and, with the comparatively small impacts occurring in railway-car operation, a steel of this kind undoubtedly could be operated with a static stress of 50,000 p.s.i.

Under sudden loads and impacts, an automobile spring may deflect twice as much as it does under a static load, or perhaps even more, if it is not limited by stops. Hence, a spring of this type should have an endurance limit more than twice as high as the static stress, that is, one of about 120,000 p.s.i., if it is computed for a static stress of 50,000 p.s.i. It is questionable whether highly hardened plain carbon steels will meet this requirement, and if they do, their ductility will be very low. Silico-manganese steels, when properly heat-treated, are better in this respect, and steels containing vanadium are likewise characterized by their toughness. Such steels have therefore found widespread application for automotive springs.

350. Example of Automotive-Spring Computation.

Assume that a car weighing 3600 lb. is supported by semi-elliptic springs with the load equally distributed on the front and rear axles. Considering the available space and the proportions as commonly used in automobile practice, it is decided to make the springs 54 in. long and 2 in. wide. Determine the leaf thickness and the number of leaves, if the deflection at rest is assumed to be 4 in. and the allowable stress 50,000 p.s.i.

For a first approximation we shall compute the spring as if all leaves were graduated. According to the specifications, there will be a load of 900 lb. on each spring. Then from formulas (4) and (3), if E is taken as 30,000,000, we have

$$t = \frac{L^2 s_b}{4Ey} = \frac{54^2 \times 50{,}000}{4 \times 30{,}000{,}000 \times 4} = 0.3 \text{ in.}$$

and

$$n = \frac{3FL}{2bt^2 s_b} = \frac{3 \times 900 \times 54}{2 \times 2 \times 0.3^2 \times 50{,}000} = 8 \text{ leaves.}$$

Let us now assume that there is one extra full-length leaf that carries the spring shackle, and seven graduated leaves. We have then in formula (5), $r = n'/n = 1/8 = 0.125$. Hence

$$t = \frac{L^2 s_b}{2(2 + r)Ey} = \frac{54^2 \times 50{,}000}{2(2 + 0.125)30{,}000{,}000 \times 4} = 0.285 \text{ in.}$$

and

$$n = \frac{3FL}{2bt^2 s_b} = \frac{3 \times 900 \times 54}{2 \times 2 \times 0.285^2 \times 50{,}000} = 9 \text{ leaves.}$$

We notice that the introduction of extra full-length leaves stiffens the spring, so that it is necessary to thin individual leaves and to increase the total number of leaves. Our initial assumption of one extra full-length leaf to seven graduated leaves does not work out and a re-computation would be necessary. It is also recommended that the thickness be adjusted to a standard Birmingham wire gauge,

although to avoid misunderstandings the actual thickness should also be given. The thickness 0.285 in. just determined happens to be almost exactly No. 2 B.W.G., which is 0.284 in., and incidentally, this is an acceptable thickness under past S.A.E. practice for springs 2 in. wide. If, therefore, the deflection agrees quite closely with the 4 in. originally assumed, we may consider the design as satisfactory. We would now have $n'/n = 1/9 = 0.111$, and

$$y = \frac{54^2 \times 50{,}000}{2(2 + 0.111)30{,}000{,}000 \times 0.284} = 4.06 \text{ in.}$$

This result is so nearly equal to the stipulated deflection that no re-computation is necessary.

351. Coil Springs. Springs of this type are formed by coiling a bar or wire into a helix. Loads are applied in an axial direction and the spring may be used either as a tension or compression member. The wire section of a coil spring, as shown in Fig. 399, is manifestly subjected to a torsional shearing stress resulting from the torque Fr. However, there is also a transverse shear equal to Fk/A, where A is the cross-sectional area and k is a factor which depends upon the slope of the coils. In addition, a direct tensile (or compressive) stress is induced by a component of F acting longitudinally along the wire, since the coils do not lie in planes at right angles to the load. Finally, the twist in the wire, and hence its stress at the inner surfaces of the coils, is greater than it would be in a straight wire of equivalent length, because the length of the inside fiber when coiled is shorter.

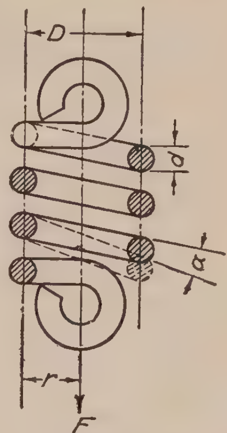


Fig. 399. Coil Spring, Tension Type.

With the ordinary proportions of coil springs, the helix angle or slope of the coils is relatively small. It has therefore been common practice to consider only the torsional shear in the wire and to neglect both the transverse shear and the direct stress. For any section across the spring, the resisting moment of the coiled wire may be considered in the same manner as the resisting moment of a straight wire acted on by the torque Fr. We would then have, for the wire of circular cross-section, $Fr = s_s\pi d^3/16$, where F is the axial load on the spring in lb., r the mean radius of the coils in in., s_s the shear stress in p.s.i., and d the wire diameter in in. If D is the mean coil diameter in in., and $r = D/2$, then the load capacity of the spring is

(6) $$F = \frac{s_s\pi d^3}{8D}.$$

To compute the deflection of the spring we observe that when the wire section twists through a certain small angle α (Fig. 399) the center of the spring is thereby lowered approximately the amount $r\alpha$, with the angle α measured in radians. The angular deflection of a round wire of diameter d under a torque Fr is $\alpha = 32Frl/\pi d^4G$, where l is the length of the wire in in. and G the modulus of elasticity for torsional shear (see page 15). The length l of a coil spring is approximately equal to $\pi Dn = 2\pi rn$, where n is the number of active coils. Substituting this value for l and multiplying the angular deflection by r, we have

$$(7) \qquad r\alpha = y = \frac{64Fr^3n}{d^4G} = \frac{8FD^3n}{d^4G},$$

where y is the deflection in in. of a coil spring under a load F.

According to A. M. Wahl,* who has performed some valuable research work on springs, this latter formula gives the deflection with sufficient accuracy for practical purposes, but he modifies the load formula by a factor K which has the following value:

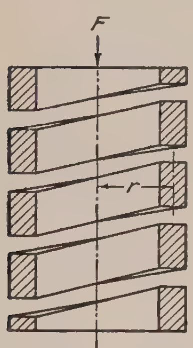


Fig. 400. Compression Spring of Rectangular Wire.

$$(8) \qquad K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C},$$

where $C = D/d$ for round-wire springs. For square-wire and rectangular-wire springs, the factor C, called the *spring index*, has the value D divided by the radial thickness of the wire in in.

For round-wire, square-wire, and rectangular-wire springs we then have the following formulas:

	Round Wire	Square Wire	Rectangular Wire
Total deflection of spring in in.	(7) $y = \frac{8FD^3n}{d^4G}$.	(9) $y = \frac{5.57FD^3n}{t^4G}$.	(10) $y = \frac{2.45FD^3n}{(b - 0.56t)t^3G}$.
Load on spring in lb.	(11) $F = \frac{s_s\pi d^3}{8DK}$.	(12) $F = \frac{s_st^3}{2.40DK}$.	(13) $F = \frac{2s_sb^2t^2}{DK(3b + 1.8t)}$.

(For rectangular wire, b is the larger and t the smaller of the dimensions.)

In these formulas, the value to introduce for n is the number of *active* turns. For compression springs with ends ground flat, as in Fig. 401, this number can be taken as two less than the total number of turns. For tension springs ending in hooks, the deflection of the hooks may add to the deflection derived from the actual number of

* See A.S.M.E. Transactions under the following reference numbers: APM-51-17, 1929, p. 158; APM-52-18, 1930, p. 217; A-35, 1935.

TABLE 65

COIL-SPRING STEELS

S.A.E. No.	Kind of Steel	Carbon Range	Manganese Range	Chromium Range	Vanadium Range	Phosphorus Max.	Sulphur Max.
1055 [a]	Carbon	0.50–0.60	0.60–0.90	——	——	0.040	0.055
1060	Carbon	0.55–0.70	0.60–0.90	——	——	0.040	0.055
1065 [a]	Carbon	0.60–0.75	0.60–0.90	——	——	0.040	0.055
1090	Carbon	0.85–1.00	0.60–0.90	——	——	0.040	0.055
5150	Chromium	0.45–0.55	0.60–0.90	0.80–1.10	——	0.040	0.050
6150	Chromium-vanadium	0.45–0.55	0.60–0.90	0.80–1.10	0.15–0.18	0.040	0.050

[a] These steels are also furnished with a manganese range of 0.90–1.20 per cent.

turns. If an allowance is made for this deflection, the number of turns to introduce for n can be taken slightly greater than the actual number of active turns.

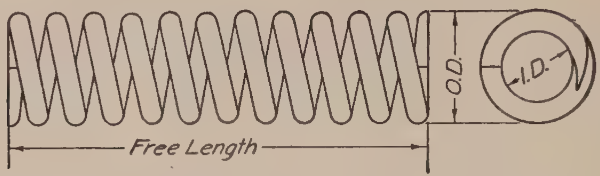


FIG. 401. COMPRESSION SPRING WITH ENDS GROUND FLAT.

For the most economical use of the material and the most favorable stress conditions, the ratio of the outside coil diameter to the thickness of the wire should fall approximately within the range of 5 to 15. The Raymond Mfg. Co. recommends 8 as a desirable ratio. Springs are very unfavorably stressed if this ratio is too low and 3 is given as the absolute low limit.

352. Coil-Spring Materials and Permissible Stresses. The most commonly used material for coil springs is medium or high-carbon steel. Where corrosive action may be serious, brass and especially phosphor-bronze are used. Representative steels as classified by the Society of Automotive Engineers are given in Table 65.

The stresses listed may be used directly for compressive loads, reduced one-fifth for tensile loads, and reduced two-thirds for continuously varying loads.

TABLE 66

MAXIMUM ALLOWABLE STRESSES FOR COIL SPRINGS (Raymond Mfg. Co.)

KIND OF MATERIAL AND SIZE OF WIRE	SHEARING STRESS (s_s) p.s.i.	TRANSVERSE MODULUS OF ELASTICITY (G)
Carbon-steel, 9/16 to 0.2437 in.	90,000	11,500,000
" " 0.2253 to 0.1055 in.	100,000	11,500,000
" " less than 0.1055 in.	120,000	11,500,000
Music wire	130,000	11,500,000
Brass and phosphor-bronze	60,000	8,000,000

353. Design of Coil Springs. If a coil spring is used merely for cushioning purposes, the computations proceed in the manner already shown in the example of a leaf spring. However, coil springs are often used to move machine parts with a positive action. Valve springs represent applications of this type.

A valve spring is required to seat the valve within a certain time. In slow-moving pumps or steam engines, there is usually ample time for this closure, and pressure differences in the valve passages help in the process. In high-speed combustion engines, the time allowed for the valve events is very short, and careful computation is necessary to make sure that the valve spring is strong enough for the purpose. The following example will illustrate the principles involved.

EXAMPLE. We will assume a gasoline engine operating at a maximum speed of 2000 r.p.m. and equipped with overhead valves, as shown in Fig. 402. The valve lift is 0.36 in. When the valve is closing, the spring must be able to impart an acceleration to the valve parts equal to that fixed by the cam outline. Otherwise the valve mechanism would not be in contact with the cam and there would be lost motion and hammering.

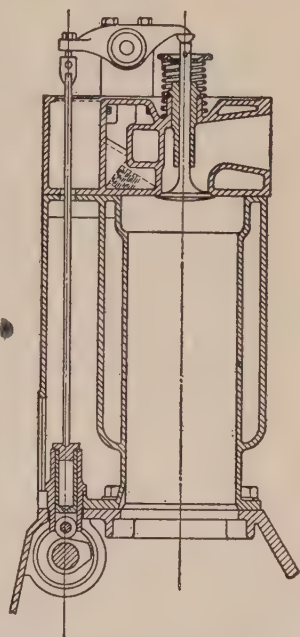


FIG. 402. ENGINE WITH OVERHEAD VALVES.

To favor the spring let us assume that 60 per cent of the time for the closing stroke is available for acceleration and 40 per cent for deceleration. We will assume, however, that acceleration takes place during half the lift stroke, deceleration during the other half, and that the closing action takes place during 46 deg. of cam-shaft motion. Since the cam shaft revolves only half as fast as the crank shaft, the time occupied in closing is $46 \times 60/(360 \times 1000) = 0.0077$ sec. In 60 per cent of this time or 0.0046 sec., the follower travels 0.18 in. If the cam is assumed to be of the constant acceleration type, we find the acceleration a from the formula $a = 2S/t^2$, where S is the distance and t the time. We then have

$$a = 2 \times 0.18/0.0046^2 = 17{,}000 \text{ in. per sec.}^2$$
$$= 1420 \text{ ft. per sec.}^2.$$

The moving parts to be actuated by the spring in an overhead-valve mechanism consist of the follower and follower rod, the rocker arm, the valve and the valve spring. Since one end of the spring is stationary and only the free end moves, and likewise since the rocker arm does not reciprocate at its pivot point, only half the mass of the rocker arm and the spring is considered as reciprocating with the full valve travel. If we assume a total reciprocating weight of 2 lb., the force necessary to impart the required acceleration is $2 \times 1420/32.2 = 88$ lb.

Since the acceleration is constant throughout the acceleration period, this force must be available up to the point where deceleration begins. At this point, however, the spring has stretched 0.18 in. from its most compressed condition. Let us assume that the valve in the closed position is to be held by a spring force of 40 lb. This means that while the spring extends 0.18 in., the spring pressure changes from 88 to 40 lb., a difference of 48 lb. Since there is a linear relation between spring force and the deflection, the force in the most compressed condition will be $88 + 48 = 136$ lb.

Assuming a mean coil diameter of 2 in. and a stress of 30,000 p.s.i., we have from formula (6) the wire diameter

$$d = \sqrt[3]{\frac{8FD}{\pi s_s}} = \sqrt[3]{\frac{8 \times 136 \times 2}{\pi \times 30{,}000}} = 0.284 \text{ in.}$$

The nearest commercial wire size is Washburn and Moen No. 1 ($d = 0.283$ in.).

The number of coils is determined by means of formula (7), which gives the total deflection from zero load to full load. Since there is a linear relation between deflection and load, we can determine the deflection increase for any additional load. Since an added load of 48 lb. causes a deflection increase of 0.18 in., we have

$$n = \frac{yd^4G}{8FD^3} = \frac{0.18 \times 0.283^4 \times 11{,}500{,}000}{8 \times 48 \times 2^3} = 4.3 \text{ active coils.}$$

Assuming two inactive coils, the total number of turns will be 6.3, or for practical purposes, approximately 6.

In many cases, it is best to leave the final design to the judgment of the spring manufacturer, in order that the best proportions may be selected for the particular materials under consideration. It is especially important to check the spring in the closed condition to make sure that it is not stressed beyond the elastic limit. To illustrate, we will assume that the torsional elastic limit of the material considered is 120,000 p.s.i. This value is four times the stress induced in the spring when under its maximum load. In consequence, the deflection from the free length to the closed state could be four times as great as the deflection to the maximum working load. In this spring it is very unlikely that the material could be over-stressed.

354. Radially Tapered Disc and Belleville Springs. Flat-tapered disc springs, as shown in Fig. 403, and dish-shaped disc springs, known as *Belleville* springs (Fig. 404), are useful in attaining comparatively high deflections and spring loads in limited spaces. Other advantages listed by Brecht and Wahl * in a paper on the tapered disc spring are as follows:

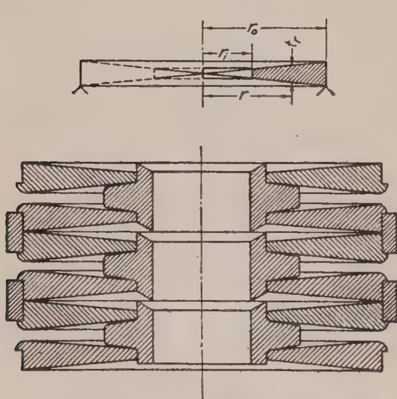


Fig. 403. Flat-Tapered Disc Spring.

1. The disc spring is adjustable in height and flexibility by adding or taking off discs.

2. It will withstand lateral loading as well as axial loading.

3. It can be so designed that no disc is overloaded. Failure of one disc will not cause the complete loss of flexibility, nor will it increase the load on the remaining discs. A failure of one disc would not require replacement of the whole spring.

* Transactions A.S.M.E., vol. 52, 1930, APM–52–4, p. 45.

4. Helical springs of large size are difficult to heat-treat uniformly —a disadvantage that can be overcome with the disc spring.

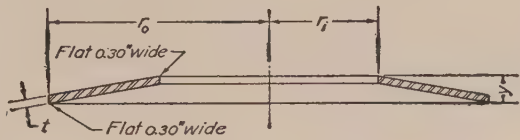
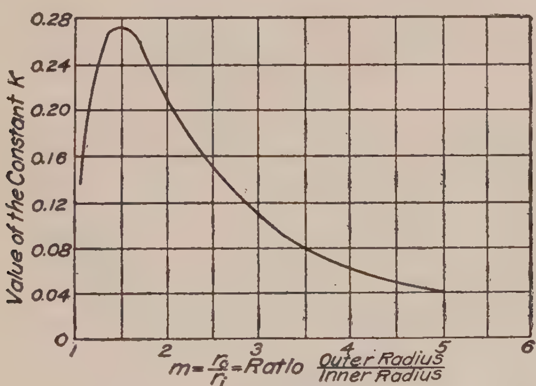


FIG. 404. BELLEVILLE SPRING.

The most comprehensive load equation applicable for radially-tapering disc springs is

$$F = \frac{Et_i^3 y}{Kr_o^2}\left[1 + \frac{1.5y^2}{t_i^2(m^2 + m + 1)}\right], \tag{14}$$

where F is the total axial load in lb., t_i the disc thickness in in. at the inner radius r_i, r_o the outer radius of disc in in., y the axial deflection in in. of one disc, $m = r_o/r_i$, and K is a constant to be taken from the curve in Fig. 405.



Transactions A.S.M.E., vol. 52, 1930

FIG. 405. DEFLECTION CONSTANT FOR RADIALLY-TAPERED DISC SPRINGS.

The maximum stress in the disc is of tensile nature and is equal to

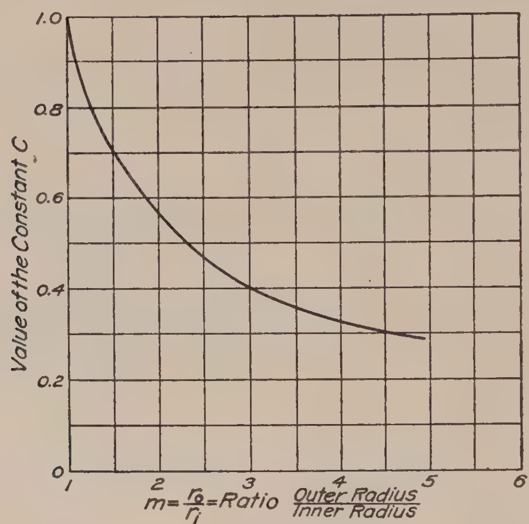
$$s_b = C\frac{F}{t_i^2}, \tag{15}$$

where C is a constant given graphically in Fig. 406.

For Belleville springs, the formula given by D. A. Gurney * is

$$y = \frac{4F}{3\pi Et^3}\left(r_o^2 - r_i^2 + r_o r_i \log_e \frac{r_o}{r_i}\right), \tag{16}$$

where y is the deflection in in., of one disc, F the load in lb., E the modulus of elasticity in tension, t the thickness in in., r_i the inside, and r_o the outside radii in in.



Transactions A.S.M.E., vol. 52, 1930

FIG. 406. STRESS CONSTANT FOR RADIALLY-TAPERED DISC SPRINGS.

No formula for the stress is given. The statement is made that with a heat-treated chromium-vanadium steel of 102,000 p.s.i. tensile strength and the surprisingly low modulus of elasticity of 23,000,000 p.s.i., the elastic limit of the material would not be reached if the following relations between outside radius and dish angle were observed:

$r_o = 6t$, dish angle = 4 deg.
$r_o = 9t$, dish angle = 5 deg.
$r_o = 12t$, dish angle = 6 deg.

Since a spring steel with a tensile strength of only 102,000 p.s.i. is an unusually weak one, it is doubtful if failure could occur if these limits of deflection are observed.

355. Springs for Transmitting Torques. Two types of springs are used for the transmission of torques, namely, closely-coiled helical

* Transactions A.S.M.E., vol. 51, 1929, APM–51–2, p. 13.

springs, which are used, for instance, in flexible shafts, and spiral springs, which are used in watches, spring motors, etc.

For torque applications of this type, approximate formulas are in use; under certain circumstances the results may be considerably in error. While only the approximate formulas in common use will be given here, it is well for the student to know under what circumstances inaccuracies are to be expected. Consequently, computed values should be checked by actual tests.

356. Helical Springs in Torsion. When a helical spring is subjected to a twisting moment, the wire is under a bending action due to the change in curvature of the coils. The assumption is made that the coils, which are sharply curved, behave as if they were equivalent to a straight beam, equal in length to the wire in the spring. With this assumption, we have

$$\theta = \frac{M_t l}{EI} = \frac{64 M_t D n}{E d^4}, \tag{17}$$

where θ is the total angle of twist in radians, M_t the twisting moment in in. lb., l the length of the wire in in., E the modulus of elasticity in bending, I the rectangular moment of inertia of the wire, D the mean coil diameter in in., n the number of coils, and d the wire diameter in in.

For the bending stress in the wire, we have

$$s_b = \frac{32 M_t}{\pi d^3}. \tag{18}$$

For a spring made of square wire of side a, the corresponding formulas are

$$(19) \qquad \theta = \frac{12\pi M_t D n}{E a^4}, \qquad \text{and} \qquad (20) \qquad s_b = \frac{6 M_t}{a^3}.$$

Sometimes there is an interest in the amount of work stored in stressed springs of this character. For a circular section the work in in. lbs. is equal to $s_b^2/8E$ per cu. in. of wire, and for a rectangular section, $s_b^2/6E$ per cu. in.

357. Spiral Springs. In the case of spiral springs, possible inaccuracy of computations arises from uncertainty as to the exact way in which the ends are held and the torque applied, as well as from uncertainty as to how perfectly the coils remain apart during the twist.* Again in this instance, it will suffice to give the commonly

* A discussion of this matter is contained in a paper by J. A. Van Den Broek, Transactions A.S.M.E., vol. 53, 1931, APM–53–18, p. 247.

used formula for a flat spiral watch-type spring, fixed at the outer end and wound by means of an arbor at the center (Fig. 407). The turning moment applied at the center o produces a torque $Fr = M_t$, where F is the reactive force at the fixed end A and r is the radius to the arbor center. The maximum bending stress occurs at the point B, where the bending moment is approximately $2Fr = 2M_t$. We then have

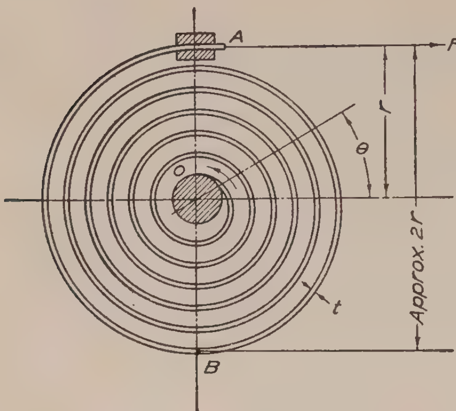


FIG. 407. SPIRAL SPRING.

$$\theta = \frac{M_t l}{EI}, \tag{21}$$

and

$$s_b = \frac{12M_t}{bt^2}, \tag{22}$$

where θ is the winding angle in radians, M_t the torque in in. lb., l the length of the spring in in., I the moment of inertia of the spring section, s_b the bending stress in p.s.i., b the width, and t the thickness of a flat spring, both in in. Since the maximum bending moment and maximum bending stress occur at only one point, the work stored in the spring per unit of volume is only $s_b^2/24E$, or one-fourth of the work stored in a helical spring.

EXAMPLE 1. What torque can be transmitted by a flexible shaft if it consists of a coiled spring having a wire diameter of 0.05 in.?

We have from formula (18) $M_t = \pi d^3 s_b/32$. With a bending stress of 50,000 p.s.i., the permissible moment is $\pi \times 0.05^3 \times 50{,}000/32 = 0.6$ in. lb. torque.

EXAMPLE 2. Determine the length of a spiral spring that will give six complete turns while the stress in the wire is being reduced from 100,000 p.s.i. to zero.

We have from formula (22) the stress $100{,}000 = 12M_t/bt^2$; consequently, $M_t = 100{,}000bt^2/12$. Six complete turns are equal to $6 \times 2\pi = 12\pi$ radians. We have from formula (21) the wire length

$$l = \frac{\theta EI}{M_t} = \frac{12\pi \times 30{,}000{,}000 \times bt^3 \times 12}{12 \times 100{,}000 \times bt^2} = 11{,}300t \text{ in.}$$

We observe that the length of the spring depends on the thickness of the wire but not on the width. If we assume a thickness of 0.050 in., the length required is 565 in.

In solving problems, remember that the deformation of a spring is directly proportional to the applied force, or $F = ky$, where k is the spring constant. Thus $F_2 - F_1 = k(y_2 - y_1)$; where the subscripts refer to corresponding values.

PROBLEMS

1. A simple cantilever spring, made of 0.02 in. thick brass, is 1/2 in. wide and 2½ in. long. What is the maximum stress induced and the maximum deflection under a load of 0.1 lb.?

2. A simple phosphor-bronze cantilever spring 3 in. long is to have a deflection of 3/4 in. for a load of 2 lb. Select the necessary width and thickness and check the maximum stress induced. What would be the calculated induced stress if the deflection were increased 25 per cent? Would this increased stress be actually attained?

3. An alloy-steel cantilever spring is made of six graduated leaves each 2 inches wide and 3/8 in. thick. Determine the deflection and the maximum stress if the spring length is 27 inches and the load 1600 lb.

4. An alloy-steel cantilever spring is made up of two full-length leaves and six graduated leaves, each 1¾ in. wide. If the spring length is 35 in. and the load is 500 lb., determine the necessary thickness to give a deflection of 3 in. What is the maximum stress?

5. A semi-elliptic automobile spring 58 in. long carries a total load of 1600 lb. The spring is composed of 10 leaves, two of which are full-length, each 2½ in. wide. Determine the necessary thickness and resultant stress to give a deflection of 2½ in.

6. A semi-elliptic alloy-steel spring has a length of 40 in. and carries a load of 10,000 lb. It is composed of 18 leaves, two of which are full-length, each 3½ in. wide. For a maximum induced stress of 70,000 p.s.i. determine the thickness of the leaves and the maximum deflection.

7. A helical spring is made of oil-tempered wire of 11 W & M gauge (0.1205 in.), has ten active coils, and an outside diameter 27/32 in. Determine the deflection and stress under an axial compressive load of 100 lb. An empirical expression for the tensile strength of this wire is $s_t = \frac{138,000}{d^{1/4}} - \frac{1600}{d}$. Is the induced stress satisfactory?

8. A helical spring is to be made of Monel metal with a limiting stress of 50,000 p.s.i. The wire diameter is 1/16 in. and the ratio of mean spring diameter to wire diameter is 6. Determine the maximum stress induced by an axial tensile load of 10 lb. and the number of coils necessary for an extension of 1 in. $G = 9,000,000$.

9. Design a compression spring of music spring wire to give a change in length of 5/8 in. when the load is increased from 60 lb. to 100 lb. The maximum length is 8 in. The allowable torsional stress may be taken as 50,000 p.s.i. The spring index is to be 5. Fix upon the diameter of the wire and the number of coils.

10. A coil spring is made of 20 turns of wire 1½ in. × 3/4 in. (3/4 in. is the dimension parallel to the axis). The free length of the spring is 30 in. and the outside diameter of the coil is 5 in. Determine the deflection between the initial and final compressive loads of 500 and 7000 lb., respectively, the loaded length of the spring, and the maximum stress.

11. An overhead automobile-valve mechanism has an effective weight of 1¾ lb. If the engine operates at 3000 r.p.m., fix upon the spring proportions using assumptions similar to those in the example of § 353.

12. Select the proper size of square oil-tempered spring wire for a helical compression spring to support a load of 250 lb. with an accompanying deflection of 3½ in. for five active coils.

13. A semi-elliptic spring with a total effective length of 30 in. has ten leaves, two of which are full-length, which are 1/4 in. thick and 2 in. wide. It is desired to replace this spring by a helical spring of 4 in. mean diameter and of such propor-

tions that for any load it will have the same value of induced stress and the same deflection as the leaf spring. What should be the diameter of the wire and the number of turns?

14. Determine the proportions of a helical spring of round wire to give a twist of one revolution under an applied torque of 100 in. lb. The allowable tensile stress may be taken as 100,000 p.s.i.

15. Determine the proportions of a helical spring of square wire to give a twist of 90 deg. under an applied torque of 30 in. lb. The tensile stress is not to exceed 125,000 p.s.i.

16. What load will produce a deflection of 1/4 in. on a spring made up of 5 radially tapered steel discs (as in Fig. 403) having a thickness of 0.3 in. at outer edge and 0.12 at inner edge, an outer diameter of 4 in. and an inner diameter of 1½ in.? What maximum stress is induced?

17. A Belleville spring is to be made up of steel discs 0.20 in. thick, having an outside diameter of 2.5 in., an inside diameter of 1 in., and a dish angle of 5 deg. How many discs are necessary for a deflection of about 0.3 in. under a load of 6000 lb.?

18. A flat spiral steel spring is to give a maximum torque of 10 in. lb. for a maximum stress of 100,000 p.s.i. What thickness and length is necessary to give two complete turns of motion when the stress decreases from 100,000 p.s.i. to 25,000 p.s.i.? Space considerations limit the width of wire to 1/2 in.

CHAPTER 25

VIBRATORY STRESSES

358. General Effect of Vibrations. Loads that are applied with suddenness or impact cause stresses very much in excess of those due to steady loads of equal magnitude. If the action of the load is periodic, or vibratory, the stress may be of even greater magnitude. Suppose a rod, as shown in Fig. 408, is set vibrating, and that every time it reaches its extreme positions a periodic force deflects it slightly farther. By such timing of the impulses, breakage can result from a very small force. The only condition that is necessary is *resonance*, that is, coincidence between the natural period of frequency of the rod and the period of application of the force.

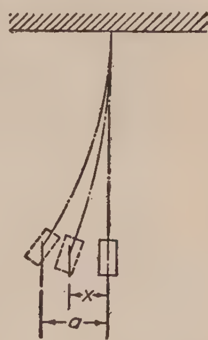


Fig. 408. Vibrating Rod.

Very serious breakages of multi-throw crankshafts of internal-combustion engines have occurred from such resonance between the natural period of the shaft and the combustion impulses in the cylinders. Resonance of the frame of a building and the operating period of a compressor or pump causes vibrations that are unpleasant, or even dangerous.

One of the first technical vibration problems scientifically investigated concerned the vibration of impulse steam turbines. For a long time the so-called *critical speeds* of shafts of high-speed rotating machinery were dealt with not from the point of view of vibratory resonance, but from the viewpoint of excessive deflection set up by centrifugal force at certain speeds.

359. Critical Speeds. The derivation of formulas from the action of centrifugal force is simple. Consider a shaft, as in Fig. 409, of negligible weight with a heavy mass M at some point between the supports. The center of gravity of this mass may not coincide with the center line of the shaft by a small amount e. In consequence, when the shaft rotates, there will be a centrifugal force which will deflect the shaft the amount y. The magnitude of this centrifugal force then is $M\omega^2(y + e)$, where ω is the angular velocity in radians per sec. As all bending formulas are based upon Hooke's law, the deflection is directly proportional to the deflecting force and to the

resisting force. For the deflection y, the force resisting deflection is Ky, where K is a constant.

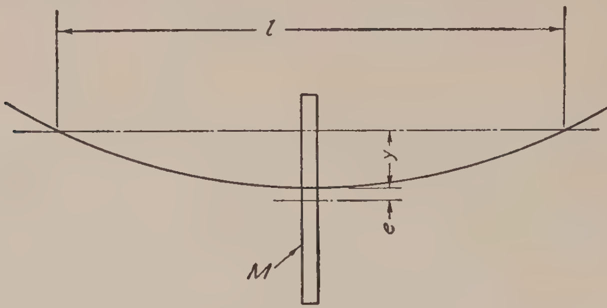


FIG. 409. DEFLECTION OF A ROTATING SHAFT LOADED WITH A SINGLE HEAVY MASS. (ROTATING BELOW CRITICAL SPEED.)

Hence at equilibrium between the centrifugal force and the resisting force in the shaft we have $M\omega^2(y + e) = Ky$, so that

$$y = \frac{M\omega^2 e}{K - M\omega^2} = \frac{e}{\dfrac{K}{M\omega^2} - 1}.$$

Unless e is zero, the deflection y becomes infinite when $K = M\omega^2$. The corresponding value of ω is called the critical speed, ω_c. We have

(1) $$\omega_c = \sqrt{\frac{K}{M}}.$$

It is important to notice that if ω exceeds ω_c, the deflection again becomes *finite*, although negative, that is, in the direction opposite to

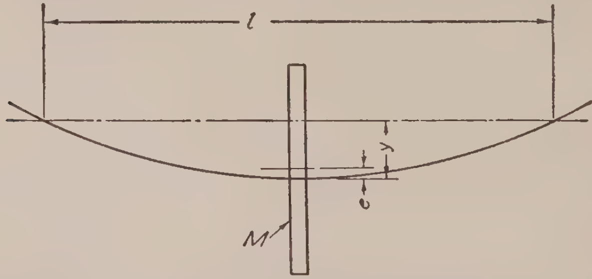


FIG. 410. DEFLECTION OF A ROTATING SHAFT LOADED WITH A SINGLE HEAVY MASS. (ROTATING ABOVE CRITICAL SPEED.)

e from the shaft axis. This situation is shown in Fig. 410. If ω is very great, K becomes negligible in comparsion with $M\omega^2$ and $y = -e$ (from $y = e/[(K/M\omega^2) - 1]$ when ω is put equal to infinity). The

deflection is then such that the mass M rotates, not around the axis of the shaft, but around its own center of gravity.

It is therefore possible to run machinery smoothly and stably above the critical speed, with the shaft in a flexed condition. Since in the region of the critical speed there is great vibration and danger of breakage, it is good practice to make the critical speed of the shaft intentionally low when the running speed is high. To attain this condition, the stiffness constant K should be made small, that is, the shaft should be made very slender.

The introduction of such so-called *flexible* shafts was one of the most brilliant and daring advances in mechanical design, and was due to Gustaf de Laval, the inventor of the impulse steam turbine.

360. Higher Critical Speeds. A weightless shaft with only one concentrated mass can deflect in only one way, as shown in Fig. 409.

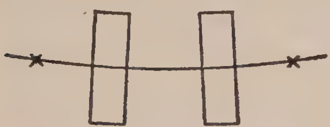


FIG. 411. DEFLECTION OF A SHAFT WITH TWO MASSES. (FORM A.)

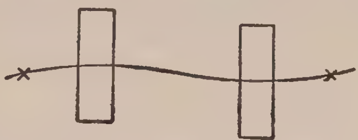


FIG. 412. DEFLECTION OF A SHAFT WITH TWO MASSES. (FORM B.)

With two masses there are two possible ways (Figs. 411 and 412); with three masses there are three (Figs. 413, 414, and 415); and so on.

It is evident that the stiffness constant K is higher for bending into curves with counterflexures than for bending into a single sweep. Hence for several concentrated masses there will be a first, a second, a third, and perhaps other critical speeds, higher than the lowest. If the shaft has only an evenly distributed load, for instance its own weight, it can bend, theoretically

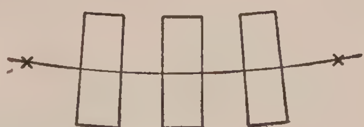


FIG. 413. DEFLECTION OF A SHAFT WITH THREE MASSES. (FORM A.)

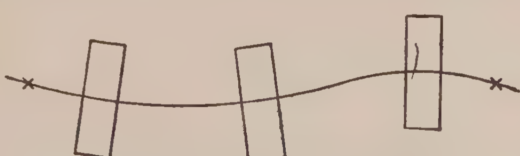


FIG. 414. DEFLECTION OF A SHAFT WITH THREE MASSES. (FORM B.)

at least, in an infinite number of ways, and there are an infinite number of critical speeds.

Very often it may be easy to avoid the first critical speed, but difficult to avoid the second, or higher critical speeds. It must therefore be ascertained whether any of these speeds fall at, or near, the running speed of the machine, or near the periodicity of any pulsating forces in the machine.

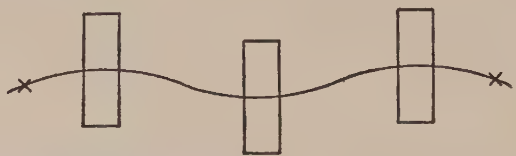


FIG. 415. DEFLECTION OF A SHAFT WITH THREE MASSES. (FORM C.)

For the computation of the critical speed of a shaft with several discs, Dunkerley's * rule is most convenient. The rule states that if several rotating masses mounted on a shaft would individually, with all others removed, give critical speeds ω_1, ω_2, ω_3, etc., then the actual critical speed is

$$(2)\quad \omega_c = \frac{\omega_1\omega_2\omega_3\omega_4 \cdots \omega_n}{\sqrt{(\omega_2\omega_3 \cdots \omega_n)^2 + (\omega_1\omega_3 \cdots \omega_n)^2 + (\omega_1\omega_2\omega_4 \cdots \omega_n)^2 + \cdots}}.$$

For two discs, this formula becomes

$$(3)\quad \omega_c = \frac{\omega_1\omega_2}{\sqrt{\omega_2^2 + \omega_1^2}}.$$

Let us compute an example for three discs, one of which gives a critical speed of 1000 r.p.m., another of 2000 r.p.m., and a third of 3000 r.p.m. We have

$$\omega_c = \frac{1000 \times 2000 \times 3000}{\sqrt{2000^2 \times 3000^2 + 1000^2 \times 3000^2 + 1000^2 \times 2000^2}} = 857 \text{ r.p.m.}$$

To show what happens with distributed loads, we may cite the case of a shaft of uniform diameter freely supported at the ends. The lowest critical speed for such a shaft is

$$(4)\quad \omega_c = \sqrt{\frac{\pi^4 IE}{ml^4}},$$

but there are other critical speeds 4, 9, 16, 25, $\cdots$ times the lowest. In this formula I is the moment of inertia, E the modulus of elasticity, m the mass per unit of length, and l the length.

* From the article *The whirling and vibration of shafts*, Philosophical Transactions of the Royal Society, vol. A, 1894, p. 279.

A shaft guided rigidly in the bearings has a lowest critical speed of

$$\omega_c = \sqrt{\frac{81\pi^4 IE}{16ml^4}}, \tag{5}$$

and higher critical speeds 2.8, 5.4, 9, ⋯ times the lowest.

On the other hand, a uniformly loaded overhanging shaft will hardly bend to a complicated curve and has a critical speed * of

$$\omega_c = 3.494\sqrt{\frac{IE}{ml^4}}. \tag{6}$$

Example. Determine the critical speed of a shaft, 1 in. in diameter, 18 in. long, carrying a 50 lb. disc in the middle. The bearings are pivoted and the shaft may be regarded as freely supported at the ends.

Reducing all dimensions to inches, we have $M = 50/(32.2 \times 12) = 0.13$. The moment of inertia I of the shaft $\pi d^4/64 = \pi/64$. The deflection formula for a beam freely supported at the ends is $y = Fl^3/(48EI)$. Hence we have for the stiffness constant, $K = 48EI/l^3 = (48 \times 30{,}000{,}000 \times \pi)/(18^3 \times 64) = 12{,}200$. Substituting in formula (1), we have

$$\omega_c = \sqrt{\frac{12{,}200}{0.13}} = 306 \text{ rad. per sec.,} \qquad \text{or} \qquad 306 \times 60/2\pi = 2920 \text{ r.p.m.}$$

While the deflection, theoretically speaking, becomes infinite only at the critical speed, there is really a condition of unsatisfactory operation through a considerable range of speeds near it. At 300 rad. per sec. and with the data used in the example, formula (1) gives a deflection of $25e$, and at 290 rad. per sec. a deflection of $8.85e$.

361. Critical Speed and Deflection. The connection between deflection and spring constant K makes it possible to compute critical speed from the static deflection of the shaft under weight loads equal to those of the rotating masses. For a mass M, we have $Mg = W$. We have also, if the shaft is bent by the load W, a deflection y_0 in in. such that $Ky_0 = W$. Hence from formula (1),

$$\omega_c = \sqrt{\frac{g}{y_0}} \text{ rad. per sec.,}\dagger \tag{7}$$

or

$$N_c = 187.7\sqrt{\frac{1}{y_0}} \text{ r.p.m.,}$$

where g, the acceleration due to gravity, is in in. per sec. per sec., and N_c is the critical speed in r.p.m.

* This value is taken from Stodola's *Dampfturbinen*, 4th ed., p. 293. Timoshenko, using a somewhat different derivation, arrives at the factor 3.515. See *Vibration Problems in Engineering*, 1st ed., p. 234.

† A form useful where the total deflection y under each load is known is

$$\omega_c = \sqrt{\Sigma Wyg/\Sigma Wy^2}.$$

These formulas apply whether the shaft is horizontal, vertical, or inclined. In any case, it is only necessary to compute the deflection under loads equal to the weights of the rotating masses.

For instance, in the shafting example computed above, the deflection is found to be 4.1×10^{-3}, and the critical speed is $\sqrt{\dfrac{32.2 \times 12}{4.1 \times 10^{-3}}} = 306$ rad. per sec., as already found.

362. General Theory of Vibration. In the elementary approach to vibrations just given it was made to appear that the critical speed was due to a certain eccentricity e of the rotating masses and that no critical speed would occur if this eccentricity were zero. Actually no eccentricity is necessary. Any periodic force in step with the natural period of a body will lead to excessive deflections, as has already been set forth. The attitude may be taken that any rotation can be regarded as made up of two vibrations acting at right angles to each other; that centrifugal force is merely a force at all times in step with the rotation that produces it; and that therefore the centrifugal force produces an infinite deflection as soon as the speed of rotation coincides with the natural period of vibration of the shaft.

In fact, if the equations of two vibrations at right angles are $x = r \cos pt$ and $y = r \sin pt$, where t is time and p a proportionality factor, we have $x^2 + y^2 = r^2(\cos^2 pt + \sin^2 pt) = r^2$. This is the equation of a circle of radius r. It shows that the resultant of x and y is the radius r of the circle and that this radius rotates with a uniform angular velocity p, its position at any time being determined by the angle pt.

To show that $x = r \cos pt$ and $y = r \sin pt$ are not merely periodic trigonometric functions, but equations for the vibratory deflections of a body obeying Hooke's law, let us derive the fundamental equation for the simple vibration of a mass M suspended at the end of an elastic rod, as shown in Fig. 408. The rod itself, in first approximation, is assumed to be weightless. Suppose we deflect such a rod by the amount a from its position at rest, and then release it. The force on M at displacement x is the spring force Kx in the rod, x being the momentary deflection, and K the spring constant. On the basis of Newton's second law, we have the relation

$$\text{(a)} \qquad M\frac{d^2x}{dt^2} = -Kx, \qquad \text{or} \qquad \frac{d^2x}{dt^2} + \frac{K}{M}x = 0.$$

The negative sign is introduced because the velocity increases as x decreases. For convenience, substitute for K/M a quantity p^2.

Assume that

$$\text{(b)} \qquad A \cos pt + B \sin pt = x$$

is a solution of this equation. By differentiating twice we get $-Ap^2 \cos pt - Bp^2 \sin pt = d^2x/dt^2$. The left-hand member is manifestly equal to $-p^2x$, and the sum of d^2x/dt^2 and p^2x is zero as the original equation (a) demands.

The quantities A and B are constants of integration. To see what they signify, replace them with two new constants, an angle α and a factor c, so that $A = c \sin \alpha$, and $B = c \cos \alpha$. We have then

$$\text{(8)} \qquad \tan \alpha = \frac{A}{B},$$

$$\text{(9)} \qquad c = \sqrt{A^2 + B^2}.$$

Introducing these values for A and B, we find readily in equation (b)

$$\text{(10)} \qquad x = c \sin (pt + \alpha).$$

The quantity c is called the *amplitude*. It is the maximum value of x. The angle α is called the *phase angle*. It gives the value of x at zero time. If x is zero when t is zero, then α is zero and the vibration is expressed simply by $x = c \sin pt$, which is what we set out to demonstrate. Observing now that p, as was already set forth, is merely the angular velocity of the rotating radius, and that by definition

$$p = \sqrt{\frac{K}{M}},$$

we find that this angular velocity is exactly equal to the critical speed as derived from centrifugal force.

In ordinary vibrations, however, it is customary to speak of free period or natural frequency of a body, rather than of critical speed. The full period of a cosine or sine function comprises the angle 2π. Hence, for a full period τ, we have $p\tau = 2\pi$, or

$$\text{(11)} \qquad \tau = \frac{2\pi}{p} = 2\pi \sqrt{\frac{M}{K}}.$$

The reciprocal of this quantity is the frequency in periods per second. Denoting this frequency by ν, we have

$$\text{(12)} \qquad \nu = \frac{1}{2\pi} \sqrt{\frac{K}{M}}.$$

We notice that the relation between the critical speed and these quantities is

(13) $$p = \omega_c = 2\pi\nu = \frac{2\pi}{\tau}.$$

363. Damped and Forced Vibrations. Damping of some sort is present in all mechanical vibrations occurring in nature. To account for it mathematically the assumption is generally made that the damping resistance is proportional to the velocity. Thus the differential equation for vibrations with damping is taken as

(14) $$M\frac{d^2x}{dt^2} = -f\frac{dx}{dt} - Kx,$$

where f is a damping factor. With the notation $f/M = 2n$, $p^2 = K/M$, and $p_1^2 = p^2 - n^2$, the solution of the differential equation has the form

(15) $$x = e^{-nt}(A \sin p_1t + B \cos p_1t) = e^{-nt}c \sin (p_1t + \alpha),$$

and the length of the period is

(16) $$\tau = \frac{2\pi}{p_1} = \frac{2\pi}{\sqrt{p^2 - n^2}}.$$

In other words, the amplitude decreases gradually, and the period is somewhat longer than the undamped period.

Damping is usually neglected in the practical computation of the natural period, or critical speed, of machine parts. The equations are given here mainly for their fundamental importance in general vibration theory.

Of tremendous importance in machine design are, however, *forced vibrations*. Forced vibrations occur when there is a periodically variable force Q supplementing the spring force. We have, therefore,

(17) $$M\frac{d^2x}{dt^2} = -Kx + Q.$$

Dividing by M and introducing for Q/M the assumed variable function $q \cos mt$, we have

(18) $$\frac{d^2x}{dt^2} + p^2x = q \cos mt.$$

Integrating this equation, we find

(19) $$x = A \cos pt + B \sin pt + \frac{q}{p^2 - m^2} \cos mt.$$

To check the correctness of this solution, differentiate the x value on the right side of the equation twice and add to this value p^2 times the value of x. It will be found that the sum equals $q \cos mt$, as equation (18) demands.

Equation (19) consists of two parts, one part, $A \cos pt + B \sin pt$, which is due to the regular period of free vibration, $2\pi/p$, and another part, $q \cos mt/(p^2 - m^2)$, which is due to the impressed force with a period of $2\pi/m$.

The amplitude of this forced vibration is $q/(p^2 - m^2)$, and it is observed that this amplitude becomes infinite when $p = m$, that is, when the period $2\pi/m$ of the impressed force equals the natural period, $2\pi/p$, of the free vibration.

In the case of rotating machinery, then, it is not necessary that there be an eccentricity of the center of gravity in order that a critical speed may develop. It is sufficient that any periodic force acting on the machinery have a frequency equal to that of some of the parts.

Equation (19) enables us also to ascertain the extent to which the amplitudes (hence the deflections and the stresses in the parts) are increased by the vibration.

If the frequency of the impressed force is small, the frequency factor m is small. If the force is static, then m is zero and the amplitude is then $q/p^2 = Q/K$. The ratio of the deflections and the stresses with vibration to those without vibration is known as the amplification factor, and is equal to

$$\text{(20)} \qquad \frac{p^2}{p^2 - m^2} = \frac{1}{1 - m^2/p^2} = \frac{1}{1 - \nu^2/\nu_0^2} = \frac{1}{1 - \tau_0^2/\tau^2}.$$

In these expressions ν_0 and τ_0 are the frequency and period of the free vibration, and ν and τ are those of the impressed force.

These equations make it clear how extremely serious it is to assume that the danger from vibrations arises only at the critical speed or the natural period. Suppose the ratio of $\tau_0/\tau = 0.7$, which might mean that the operating speed of a machine is about 30 per cent below the natural period. Many machine designers would feel that this would eliminate all danger of breakdown from vibration. Actually since $0.7^2 = 0.49$, the stresses due to the vibratory forces would be twice as high as stresses from static forces of the same magnitude. This might very easily carry them beyond the endurance limit or the elastic limit, with the consequence that final breakdown would be inevitable.

364. Torsional Vibrations. In deriving the formulas for vibrations we have dealt with linear vibrations. Actually, in connection with shafting particularly, torsional vibrations are at least as important.

In the case of a rod anchored at one end and carrying a heavy mass at the other, the formula for the torsional vibration is exactly similar to that for the linear vibration. The only thing to remember is that for rotation Newton's second law takes the form

$$I\frac{d^2\phi}{dt^2} = -M_t \tag{21}$$

where the negative sign is used as on page 456, ϕ is the angular displacement, M_t is the torque causing it, and I is the moment of inertia of the rotating body. We also have the relation

$$K_1\phi = M_t, \tag{22}$$

where K_1 is a torsional spring constant. For a round, solid shaft of diameter d and length l, $K_1 = \pi d^4G/(32l)$, where G is the modulus of elasticity in shear.

The formula for the critical speed will have exactly the same form as for transverse vibrations, only with the substitution of K_1 for K and I for M. Hence we have

$$p = \sqrt{\frac{K_1}{I}}. \tag{23}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{K_1}{I}}. \tag{24}$$

$$\tau = 2\pi\sqrt{\frac{I}{K_1}}. \tag{25}$$

As an illustration, the frequency and period for a shaft of uniform diameter d are

$$\nu = \frac{1}{2\pi}\sqrt{\frac{\pi d^4G}{32Il}}, \qquad \tau = 2\pi\sqrt{\frac{32Il}{G\pi d^4}}. \tag{26}$$

Formula (26) may be applied to a rotating shaft provided one end is fastened to a very heavy mass rotating at uniform speed. Such a mass may be a very heavy flywheel or the rear-axle drive of an automobile, since the uniform forward motion of the car forces a uniform speed of rotation on the axle. In this case a mass some distance away from the rotating anchorage mass rotates with a velocity that swings periodically above and below an average speed equal to the uniform speed of the other end.

In the general case, however, the masses carried by a rotating shaft are not so large as to be unaffected by the vibration of one of them. Consider for instance the crankshaft of a single-cylinder combustion

engine driving the rotor of an electric generator. Every time there is a power impulse on the engine shaft, it tends to speed ahead of the rotor and the shaft is distorted "forward" from the rotor toward the engine end. The impulse, however, is transmitted to the rotor, which causes an increase of speed. As soon as the power impulse passes, the distortion in the shaft is relieved. Hence the crank arms and other rotating masses at the engine end swing back toward the mean position of rotation, and the negative acceleration acquired on this backward swing will make them swing below the uniform speed. The electric rotor will then tend to carry them along, and will be slowed up by this drag. Both ends therefore oscillate relatively to the mean rotating position, and they also oscillate relatively to one another, so that whenever the mass at one end tends to twist the shaft forward, that at the other end tends to twist it backward, and vice versa.

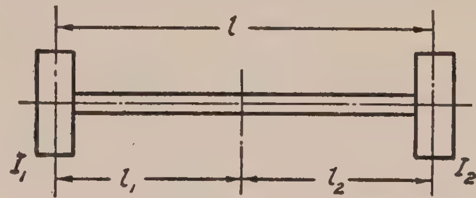


FIG. 416. ROTATING SHAFT WITH TWO MASSES.

In consequence of this reversal of twist from one end of the shaft to the other, there is one section of the shaft which twists neither forward nor backward during the rotation, but rotates with uniform speed. This section can be regarded as a point of anchorage for the masses swinging at the ends. With reference to Fig. 416, suppose it to be located the distance l_1 from the mass I_1 and the distance l_2 from the mass I_2. We have

$$l_1 + l_2 = l. \tag{27}$$

We also have for the frequencies, with the diameter d the same throughout,

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{\pi d^4 G}{32 I_1 l_1}}, \qquad \nu_2 = \frac{1}{2\pi}\sqrt{\frac{\pi d^4 G}{32 I_2 l_2}}. \tag{28}$$

Since the vibration of the shaft consists in the swing of the two masses relatively to one another, $\nu_1 = \nu_2$, and

$$I_1 l_1 = I_2 l_2, \tag{29}$$

and by virtue of (27),

$$l_1 = \frac{I_2 l}{I_1 + I_2}. \tag{30}$$

By introducing (30) into (28), we get

$$\nu = \frac{1}{2\pi}\sqrt{\frac{\pi d^4 G(I_1 + I_2)}{32 I_1 I_2 l}}. \tag{31}$$

EXAMPLE. A steam turbine and a generator rotor are joined by a 4 in. shaft 30 in. long. The steam-turbine rotor weighs 500 lb. and may be regarded as concentrated on an 18 in. radius. The generator rotor weighs 1000 lb. and may be regarded as concentrated on a 16 in. radius. What is the period of vibration of this shaft?

We have $I_1 = 500 \times 18^2/(32.2 \times 12) = 420$ lb. in. sec.^{2}, and $I_2 = 1000 \times 16^2/(32.2 \times 12) = 665$ lb. in. sec.^{2}. Hence

$$\nu = \frac{1}{2\pi}\sqrt{\frac{\pi 4^4 \times 11{,}500{,}000(420 + 665)}{32 \times 420 \times 665 \times 30}}$$
$$= 30.7 \text{ vibrations per sec.} = 1842 \text{ vibrations per min.}$$

This vibratory speed may be very close to the running speed of the turbine.

365. Torsional Vibrations with Multiple Masses. It will be observed that a shaft with two oscillating masses has only one torsional vibration frequency. With three masses there are two, with four masses three, and so on.

The ability to determine critical speeds for shafts with several masses is of great importance for any designer concerned with machinery having multi-throw crankshafts, such as combustion engines or multi-cylinder pumps, or with machinery having a multiplicity of rotating discs, such as steam turbines, centrifugal pumps, and compressors.

The young designer meeting this problem for the first time should acquire special books on vibrations.* Frequently, however, the matter is made unnecessarily difficult to the average student by an approach through a general mathematical analysis based on mechanical laws which are not always known to the practical engineer.

Actually the situation is quite simple.† Any rotating mass, unless it is at the end of the shaft, is acted on by a torque from the right and a torque from the left, and these torques, since they come through the shaft, are equal to the angular distortion in the part of the shaft through which they occur times the spring constant in that part. If at a certain mass I_2 the angular deflection is ϕ_2 and the deflections at the adjoining discs are ϕ_1 and ϕ_3 respectively, we have with the spring constants K_1 and K_2 (since the torque on one side tends to

* Such books are, for instance, Timoshenko, *Vibration Problems in Engineering*, Van Nostrand; Den Hartog, *Mechanical Vibrations*, McGraw-Hill.

† Den Hartog in his book on vibrations makes use of the simple method of approach here set forth.

increase the deflection and that on the other tends to reduce it),

$$(32) \qquad I_2\frac{d^2\phi}{dt^2} + K_2(\phi_2 - \phi_3) - K_1(\phi_1 - \phi_2) = 0.$$

Also, if there is no outside force, at the period of free vibration, the total increase in momentum must be zero. That is to say,

$$(33) \qquad I_1\frac{d^2\phi_1}{dt^2} + I_2\frac{d^2\phi_2}{dt^2} + I_3\frac{d^2\phi_3}{dt^2} + \cdots = 0.$$

By putting the deflection angles ϕ_1, ϕ_2, $\cdots$, equal to $\lambda_1 \cos(pt + \alpha)$, $\lambda_2 \cos(pt + \alpha)$, $\cdots$, and observing that $d^2\phi/dt^2$ is then equal to $-p^2\phi$, where p is the frequency factor $2\pi\nu$, we arrive at equations

$$(34) \qquad I_1\lambda_1p^2 - K_1(\lambda_1 - \lambda_2) = 0,$$

$$(35) \qquad I_2\lambda_2p^2 + K_1(\lambda_1 - \lambda_2) - K_2(\lambda_2 - \lambda_3) = 0,$$

and also

$$(36) \qquad I_1\lambda_1 + I_2\lambda_2 + I_3\lambda_3 + \cdots = 0.$$

Since the free period of vibration does not depend on the amplitude λ, we can assume any value for λ_1, and by a process of trial and error arrive at a value for p, which satisfies equations (34) and (35). A very simple tabular method for this purpose has been suggested by Holzer and may be found in the books of Timoshenko and Den Hartog already mentioned.

366. Energy Method of Determining Vibrations. Rayleigh Approximation. Engineers are very often faced with the necessity of making fairly reasonable estimates, where exact calculations are difficult or impossible. This is particularly true with vibrations. In such cases the energy method of determination, particularly with the approximation of Rayleigh, is very helpful.

The energy method of calculating a free vibration is based on the realization that in a free vibration, with no energy entering from the outside, the sum of the potential and the kinetic energies must always be constant. When the swing is at its full amplitude, the whole energy is potential. When it passes the equilibrium position it is all kinetic. The potential energy of a mass suspended on a weightless spring is $Kx^2/2$, where x is the deflection and K the spring constant. The kinetic energy is $\frac{1}{2}M(dx/dt)^2$, where M is the swinging mass and dx/dt is equal to the velocity. When x reaches the full amplitude c, the whole energy is $Kc^2/2$. We have then

$$\frac{M}{2}\left(\frac{dx}{dt}\right)^2 + \frac{Kx^2}{2} = \frac{Kc^2}{2}.$$

At the moment of maximum velocity,

$$\frac{M}{2}\left(\frac{dx}{dt}\right)^2_{\max} = \frac{Kc^2}{2}. \tag{37}$$

If, however, we put $x = c \cos pt$, then $dx/dt = - cp \sin pt$, and $(dx/dt)_{\max} = - cp$. Substituting this value into (37), we get

$$p^2 = \frac{K}{M}, \tag{38}$$

which is the value derived for p from Newton's second law (see page 457). To derive the expression for the frequency factor, p, all we need to do is to equate the maximum kinetic energy to the maximum potential energy, introducing for the velocity an expression containing p.

EXAMPLE. As an illustration of the use of this method let us consider a mass suspended at one end of an arm pivoted in the middle, the other end being anchored, as shown in Fig. 417, to a spring which may be considered weightless.

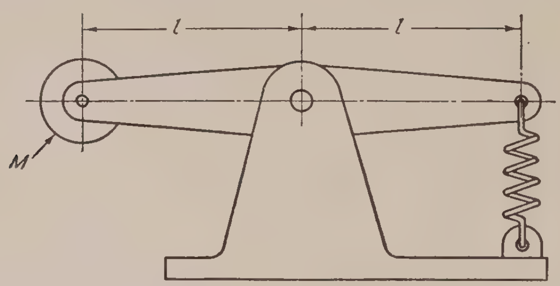


FIG. 417. PIVOTED ARM ANCHORED AT ONE END WITH MASS AT OTHER END.

In this case the kinetic energy is made up of two parts, the energy of the mass M, which is $\frac{1}{2}M(dx/dt)^2$, and the energy of the arm, which is $I(dx/dt)^2/2l^2$, l being half the length of the arm, as shown in Fig. 417, and $(dx/dt)/l$ being the angular velocity of the arm. The total kinetic energy is

$$\frac{\left(\frac{dx}{dt}\right)^2\left(M + \frac{I}{l^2}\right)}{2}.$$

Since the arm is pivoted in the middle and the amplitude is c at each end, we have

$$\frac{c^2p^2(M + I/l^2)}{2} = \frac{Kc^2}{2},$$

or

$$p^2 = \frac{K}{M + I/l^2}.$$

In this case we have tacitly assumed that the type of vibration is not changed by the addition of the arm between the mass and the spring, and this is correct if the spring is assumed weightless.

Suppose, however, that we wish to allow for the weight of the spring. In this case it is not at all certain that the various spring elements will perform a simple harmonic vibration proportional to their distance from the anchorage. If we assume that they do, we have thereby introduced an uncertain premise.

The *Rayleigh approximation* consists in making simplifying assumptions of this kind. Its value lies in the fact that these approximations give astonishingly accurate results in a great number of cases in which the assumptions certainly are very far from the truth.

Suppose, for instance,* that we have derived the formula for the vibratory speed of a weightless cantilever beam with a concentrated mass M at the end. The formula is $\omega_c = (3EI/Ml^3)^{1/2}$. Suppose we wish to allow for the weight of the beam, which may have a mass m per unit length. We now make the assumption that the deflection curve and the mode of vibration for the weighted beam are the same as those of the weightless beam. This is obviously an assumption of questionable accuracy, particularly if the beam is heavy compared with the concentrated weight. The deflection x at the distance a from the fixed end of a weightless beam of length l with a load F applied at the end is equal to

$$x = \frac{Fa^2}{EI}\left(\frac{l}{2} - \frac{a}{6}\right);$$

the deflection at the end is

$$x_{\max} = \frac{F}{EI}\frac{l^3}{3}.$$

We have

$$\frac{x}{x_{\max}} = \frac{3a^2l - a^3}{2l^3}.$$

Hence the kinetic energy of the beam alone is

$$\int_0^l \frac{m}{2}\left(\frac{dx}{dt}\right)^2\left(\frac{3a^2l - a^3}{2l^3}\right)^2 da = \frac{33}{140}\,ml\,\frac{\left(\frac{dx}{dt}\right)^2}{2}.$$

* Following Timoshenko, *Vibration Problems in Engineering*, p. 57.

Since the kinetic energy of the mass M alone is $\frac{1}{2}M(dx/dt)^2$, it appears that all we have to do to arrive at the frequency factor p, which is equal to the critical speed ω_c, is to substitute for M the quantity $M + 33ml/140$.

Suppose we apply this in the extreme case when $M = 0$. We have then

$$p = \omega_c = \sqrt{\frac{140K}{33ml}}.$$

The spring constant for a shaft loaded at the end, which is the type of bending we have assumed, is $3EI/l^3$. We have therefore

$$p = \omega_c = \sqrt{\frac{140 \times 3 \times EI}{33ml^4}} = 3.567\sqrt{\frac{EI}{ml^4}}.$$

This value differs by only about 2 per cent from the exact solution given by formula (6) on page 455. Yet in this case the deviation of the approximate assumption from the actual loading condition is about as extreme as could be imagined.

In view of this insensibility of the Rayleigh method to comparatively large errors in simplifying assumptions regarding types of deflection and modes of vibration, this method is of the very highest value to the practical engineer. It is known that it has been successfully applied to such complicated cases as that of turbine-disc vibrations.

PROBLEMS

1. Calculate the critical speed of the shaft of Problem 12, Chapter XI.
2. Calculate the critical speed of the shaft of Problem 13, Chapter XI.
3. Calculate the critical speed of the shaft of Problem 14, Chapter XI.
4. A flexible shaft of 1¼ in. diameter is connected at one bearing, A, to a gear which stiffens it so that it may be regarded as clamped at this end. At the other end, B, it is freely supported in a self-aligning bearing. The span between centers of bearings is 20 in. and may be regarded as the effective beam length. The shaft carries a disc weighing 100 lb. in the middle of the span. What is the critical speed, if the weight of the shaft is neglected? The deflection of a beam with one end freely supported and one end clamped is represented by

$$y = \frac{Fa^2b^3(4a + 3b)}{12EIl^3},$$

where F is the load, a the distance from the freely supported end to the load, b the distance from the clamped end to the load, l the length, and y the deflection under the load.

5. In the preceding problem, the single disc is replaced by two discs, each weighing 50 lb., and each located 7 in. from the nearest bearing center. (a) Compute the critical speed caused by each disc, and then, by means of Dunkerley's formula, the critical speed of the whole system. (b) Compute the deflection at each disc by

adding the deflections at these points caused by each disc separately, and compute the critical speed from these deflections.

The deflection formula for a beam freely supported at one end, clamped at the other, and carrying a load F at the distance a from the freely supported end is

$$y(EI) = -\frac{Rx}{6}\left(l^2 - x^2\right) + \frac{Fx}{6l}\left(l - a\right)^3$$

from $x = 0$ to $x = a$, and

$$y(EI) = -\frac{Rx}{6}\left(l^2 - x^2\right) + \frac{Fx}{6l}\left(l - a\right)^3 - \frac{F}{6}\left(x - a\right)^3$$

from $x = a$ to $x = l$. The reaction at the free end has the value

$$R = \frac{Fb^2(3a + 2b)}{2l^3}.$$

6. Compute the bending stress at the disc and at the gear for the shaft in Problem 4. To what value would this stress be increased if the machine were run at 90 per cent of the critical speed?

7. For the shaft in Problem 4 assume the gear to weigh 80 lb. and to have its mass concentrated on a 3 in. radius, and the 100 lb. disc to have its mass concentrated on an 8 in. radius. What would be the frequency of the torsional vibration if the distance between the gear and disc is 12 in.?

8. The sprung part of an automobile weighs 3000 lb. and the spring deflects 4 in. under this load. What would be the natural frequency of the car if it merely oscillated up and down, remaining parallel to its original position?

9. Suppose a rocking motion of the car in the preceding problem could be taken as a swing of the rear end about the front axle as a pivot. The wheel base is 110 in., and for a very rough estimate of the order of magnitude of the frequency of rocking motion, the weight of the car may be uniformly distributed over the wheel base. Derive the moment of inertia by integrating the expression

$$\int_0^{110} mr^2dr,$$

and compute the torsional spring constant by observing that a force of 1500 lb. on a radius of 110 in. rotates the rear end of the car through an arc of 4 in. Then compute the frequency of angular vibration.

10. Suppose the formula for the deflection of a beam freely supported at one end, clamped at the other, and loaded w lb. per in. of length is

$$y = \frac{wl^4}{48EI}\left(\frac{x}{l} - 3\frac{x^3}{l^3} + 2\frac{x^4}{l^4}\right)$$

where x is measured from the freely supported end.

Suppose the weight of the shaft in Problem 4 is 0.25 lb. per in. of length. What would be the natural frequency of the shaft if the influence of the weight of the shaft itself is allowed for by Rayleigh's approximation?

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